Nonlinear dynamos driven by shear flows

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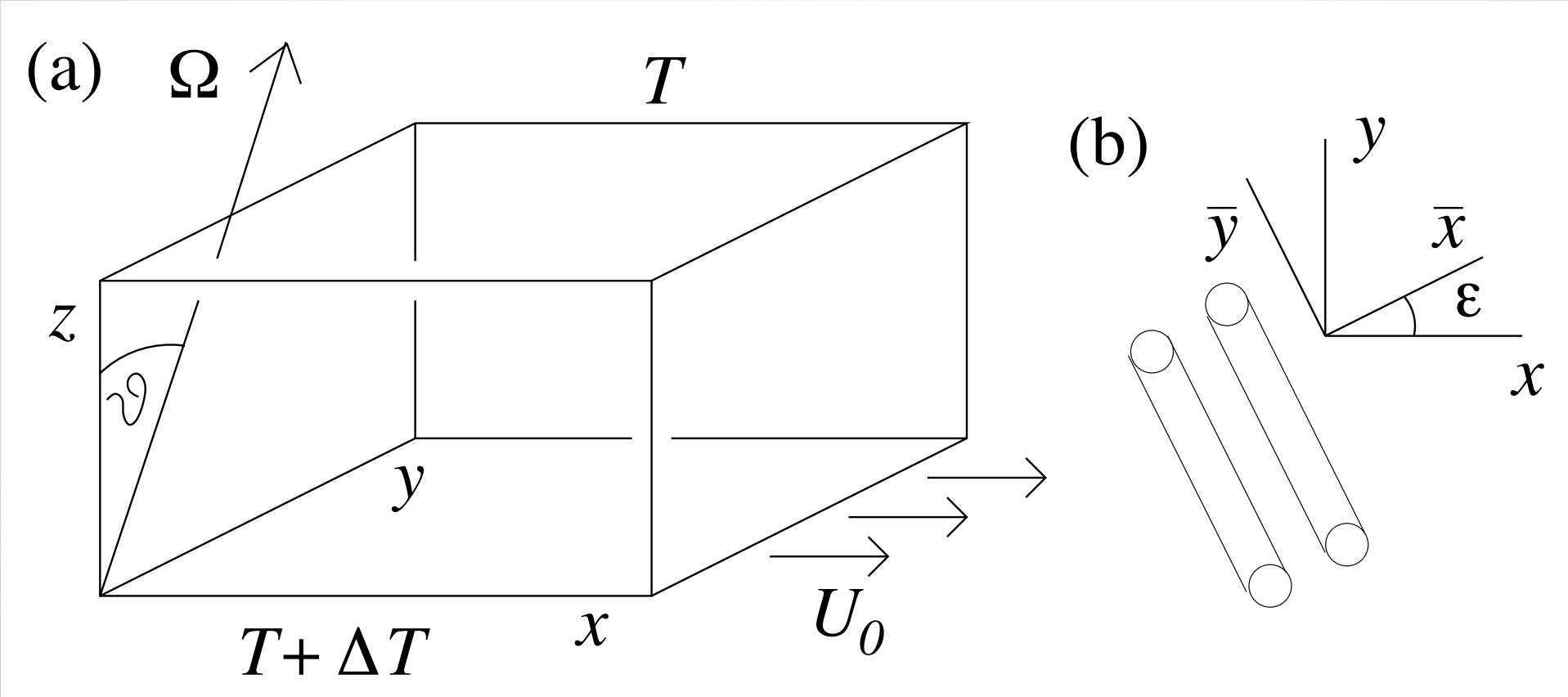
Andrew Soward, keke Zhang,
Yannick Ponty.

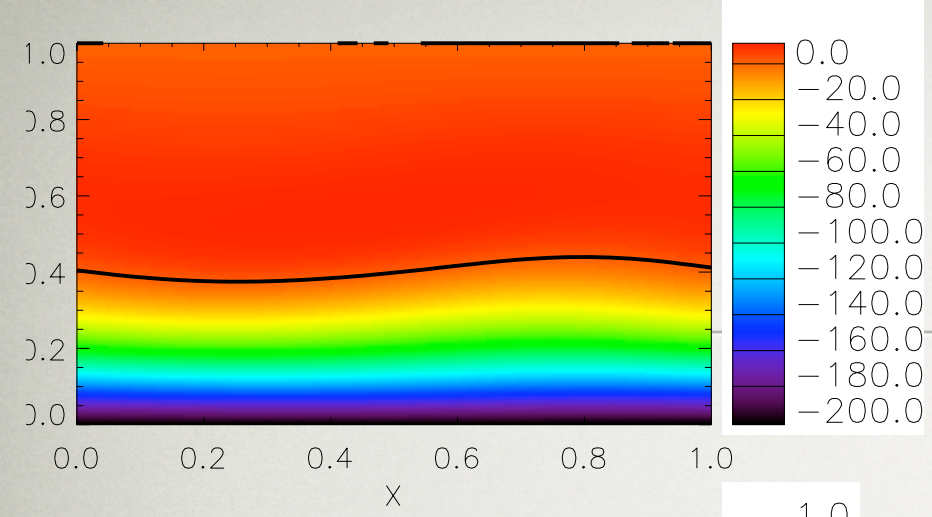
with thanks to the LEVERHULME TRUST.

St Andrew's
UK-MHD-06

Geometry: flows driven by shear and/or convection

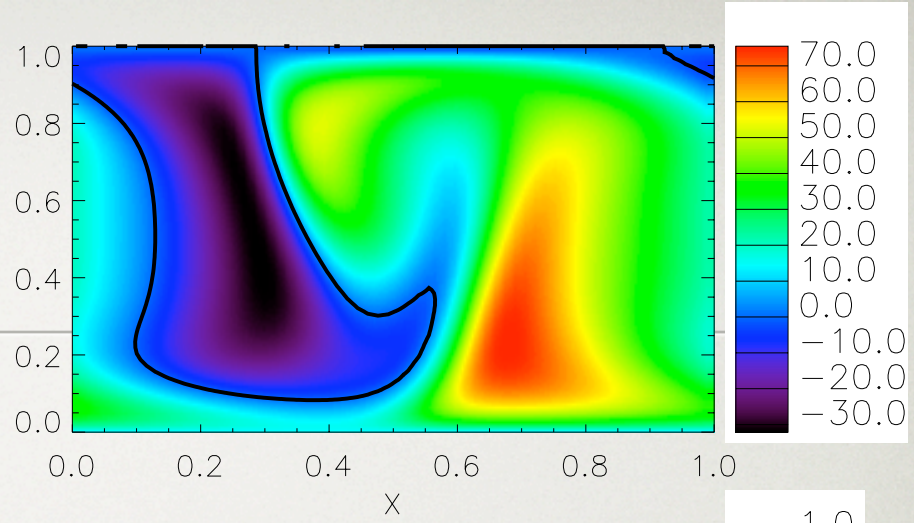


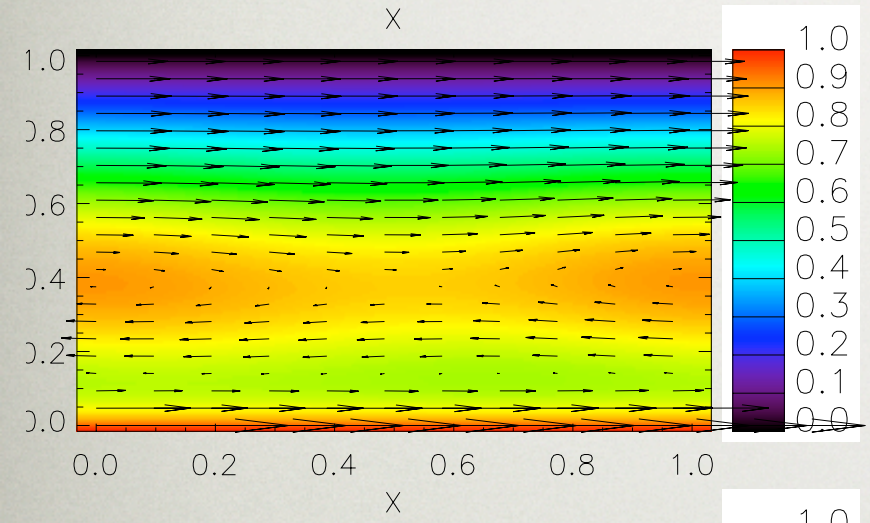
Flow driven by shear or convection in a rotating frame. Ekman layer can become unstable giving cat's eye rolls (Ponty, Gilbert, Soward 2001).



