SPECIAL CONTRIBUTIONS.

STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.

V. RELATIONS BETWEEN THE GENERAL CIRCULATION AND THE CYCLONES AND ANTICYCLONES.

By Prof. FRANK H. BIGELOW.

UNEQUAL DISTRIBUTION OF CYCLONES IN NORTH AMERICA AND EUROPE-ASIA.

We have arrived at the following proposition as the result of the discussion of Ferrel's and Oberbeck's analysis of the general and local circulation, that the general cyclone and the local cyclones and anticyclones have been treated almost independently of one another, while in fact the imperfect results of the theory and the modern observations both indicate that these two classes of movement should be analyzed in close relation with each other. The evidence compels us to regard both these circulations as the common effect of the readjustment of the thermal equilibrium, which is disturbed by the radiation of the sun falling on the tropic zones, and the true meteorological problem is to trace out the successive stages in the process of this interaction through the resulting currents which circulate in the atmosphere. The results contained in this paper apply especially to the North American Continent, and it is hardly to be expected that the details will be found the same in all the other regions of the earth. Indeed there are several reasons for believing that this continent is the peculiar theater for the interchange between the heat and the cold of the Northern Hemisphere, and that the Euro-Asian Continent plays a very different role in the meteorological economy of this hemisphere. For it is well known (1) that while the American Continent is the place for the profuse generation of cyclones, Europe is the region for their dissipation, and in Asia very few cyclones occur except along the ocean borders; (2) that the velocity of motion of the atmosphere generally is about twice as great over North America as over Europe. This points to a profound difference between the actions of the atmosphere in these two regions, but one cause of it at least is easily perceived. It has been shown that the currents which are especially concerned in forming cyclones are contained for the most part within 2 or 3 miles of the ground, though their accompanying effects may extend much higher. Hence, any barriers of elevated ground, as mountain ranges, which tend to deflect the flow of the lower strata, must strongly influence the formation of the cyclones themselves, if they are to be referred to the counter flow of long horizontal currents of different temperature rather than to local vertical convective currents.

The great range of the Himalaya Mountains stretching east and west is such a barrier to the flow of the tropical and polar currents in that region, and the result is that true cyclonic movements are almost excluded from the interior of Asia. On the other hand the Rocky Mountain range, stretching north and south along the western districts of North America, favors the counter flow from the Tropics and the polar regions by deflecting the westward current of the Tropics toward the north, and the eastward drift of the higher latitudes toward the south. The same tendency is favored by the location of the high pressure belt in the latitude of 35°, which causes a high pressure area to form over the middle Atlantic Ocean, while the Rocky Mountain range breaks through the midst of it. The result is to produce a powerful anticyclonic center of action over the Atlantic Ocean, which produces a strong northward component from the West Indies toward the interior of the continent. At the same time the American Continent causes the isobars and isotherms to loop southward, and thus in consequence to draw the Siberian atmosphere in a direction nearly parallel to the Rocky Mountain range. These physical conditions are a constant incentive to the formation of countercurrents which meet on the Canadian and United States territory, with the result that 75 per cent or 80 per cent of the storms of the Northern Hemisphere are generated in these districts. It is not necessary for maintaining the temperature equilibrium of the hemisphere that the interchange of heat and cold should occur so as to have a uniform distribution over all portions of it, because if there is an excessive interchange in any place, as in North America, the general movements of the atmosphere will soon transfer the effects to all other parts. Keeping these facts in mind will facilitate an understanding of the views which will be briefly described as follows:

CRITICISM OF THE CANAL THEORY OF THE GENERAL CIRCULATION.

The immediate problem before us is this: To what extent is the canal theory of the general circulation over a hemisphere correct, and in what direction must it be modified to conform to the modern observations? Ferrel derived the following equations from a discussion of the first equation of 397a for the approximate velocities and gradients in the strata of the upper atmosphere:

$$408a. \quad \text{Velocity;} \quad v = v_0 - \frac{0.016}{(2n+\nu)} \cdot \frac{gh}{r} \cdot \frac{A_2 \sin\theta}{(1+\alpha t)};$$

$$409a. \quad \text{Gradient;} \quad G = G_0 - \frac{0.00001327}{\cos^2 i} \cdot \frac{A_2 \sin 2\theta}{(1+\alpha t)^2} \cdot \frac{P}{P_0} h;$$

when v_0 G_0 P_0 are the values at the surface, and v, G, P, the values at the height h. Since A_2 is negative, — 20.95°, it follows that v is greater than v_0, and G is greater than G_0, so that the eastward velocity and the meridional gradient velocity increase with the height. The relative eastward velocity is shown on fig. 16. The interchanging velocity along the meridians is northward above the neutral plane and southward below it as given on fig. 16.