V

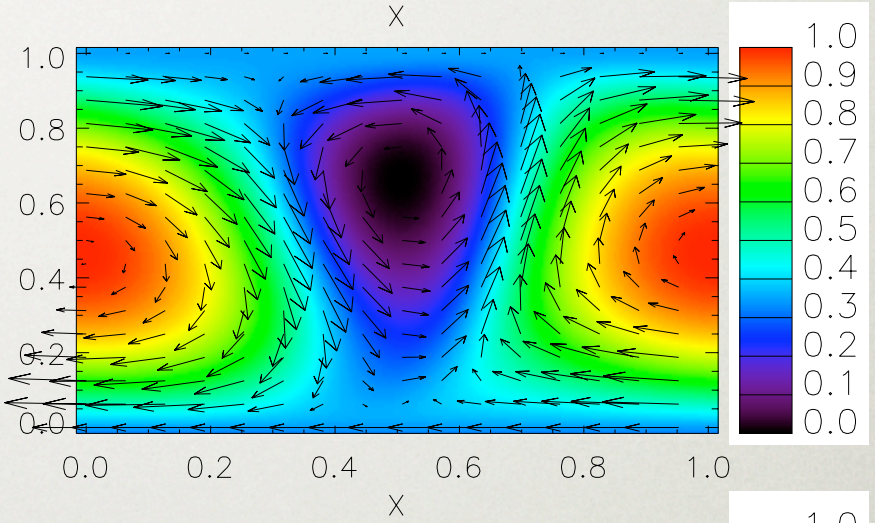
(a)

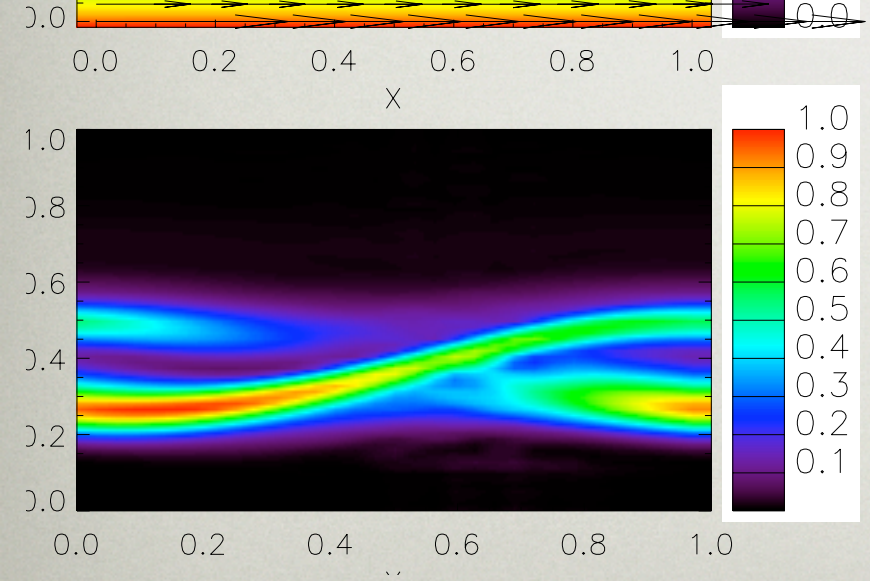




ψ

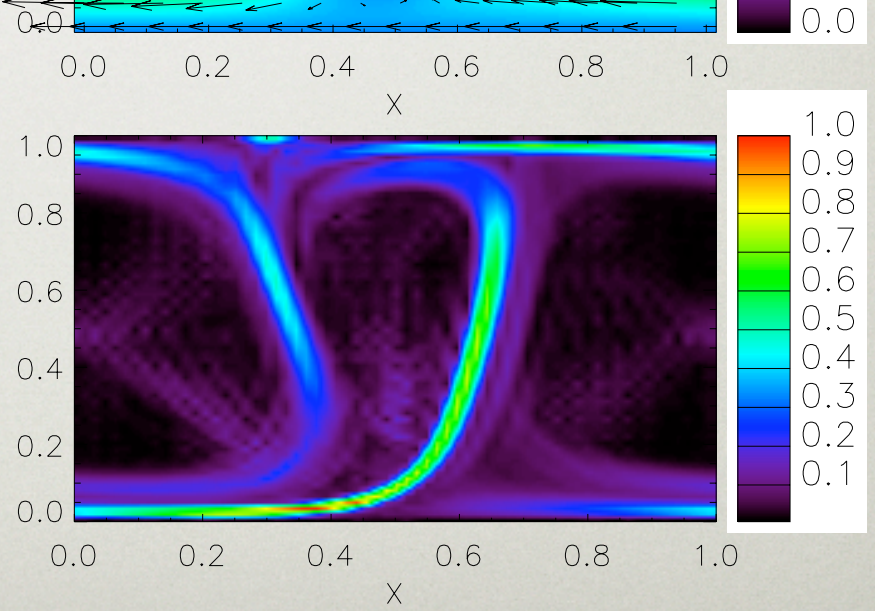
(b)





|B|

(c)



Ekman instability

Convective instability

Equations and parameters: $\{\mathrm{Re}, \tau, \mathrm{Pm}, \vartheta, \varepsilon, k_{\bar{x}}, k_{\bar{y}}\}$. .

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U} + \tau\widehat{\mathbf{\Omega}}\times\mathbf{U} = -\nabla\Pi + (\nabla\times\mathbf{B})\times\mathbf{B} + \nabla^2\mathbf{U},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{U}\times\mathbf{B}) + \mathrm{Pm}^{-1}\nabla^2\mathbf{B},$$

$$\nabla\cdot\mathbf{U} = 0, \quad \nabla\cdot\mathbf{B} = 0$$

$$\vartheta, \quad \mathrm{Re} = \frac{U_0 h}{\nu}, \quad \tau = \frac{2\Omega h^2}{\nu}, \quad \mathrm{Pm} = \frac{\nu}{\eta},$$

The Magnetic Reynolds number becomes a diagnostic defined by:

$$\mathrm{Rm} = \mathrm{Pm}U, \qquad U \equiv \sqrt{2E_K}, \quad E_K = \tfrac{1}{2}\langle \mathbf{U}^2 \rangle$$

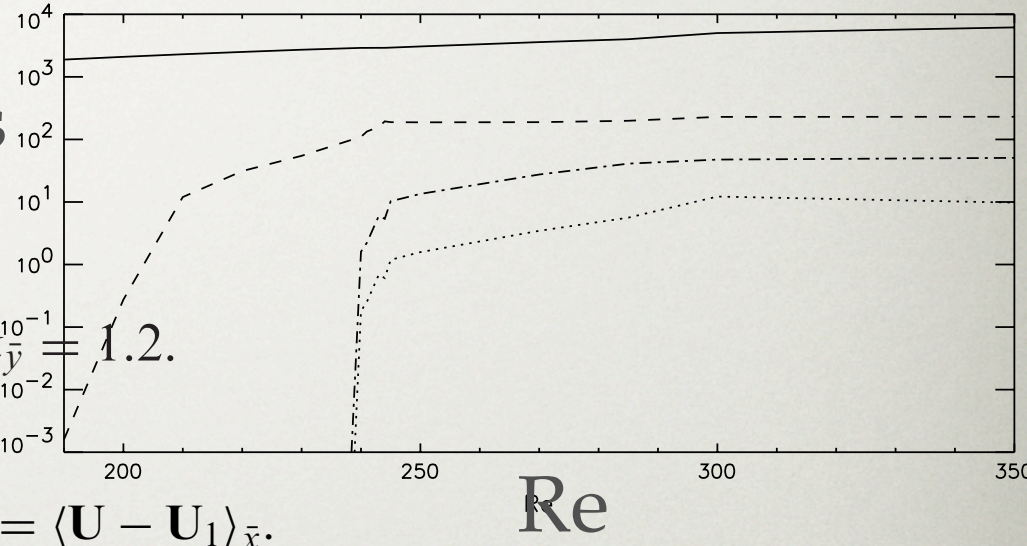
Kinetic Energy & dynamo growth rates

Energy in different components of Ekman flow for:

$$\tau = 60, \quad \vartheta = 45^\circ, \quad \varepsilon = 73.5^\circ, \quad k_{\bar{x}} = 3.44, \quad k_{\bar{y}} = 1.2.$$

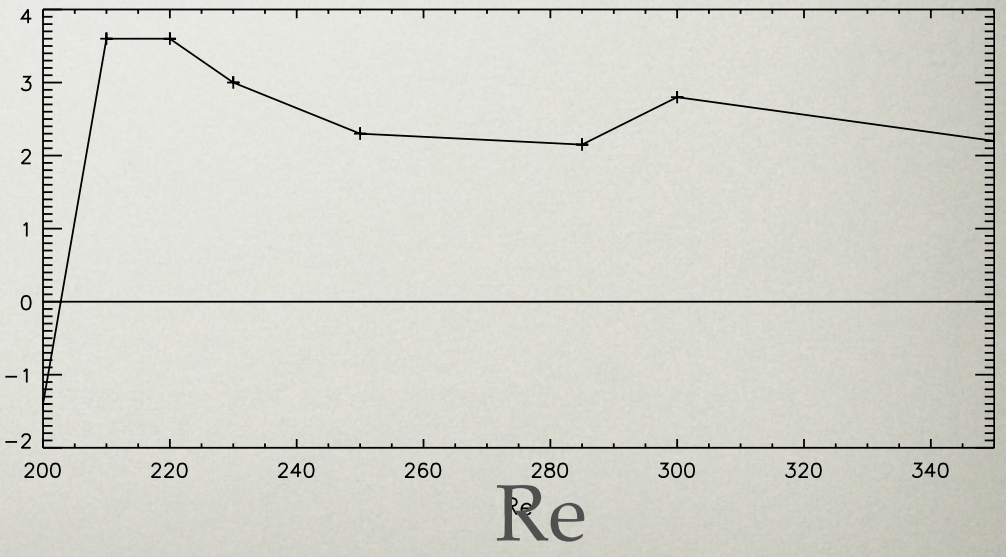
$$E_K = E_{K1} + E_{K2\bar{x}} + E_{K2\bar{y}} + E_{K3}$$

$$\mathbf{U}_1(z) = \langle \mathbf{U} \rangle_{\bar{x},\bar{y}}, \quad \mathbf{U}_{2\bar{x}}(\bar{x}, z) = \langle \mathbf{U} - \mathbf{U}_1 \rangle_{\bar{y}}, \quad \mathbf{U}_{2\bar{y}}(\bar{y}, z) = \langle \mathbf{U} - \mathbf{U}_1 \rangle_{\bar{x}}.$$

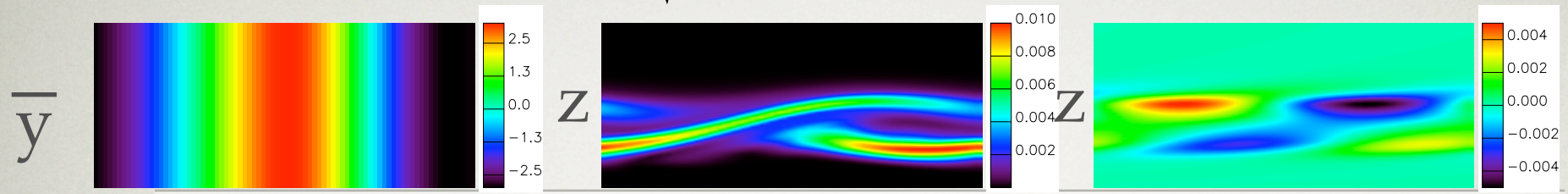


Kinematic dynamo growth rates at:

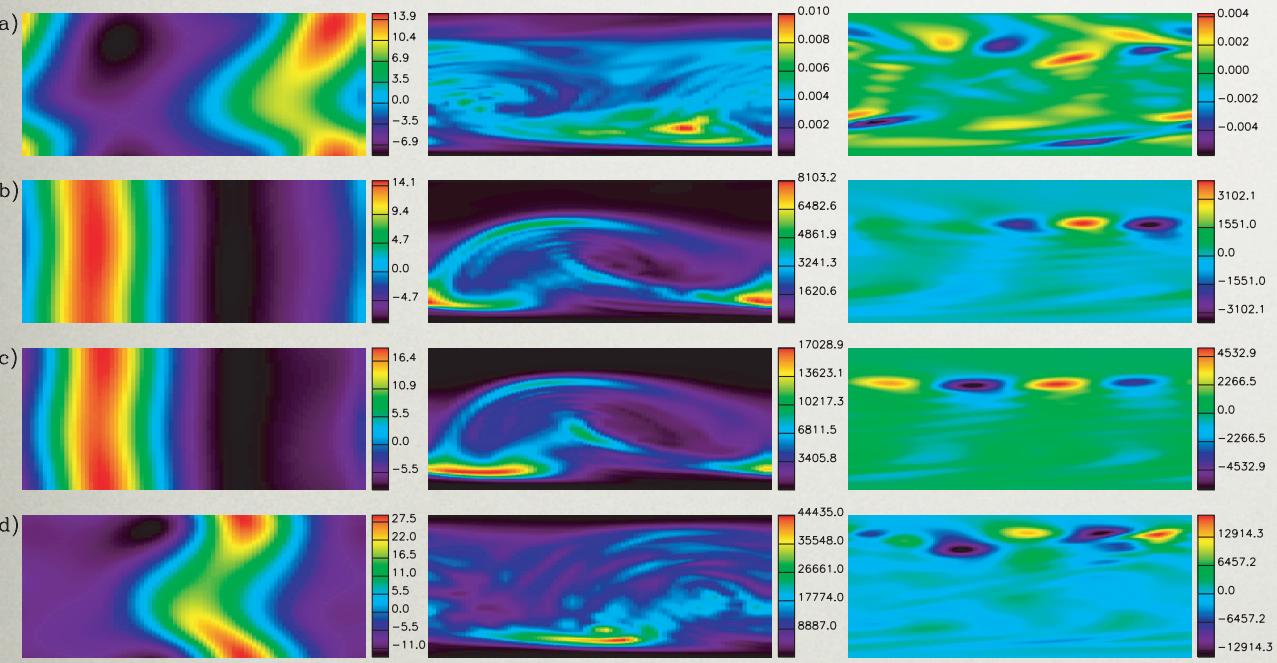
$\mathrm{Pm} = 50.$



$U_z\left(\bar{x}, \bar{y}, \frac{1}{2}\right)$ (left), $\overline{B}(\bar{x}, z) \equiv \sqrt{\langle |\mathbf{B}|^2 \rangle_{\bar{y}}}$ (middle), $\overline{B}_{\bar{x}}(\bar{y}, z) \equiv \langle B_{\bar{x}} \rangle_{\bar{x}}$ (right).

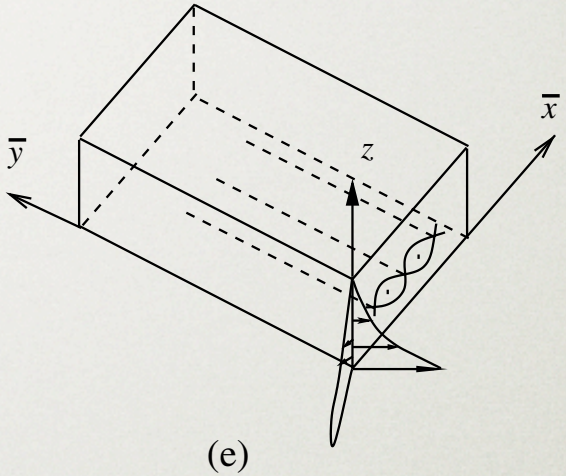


Re=210



gure 6. Snapshots of flow and magnetic field diagnostics (21) as in figure 3, in the kinematic regime at times (a) $t = 21.5$, (b) $t = 22$, (c) $t = 22.2$, and (d) $t = 22.4$.

Re=250



(e)

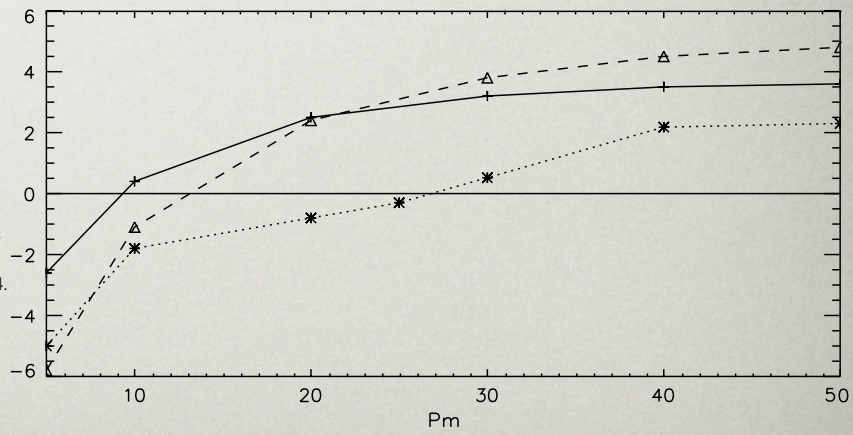
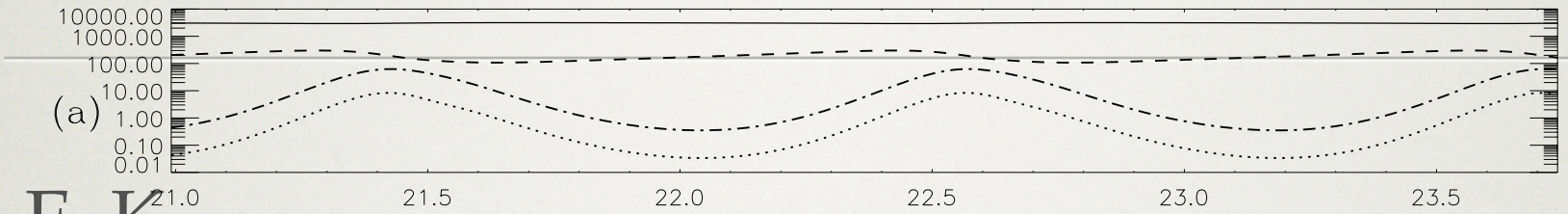
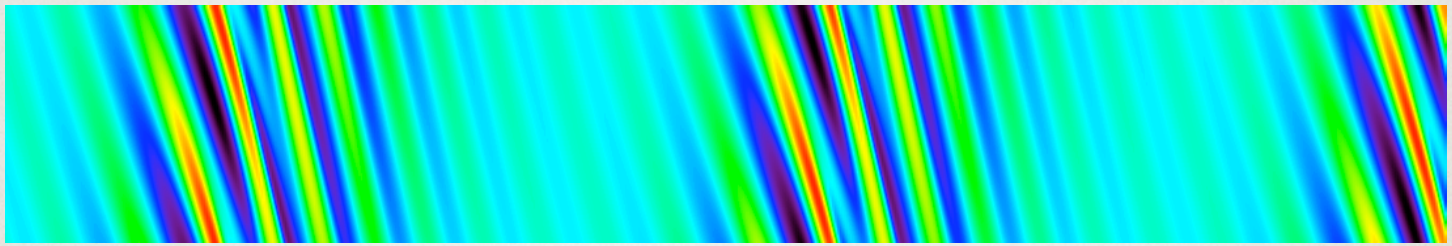