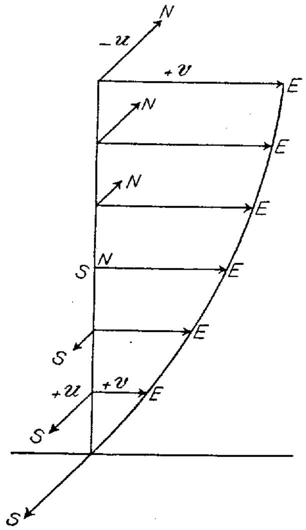

FIG. 16.—Ferrel's component currents by the canal theory.

The canal theory requires that the flow of the atmosphere should be in unbroken stream lines conforming to these precepts. The result of the principle of preservation of areas, $v\,\varpi =$ constant, as applied to the axis of rotation of the earth, is that the velocity v is excessive near the poles, and the gradients of the upper strata much too large. Ferrel sought to escape this difficulty by evaluating $\Sigma m\ (v - v_0)\ k$ for the upper strata with a northward $-u$, and $\Sigma m\ (v - v_0)\ k$ for the lower strata with a southward $+u$, and assuming that the difference of momenta is equal to the retardation of the eastward drift by the frictional resistance. It is known, however, that the frictional coefficient is a very small factor, and not capable of producing the required retardation.

Furthermore, the modern observations show, as in Paper III,[1] that the northward $-u$, and the southward $+u$, components are not distributed as the canal theory requires, but that there are approximately equal currents flowing northward and southward *on the same levels,* which reduces the difference of momenta to zero, and is fatal to the frictional theory of retardation. The counter flow of horizontal currents on the same levels, most powerful in the strato-cumulus level, does, however, constitute a dynamic mechanism quite capable of retarding the eastward drift, so as to produce the observed moderate eastward velocities of the temperate zones. This is the true source of the energy consumed in the motions of cyclones and anticyclones. Compare fig. 17.

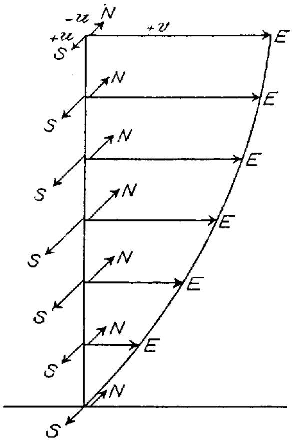

FIG. 17.—Bigelow's component currents from the Weather Bureau observations.

The general theory which has already been mentioned in my previous paper—Storms, Storm Tracks, and Weather Forecasting, Bulletin No. 20, Weather Bureau, 1897—is practically as follows: There is a sheet of the atmosphere flowing quite steadily eastward over the United States at the levels from 2 miles upward. Beneath this flows north and south, quite independently of the upper drift, a series of countercurrents which are cold and warm, respectively. The interaction of these currents sets up the anticyclonic and cyclonic whirls chiefly by dynamic action, the former causing a downward component and the latter an upward component. The cyclone receives its supply from the northwest and from the south, and thus discharges great masses of air through itself, so that the cyclonic configuration constitutes a type of persistent circulation in a vortical form of stream lines. This gyratory rotation lifts the air from one level to another, purely by its mechanical action, and thus raises the mass from the strata of lower to those of higher eastward velocity. This rapid change of eastward motion, if imparted to the air raised up, can take place only by an interchange of inertia, that which is gained by the rising air being lost by the upper eastward drift. Also the gyration of the lower strata causes deflection in the eastward drift by the composition of forces, as must be the case, since the method followed in our discussion has been to subtract the normal west-east and north-south velocities from those actually observed, the residuals being the circular cyclonic movements that are now described.