Figure 4. Magnetic growth rate σ as a function of Pm for the Ekman layer flow with para and $\mathrm{Re} = 210$ (solid), $\mathrm{Re} = 250$ (dotted). The dashed curve gives growth rates for the Taylor–C with parameters (24).

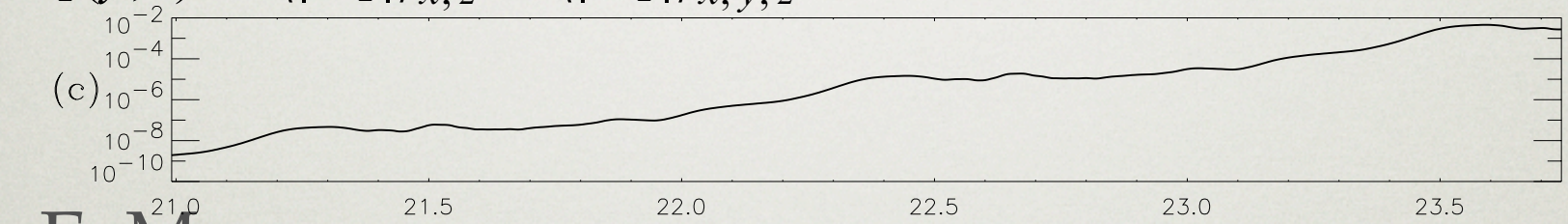
Kinematic behaviour



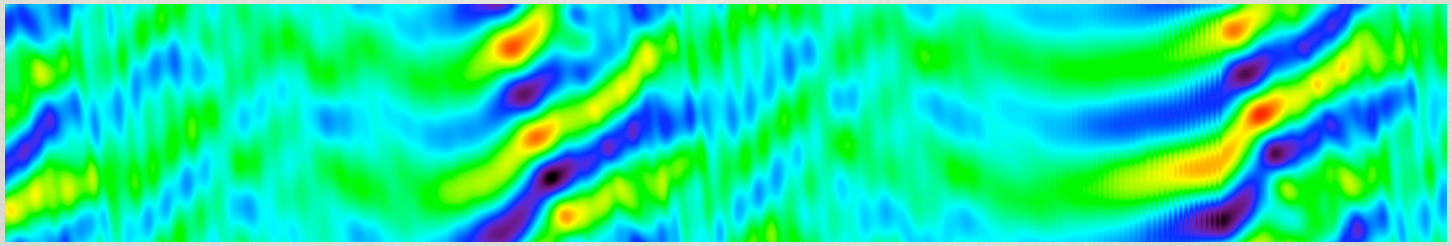
E_K



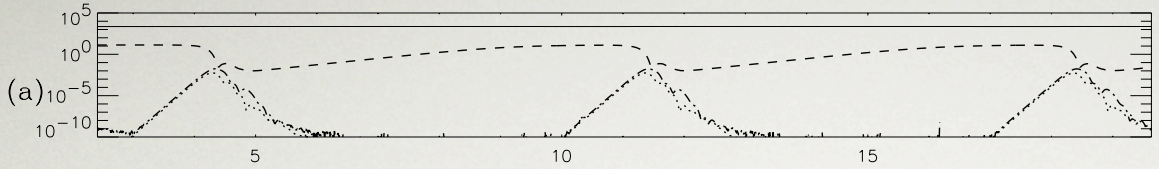
$$\hat{U}_z(\bar{y},t) = \langle |U_z| \rangle_{\bar{x},z} - \langle |U_z| \rangle_{\bar{x},\bar{y},z}.$$



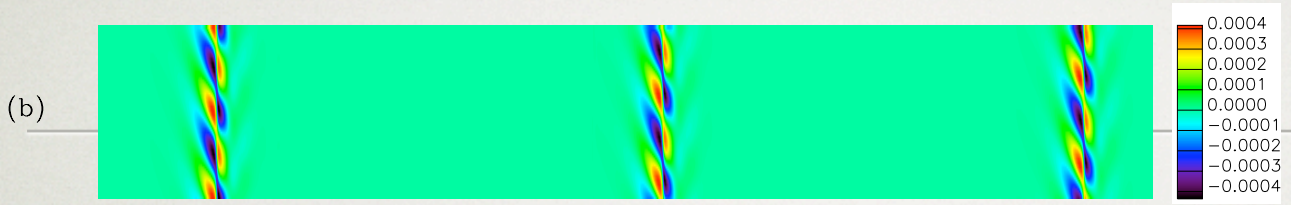
E_M



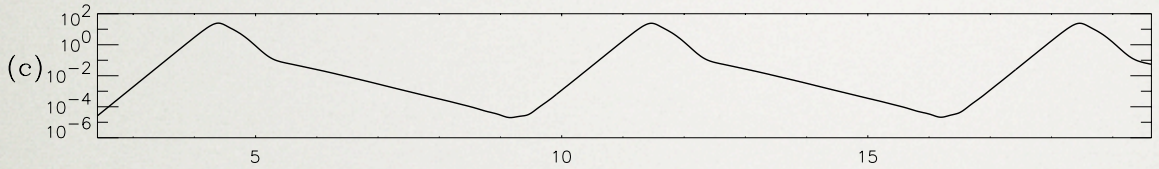
$$\hat{B}_{\bar{x}}(\bar{y},t) = \mathrm{e}^{-\sigma t} \langle B_{\bar{x}} \rangle_{\bar{x},z}$$



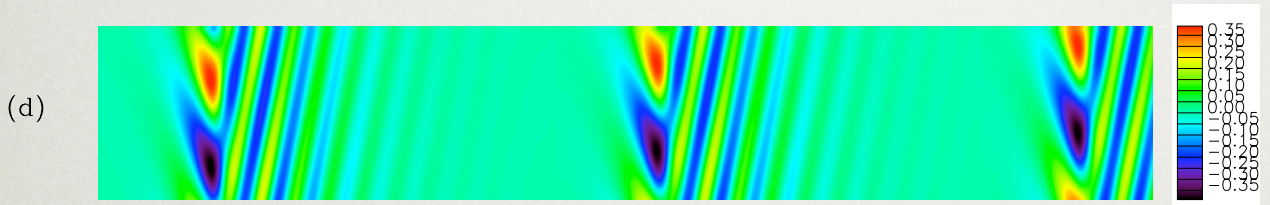
E_K



Rolls



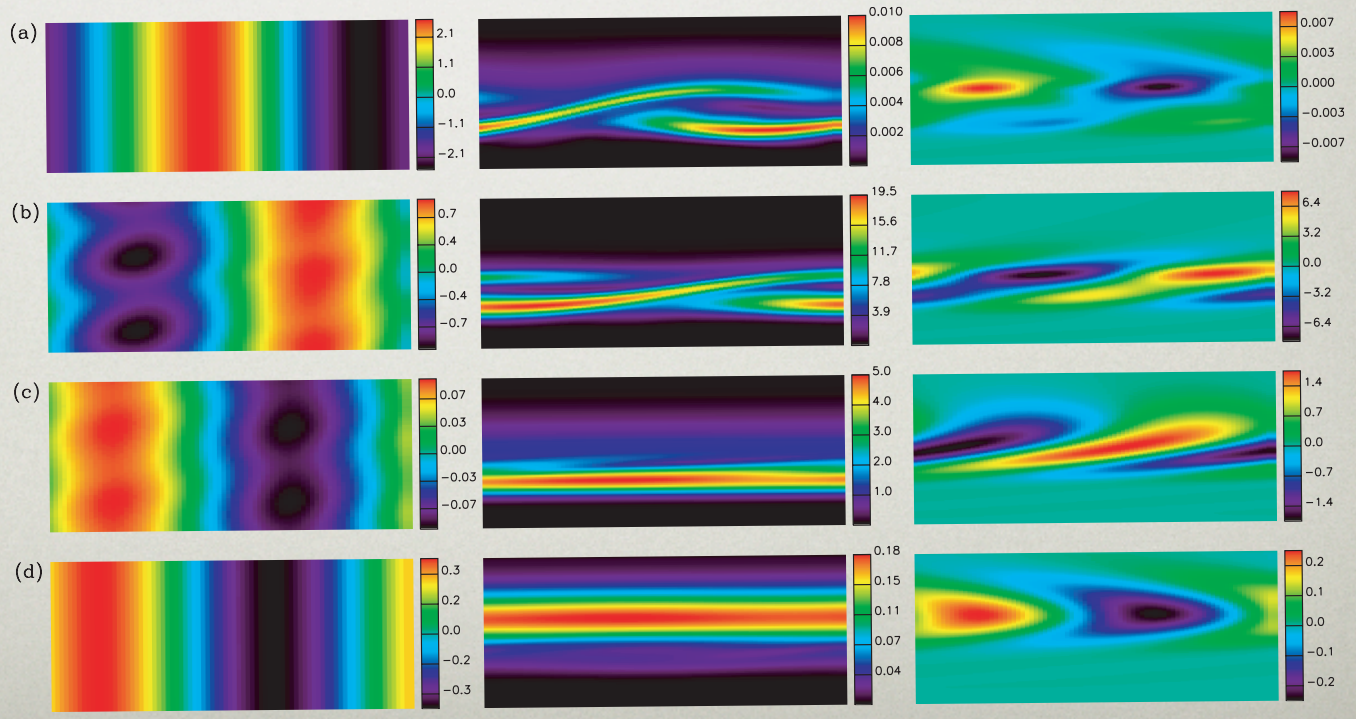
E_M

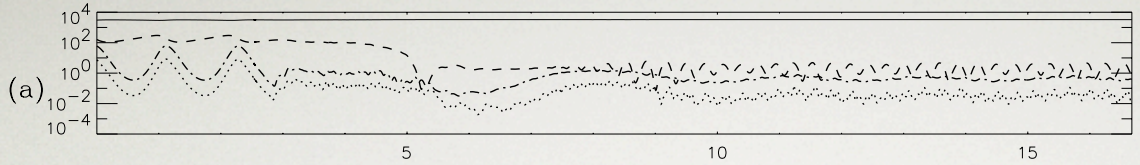


B-field

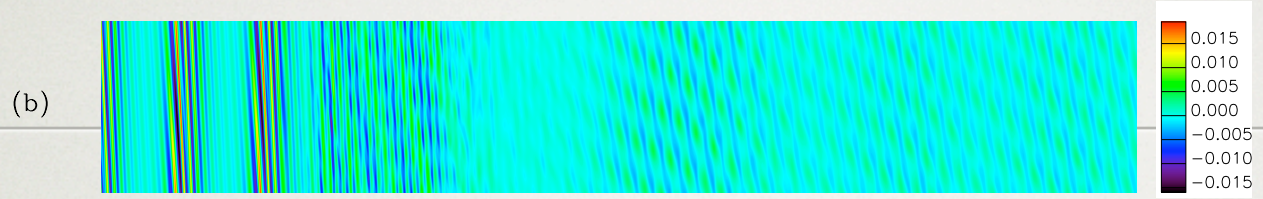
Dynamical behaviour at Re=210, Pm=50

Pmc=9

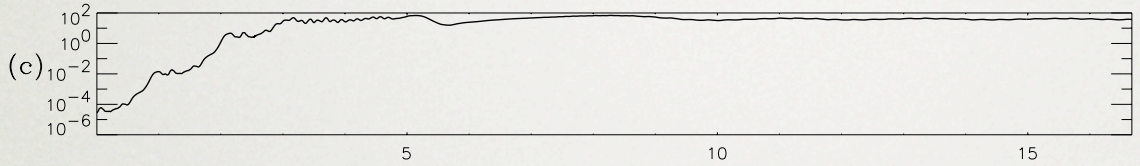




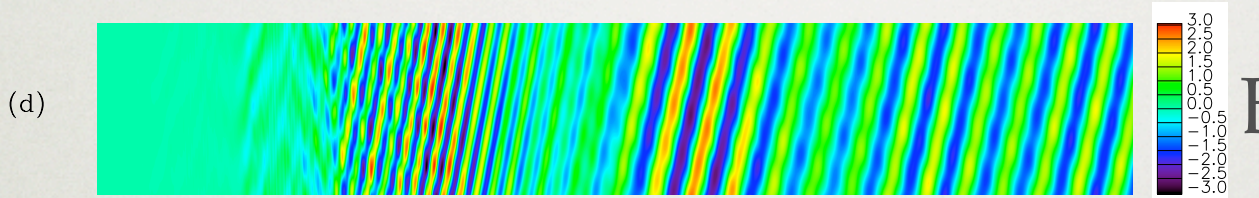
E_K



Rolls



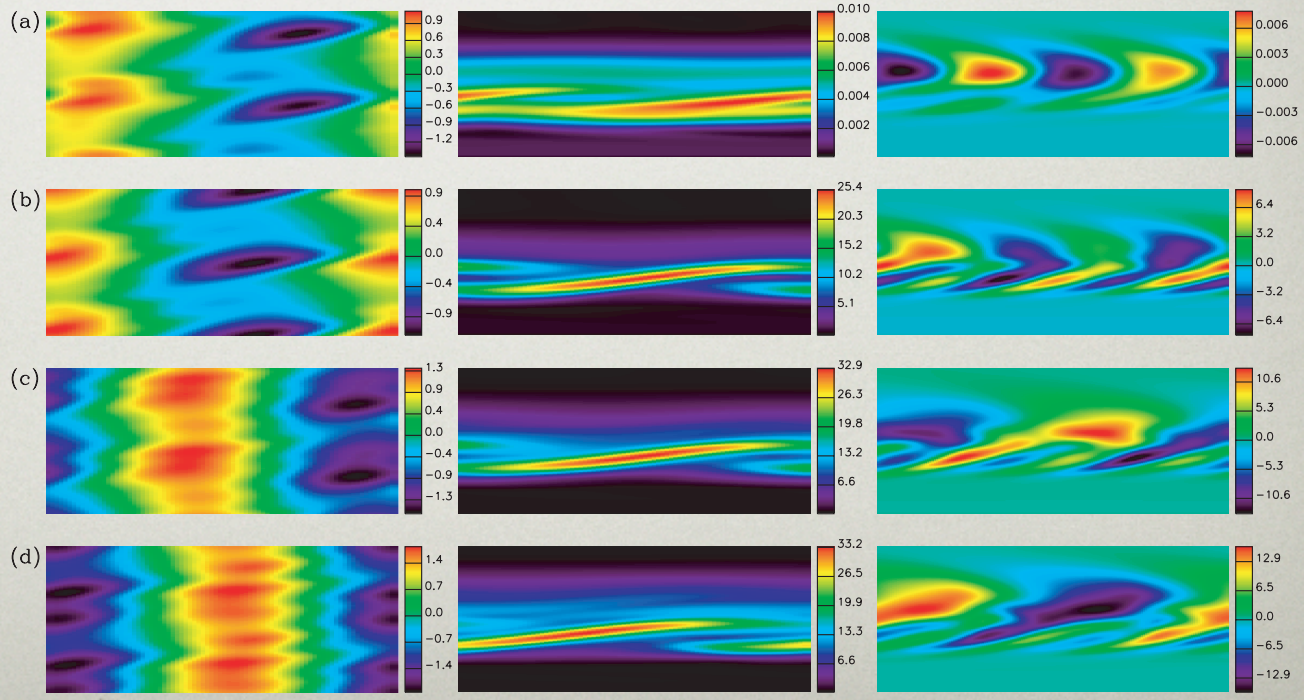
E_M



B-field

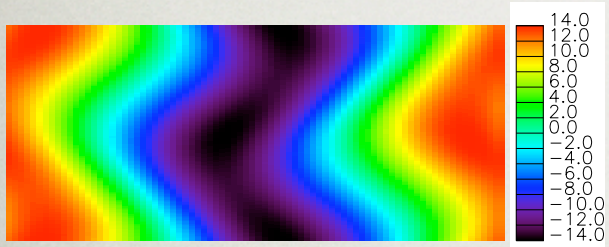
Dynamical behaviour at Re=250, Pm=50

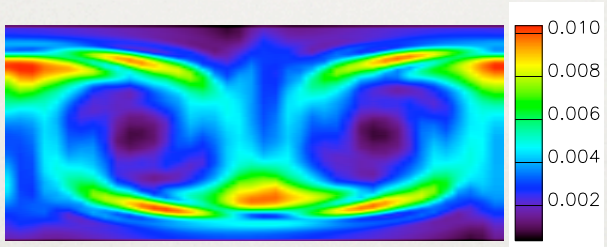
Pmc=28

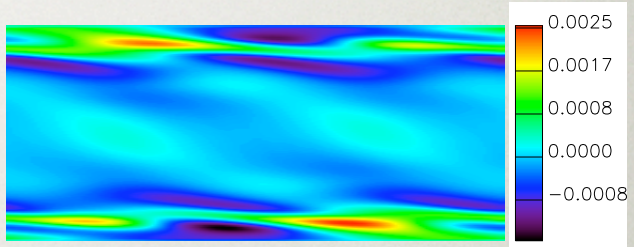


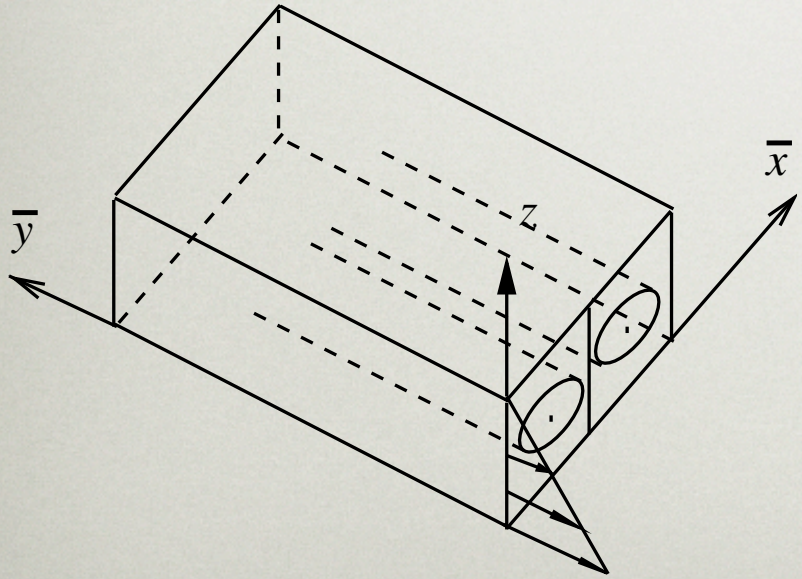
Wavy Taylor Couette: kinematic dynamo

$$\tau = 22, \quad \mathrm{Re} = 200, \quad \vartheta = 90^\circ, \quad \varepsilon = 90^\circ, \quad k_{\bar{x}} = 3.117, \quad k_{\bar{y}} = 1.$$