The point of view for the consideration of the theory of the general cyclone is thus considerably altered. (1) Instead of a circulation as in a canal, northward above and southward below, we find that the interchanging motion is largely confined to the lower strata, by means of currents not flowing above one another, but on the same level. (2) Instead of the momentum $m\ (v_0 - v)$ being determined by the difference between the eastward flow at different levels of the same latitude, we prefer the statement that cyclonic gyrations are produced by the counter flow of independent streams, and that the rapid eastward drift is retarded by mechanical inflows, from above toward the base of the anticyclone, and from below through the stream lines of the cyclone into the eastward drift. The energy upon which cyclones and anticyclones depend for their activity is to be traced to a different source from that generally assigned to it by meteorologists. The common theory is that the cyclone is due to some form of vertical convection, caused by overheating a local region, and by the latent heat produced in precipitation. Our theory would more naturally depend upon horizontal convection, by means of which temperature gradients thousands of miles in extent produce comparatively shallow streams, which flow from the north and from the south, and sustain them for considerable intervals of time. The upward and the downward discharges, together with the rotary components which make the sinuous flow of the air in the upper levels, practically tie together the upper and the lower strata, retard the eastward drift in the higher strata, and accelerate the eastward motion in the lower. This effect is readily perceived in the eastward propagation of high and low pressure areas over the United States, which is the basis of our system of forecasting and renders it possible.

It is by no means to be concluded that by suggesting this modification of the Ferrel theory, any intention exists of not recognizing fully the fact that it remains substantially correct in some of its features. There exists the eastward drift throughout the middle latitudes and the westward drift in lower levels of the tropical zone. But there is yet another reservation to be introduced. The heating of the tropical belt raises the isobars adjacent to the equator so that they slope toward the poles, and to such an extent that they almost exactly counterbalance the deflective force $2nv \cos \theta$, which is directed southward. It may properly be assumed from this that in the upper strata the directions of the isobars, the isotherms, and the stream lines of the wind motion are very nearly parallel to each other, if not coincident. The friction is evidently small, judging from theoretical conditions, and from the results of the observations, or else this could not be the case.

There is another important deduction to be drawn from this discussion regarding the flow of currents from the Tropics toward the poles in the lower strata. According to the Ferrel theory, the overheating of the Tropics is relieved by the upward expansion and overflow, but in accordance with the present view the tropical congestion is relieved by irregular streams which flow outward from the lower levels. This being the case, the poleward gradients in the upper levels are called upon to sustain much less pressure from the deflection, and evidently the tendency to excessive eastward velocities is much diminished, just because the equatorial lift of the strata can not be so great, since the expansion upward, as stated, leaks off sidewise in the lower levels. The eastward drift does not therefore increase to excessive values for these two reasons: (1) The tropical strata are not elevated up to the theoretical amount because of the escape of the currents poleward not very far from the surface of the ground; (2) the eastward drift is diminished by the operation of the vertical discharges between the lower and the higher levels produced by the purely mechanical vortex motion in the cyclones and the anticyclones.

The evidence before us is to the effect that the heating of the atmosphere is generally confined to a layer less than 5,000 meters, or 3 miles, thick. It is not intended to allude now to

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[1] MONTHLY WEATHER REVIEW, March 1902, Vol. XXX, p. 117.