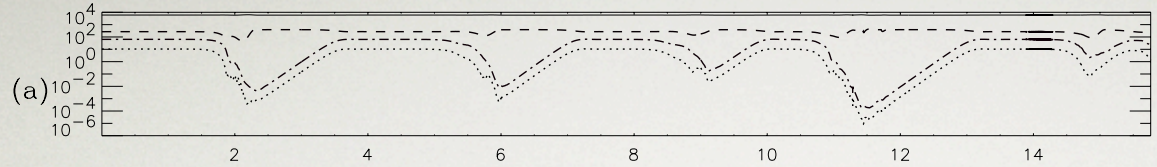




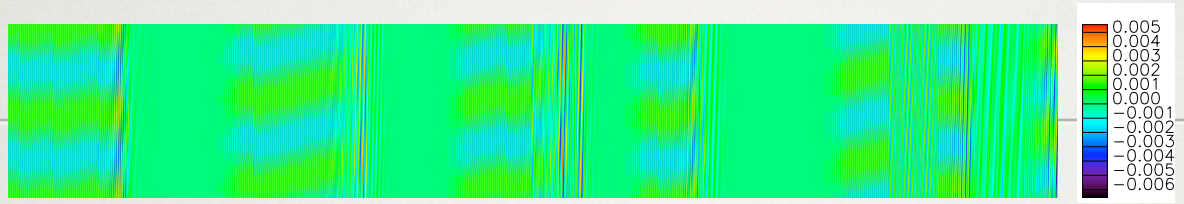


Bifurcation to steady wavy rolls at Re=190

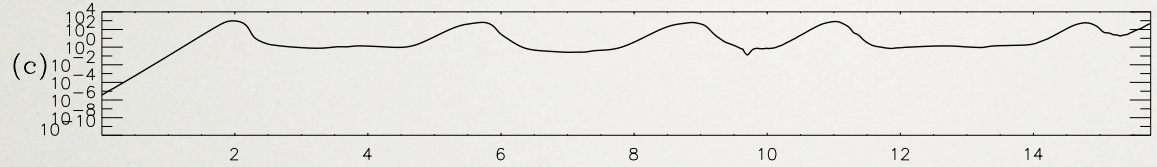
Pmc=13



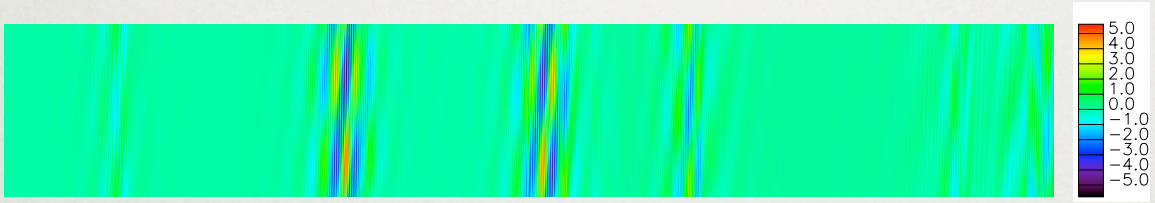
E_K



Rolls



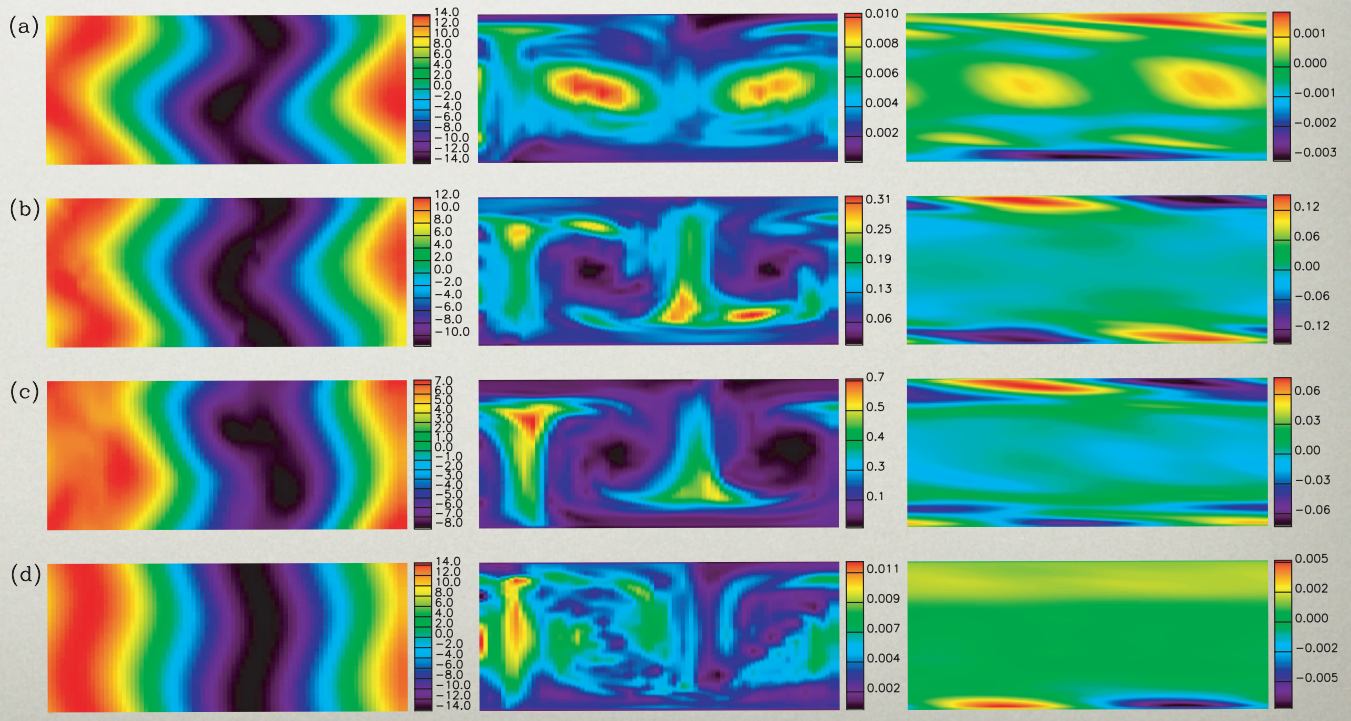
E_M



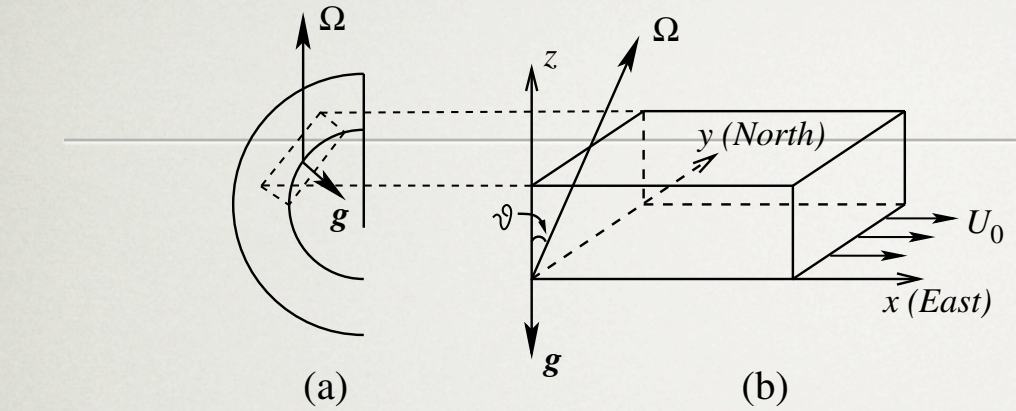
B-field

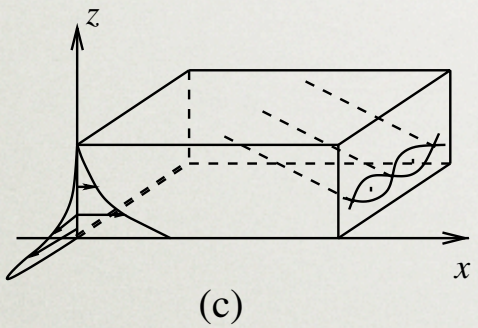
Dynamical behaviour at Re=200, Pm=50

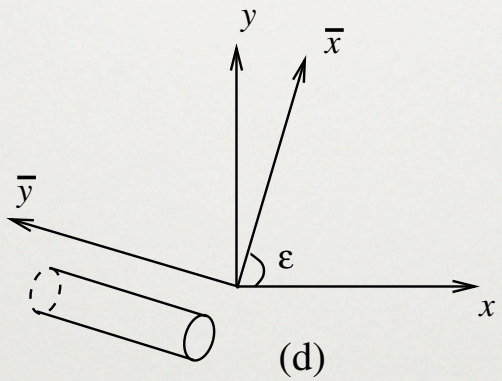
Pmc=13

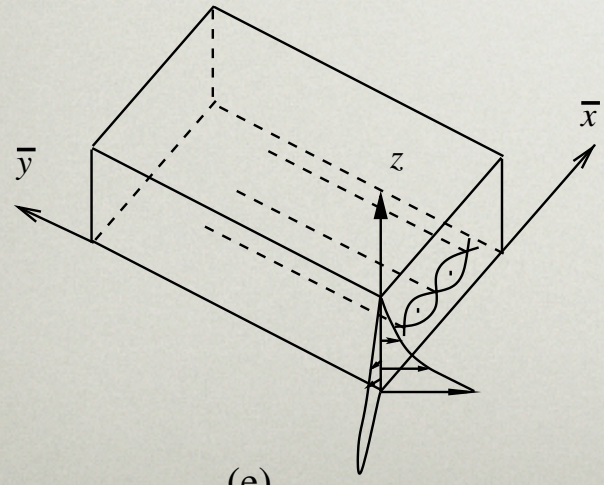


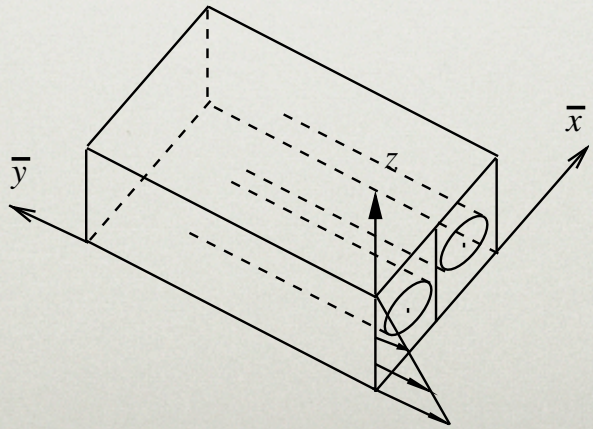