the annual range in temperature with the sun's change in latitude, but rather to the shorter periods of only a few days length which contribute to the impulse of streams from the south. There are several reasons which lead to this conclusion: (1) The trend of the preceding argument has been to show that the readjustments of disturbed temperature equilibrium take place in the lower layers of the atmosphere by means of rather spasmodic impulses, controlled partly by the temperature energy in the tropic and the polar regions, partly by the distribution of land and ocean temperatures, and by the relative radiation which takes place from them in the winter and summer season, respectively. (2) Half of the mass of the atmosphere is contained below the 5,000-meter level, and this is the layer within which the greater part of the aqueous vapor is also collected, since the vapor contents of the higher levels is in the form of fine ice crystals, which drift eastward, encircling the earth, and perhaps seldom finding their way to the ground. The fact that the dust of the Krakatoa volcano was thus carried about the earth for two or three years shows that the upper current has a history of its own, to a considerable extent independent of the 3-mile layer nearest the earth. Now, the important part which the aqueous vapor plays in the absorption of heat is well understood, and it depends upon the very high latent heat of water, which is 606.5 calories per kilogram at 0° C. The evaporation and condensation of water in precipitation is certainly confined to the lower stratum, and hence it is in harmony with this view to limit the effective heating of the air by the sun to the lowest 3 miles. (3) The diurnal variation of the temperature at the surface of the ground takes on a wide range. The temperature is above the normal in low areas, in the summer season, and in the middle of the day; it is below the normal in high areas, especially in the case of cold waves in the winter, and in the night-time. The range at the ground generally amounts to 10° or 20° F., but diminishes upward with the height and disappears at the 5,000-foot level, or even considerably below that height. This is shown very clearly in the study of the changes in the vertical temperatures, as explored by means of balloon ascensions, where the 5,000-foot level marks the convergence of the lines which represent the gradients in the forenoon and afternoon. The range in the United States is usually greater than in Europe at the ground, owing to the more pronounced nature of the cold and warm waves that move eastward over that region, but the evidence is that the diurnal lines converge at 2 or 3 miles above the ground. (4) The fact that the great eastward drift of the upper levels is underrun by a series of comparatively thin currents, which move about in every possible direction, shows that the disturbance of equilibrium is local and confined to a shallow skin near the ground, the most rapid currents belonging to the cumulus and strato-cumulus levels, which also implies that the upper regions of the atmosphere are much less affected than the lower. (5) The same conclusion is indicated very clearly by the seasonal change in the drift of the high areas over the northwestern portions of the United States, from the northwest in winter and from the southwest in summer, in conformity with the location of the permanent high areas in winter over the continent and in summer over the ocean.

From these considerations it seems evident that the upper atmosphere is but slightly disturbed in its temperature equilibrium by the effects of the solar radiation, but the solar rays pass through it with comparatively little absorption, while the larger percentage of the heat retained in the lower strata, is due to a change of the wave length. This conclusion is very important for two reasons: (1) It shows why the general circulation of the atmosphere prescribed by the Ferrel-Oberbeck theory does not seem to be confirmed by the observations. (2) It also indicates where the energy comes from which is finally expended in the generation of cyclonic circulations and in the retardation of the eastward drift by the agency of inertia rather than by friction; for if the total amount of energy falling upon the Tropics does not expend itself in an upper poleward current, because the higher strata retain a temperature of equilibrium almost undisturbed, then this energy must give rise to a series of comparatively small currents moving poleward in the lower strata—a fact which is abundantly confirmed by the observations. Also, if the friction of the upper atmosphere upon itself is very slight, then there will be but little retardation to an excessive eastward drift, tending under a steady force to have a constant acceleration. The only other available agency which will produce the same effect is the intrusion into the higher strata from the lower, or vice versa, of air moving eastward with a different velocity, which must suddenly be subject to acceleration. The discharge of the product passing through cyclonic circulations is perfectly fitted to perform this office. Hence, the theory here expounded consists of two parts as regards the eastward drift: (1) The upper poleward gradients are not built up to the amounts supposed by Ferrel, because of the lateral escape poleward of currents from the tropical belt; (2) The agency of friction as a retarder is replaced by the interchange of inertia derived from a compound circulation, the sources of the separate parts having different and independent causes. It has been important to thus carefully clear the ground for the theory of local cyclones, which will be advanced to take the place of the type proposed by Professor Ferrel on the one hand, or of that advocated by the German school on the other hand. These two theories are not in harmony with each other, and neither of them seems to be in agreement with the observations. It is very evident that the six assumptions which have been made in order to pass from the general equations of motion, 200, to the working system employed by Ferrel, 397a, must be carefully revised before we can expect to put this branch of meteorology upon a correct working basis. In particular it is not suitable to omit the variations of temperature in longitude, because in so doing the turbulence of the lower strata and the alternate streams which are implied in this variation profoundly modify the general and the local circulation. It is also necessary to correct the statement regarding the friction as the special agency of retarding accelerated flows and substitute, or rather add thereto, the inertia of currents which are rapidly changing their velocities. The conflict of turbulent countercurrents, especially at a short distance above the ground, must be rigorously considered in studying the resultant effects. Likewise the direct application of the law of conservation of areas passed over by the rotating radius vector can not apply immediately to the lower strata, though it may be much more nearly correct above the 3-mile level.

MODIFICATION OF THE CANAL THEORY.

In consequence of these considerations it seems necessary to modify the canal theory to such an extent as to be practically equivalent to an abandonment of it. If this is done it is important to trace out a chain of circumstances which will give a more correct account of the general and the local circulation of the atmosphere. The canal theory is very artificial, depending as it does upon a simple laboratory experiment, in connection with an obvious analysis of the general equations of motion. If this theory is to be preserved, then we must be assured that the atmosphere does in fact traverse the circuits prescribed, for it has been commonly assumed to conform with the facts of observation. As has been shown, observation does not bear out the requirements of the theory, and I shall, therefore, attempt to trace out in a descriptive way the circulation as it is developed over the North American Continent. The reduction of this kinematic picture to the corresponding mathematical form of dynamics is a task of very great difficulty, as may readily be inferred.

We may conceive the tropical strata to be elevated by thermal expansion relative to the polar regions, so that there is a certain average gradient slope and corresponding west-east velocity which is in equilibrium with it in accordance with the usual equations. If the canal theory of circulation is in operation we have poleward gradients in the upper strata, and equatorward gradients in the lower strata. If, instead of maintaining this circulation, there is an escape of currents from the Tropics in the lower strata in a poleward direction, then the gradients of the canal theory diminish and become much more moderate in consequence of this release of tension. Every such leakage current from the Tropics causes a break or fault in the gradient, and this must be attended by a corresponding deflection of the eastward current, through the operation of the deflecting force at right angles to it, due to the earth's rotation. Referring to the scheme of self-regulation of the circulation by the rise and fall of the gradients, fig. 18, we may consider the average gradient in the strato-cumulus level. When in its mean position, as determined by the whole set of natural circumstances which produce it, the isobars and the wind with a given direction and velocity are all practically in coincidence. The velocity is just enough to maintain the slope of pressure as measured by the gradient. If the thermal expansion of the Tropics increases, the gradient slope is elevated, the eastward velocity is increased, and this acceleration will continue to advance till an excess of energy secures for itself a way of escape. This increase of velocity is probably checked as follows:

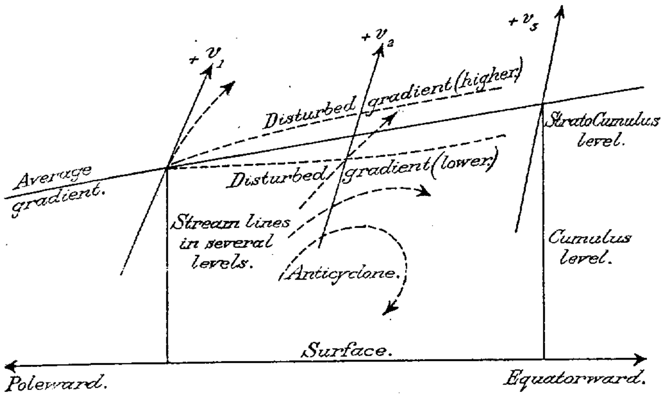

FIG. 18.—Scheme of self-regulation of the circulation by the rise and fall of the gradient.

From the Tropics in the lower levels, that is in the cumulus and strato-cumulus strata, a stream of air breaks away from the canal circuit, and pushes northward in some irregular course. This evidently causes a break in the gradient surfaces, such as is indicated, for example, by the dotted lines of fig. 18, and the slope of pressure which held the eastward drift in its position gives way. The action of the deflecting force due to the earth's rotation bends the current southward as is indicated, and there is thus made the beginning of an anticyclonic local movement. In the lowest levels, from cumulus down to the surface, the break in the average gradients may be so pronounced as to offer but little check to a complete anticyclonic gyration, such as appears at the surface of the earth. This deflection of direction causes a whirl and absorption of energy in the strata affected through the local interchange of inertia, and slows up the eastward drift by the vortex action which is produced with a downward component. The alternate rise and fall of the gradients, and the attendant anticyclonic gyrations, mark the successive efforts at self-regulation which the atmosphere as a thermal engine imposes upon itself. These continental pulsations are shown on the weather maps as the procession of high and low pressure areas which traverse the middle latitudes. In consequence of the superior velocity of eastward motion in the strato-cumulus level, this region is the first to feel the decline in gradient strength, due to the tendency of a stream to escape from the Tropics, as from the Gulf of Mexico over the United States. It thus happens that the anticyclone is not only larger in area, but it also generally precedes the cyclone in its formation. There are numerous instances in which the anticyclone overspreads the United States, while there is no important cyclone in connection with it, except possibly some depression or irregular action along the edges of the high area. Usually, however, incipient cyclones increase in intensity from these small beginnings, and they may even seem to drink up and exhaust the air which is flowing in anticyclonic circulation. This is the reason why I have heretofore described the anticyclone as preceding the cyclone in efficiency, and have thus reversed the order of action as taught by Professor Ferrel in his well-known theory.