Geometry





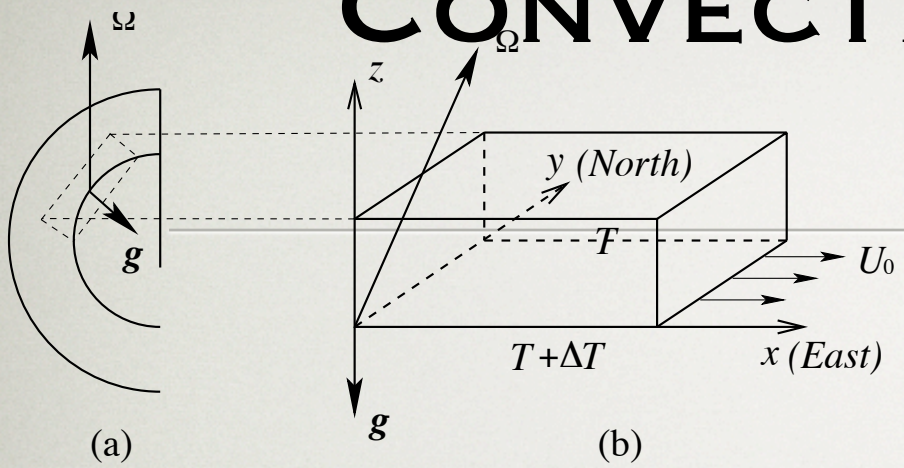






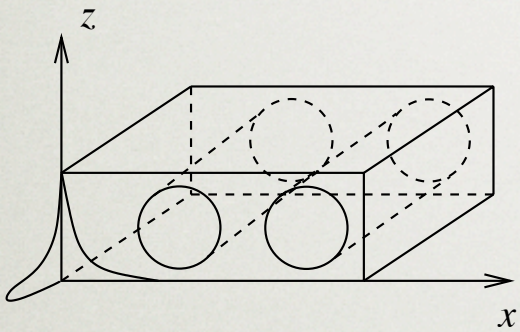
Note cat's eyes or TC rolls have axes approximately aligned with the shear.

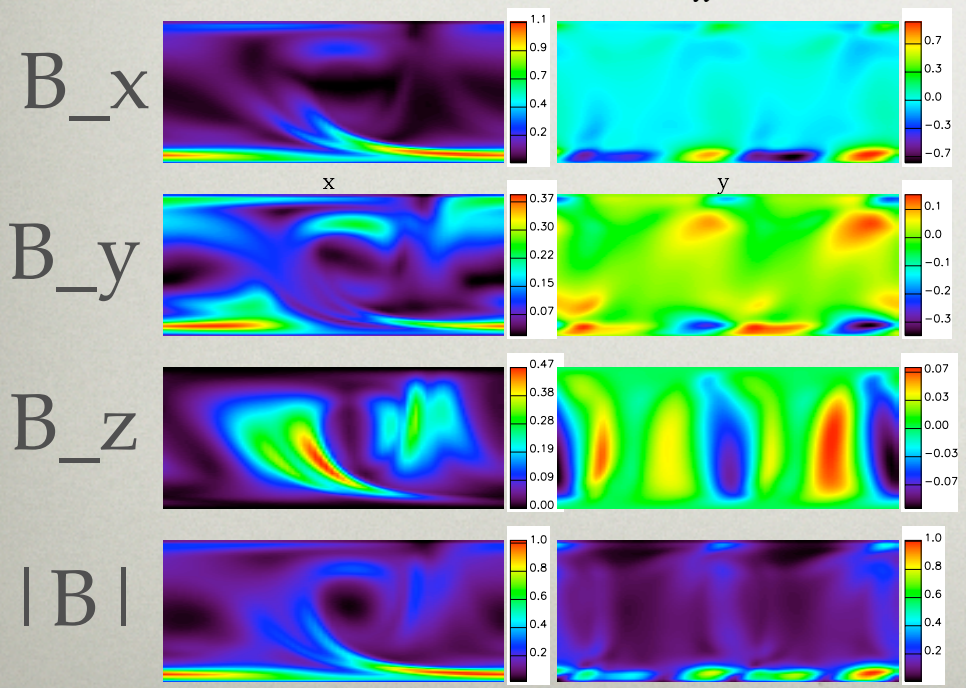
Convective example



Convective roll axes here aligned with horizontal component of rotation.

$$\mathrm{Ra} = 7500 \simeq 2\,\mathrm{Ra}_c, \quad \mathrm{Re} = 30, \quad \tau = 200,$$
$$\mathrm{P} = 1, \quad q = 50, \quad \vartheta = 67.5^\circ, \quad k_x = 4.30, \quad k_y = 1.0,$$





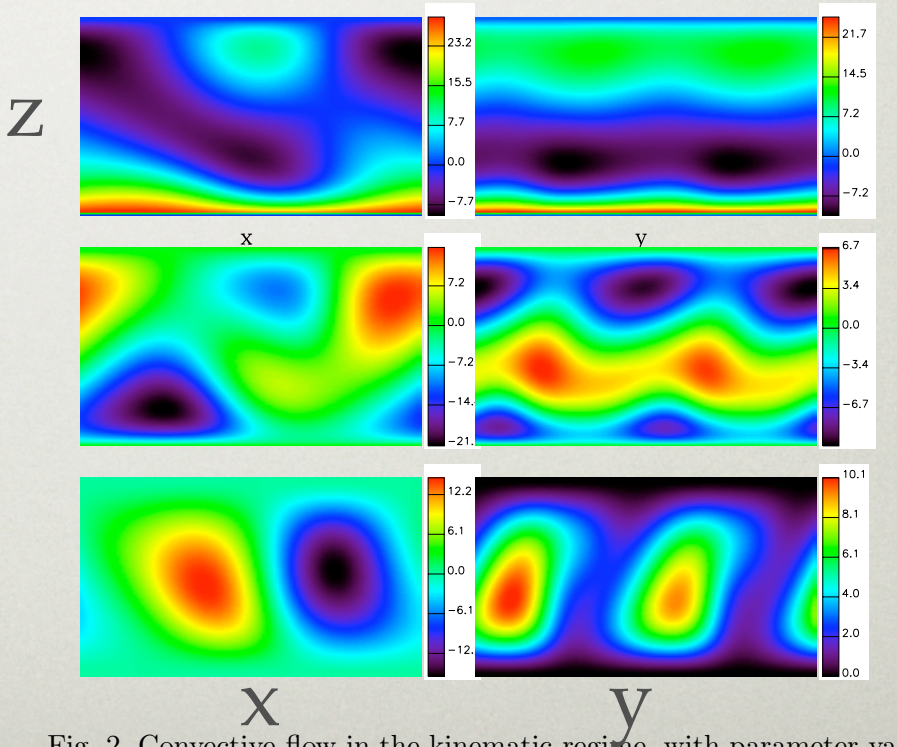
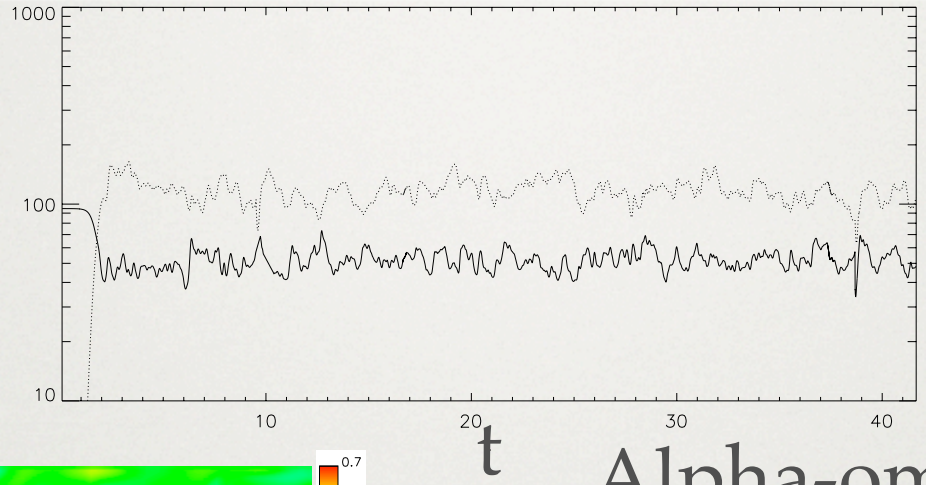
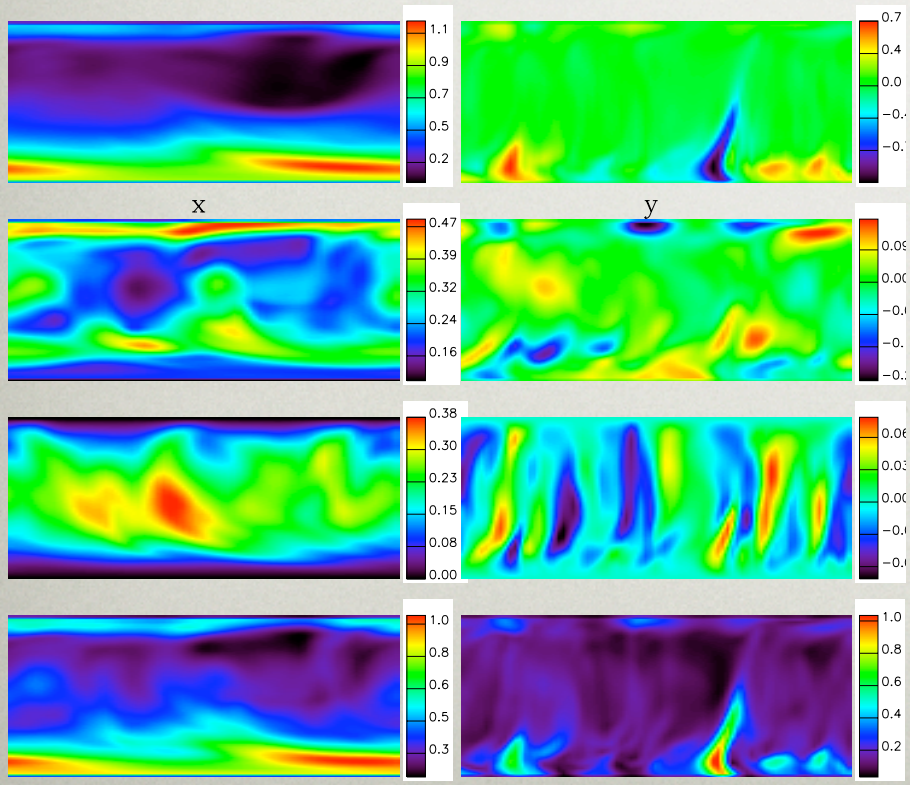


Fig. 2. Convective flow in the kinematic regime, with parameter values (3.1), (3.2). Plotted are the flow components (U, V, W) of $\boldsymbol{U}$ in the (x, z)-plane (left-hand panels) and (y, z)-plane (right-hand panels).

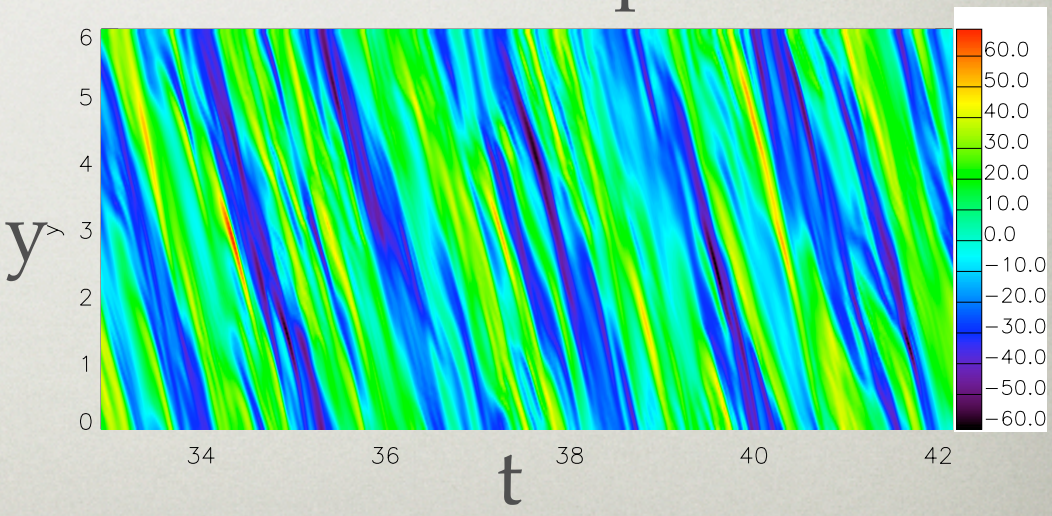
Dynamical regime

E_K, E_M





Alpha-omega dynamo: waves and phases



$$\hat{B}_x(y,t) = \langle B_x(x,y,z_{\max},t)\rangle_x$$

Conclusions

- Ekman instabilities and T-C support dynamos,
- complex, relatively incoherent field structure,
- weak magnetic energy, similar to perturbation E_K,
- relaxation oscillations common: magnetic field tends to turn off the shear instability,
- convective `interface' dynamos give coherent, strong fields (E_M~E_K) and alpha-omega waves,
- convective dynamos more likely in solar context, also in view of stratification,
- limitation: only moderately supercritical.