The warm stream from the south is deterred from mingling with the cold anticyclonic air by the difference of its temperature, and the result is that the eastern side of the anticyclone, which flows southward, and the warm escape current from the south, flowing northward, compose two counter currents. These two currents together generate the cyclone, which is a vortex with an ascending central velocity. The gyration is produced by the action of the two independent streams acting like a couple, since they each depend upon separate gradient systems for their own mechanical pressures. It is not a gyration due to frictional impact, but rather to the steady pressure on the arms of a couple, since the streams are driven by independent gradients, and are held apart by having different temperatures in the two separate currents. The air is thus raised from the lower to the higher strata in great masses, by circulating through the configuration of the cyclone, and this, too, produces a retardation of the eastward drift by an immediate interchange in the inertia, since there is a quick mingling of air having different directions and velocities. Examples of this action can be seen by studying the Charts 20 to 35, inclusive, of the International Cloud Report. Furthermore, the motions of the atmosphere result in placing strata of air having different temperatures together, side by side, so that the surface isotherms are directed from the southwest to the northeast, and in consequence the northwest portions of a cyclone are cold and the southeast portions are warm. This does not conform to the requirements of either the Ferrel or the German theories, which demand a warm local center for the generation of cyclones. Yet if two such masses of air lay alongside while they are of different temperatures, an interchange of heat contents will take place locally between them, and thus the streams will interflow in such a way as to strengthen the cyclonic gyration. This will be accompanied by distinct stratification of the air currents in the local cyclones, such as is observed in the kite and balloon ascensions, since the air of different temperatures is drawn out into thin ribbons having large discontinuous surfaces, which are favorable to the interchange of heat. It is frequently found that a great anticyclonic area as it approaches the ocean, although attended by no cyclone, will yet suddenly cause a violent whirl to form on its edge by the mere action of these adjacent masses of different temperatures. Such local storms are sometimes formed on the Atlantic coast during a single night, and they may cause vortices with hurricane velocities on the coast. The line of junction of these warm and cold currents, along the southern and southeastern parts of the low area, is the locus of the formation of the majority of the tornadoes of the United States, the counter flow setting up the gyration, which is converted into a genuine columnar vortex, through which the heated air of the lower strata escapes into the colder strata of the higher levels. The hurricanes of the West Indies similarly form along the places of the counter flow between the Atlantic high area and the southeast trade winds when at their extreme northern limit, as in August to October. The vortex then travels westward and skirts around the periphery of the high area until it is absorbed finally in the eastward drift of the higher latitudes. Such dynamic intermingling of the general and the local circulation is, therefore, not only in accordance with observations, but it is a suitable substitute for the defective canal theory of the general circulation, and also for the untenable theory of the local cyclones and anticyclones, supposed to be dependent upon the central heat produced by condensation of the aqueous vapor of precipitation. This view is attended, on the other hand, by the following disadvantage: That while the canal theory and the warm center cyclone theory lend themselves readily to mathematical treatment and to analytic solutions of considerable elegance, we are obliged to substitute for them an irregular system of stream lines in the lower strata, not at all readily put into mathematical forms. This turbulent circulation, with its self-adjusting government of the eastward flow, its interaction between the general and the local vortices, its numerous subordinate phenomena, such as tornadoes, hurricanes, and cyclones, is easy to comprehend, but hard to analyze mathematically into the exact dynamic forces of equilibrium. It is possible to construct several special typical configurations for each district of the earth, as was done for the United States in Charts 20–35, inclusive, of the International Cloud Report, and then draw the stream lines, with their velocities, in order to prepare for the computation of the dynamic forces involved. This is the true meteorological problem of the future.

THE STRUCTURE OF THE ANTICYCLONE.

We will now examine a little more closely the structure of the anticyclone and the cyclone as given by observations for the sake of the analytical problems presented by their configuration. It has been claimed by meteorologists that there is a southward component in the middle strata of the north temperate zone toward the high pressure belt, but a northward component in the upper strata and another northward component near the surface, as is indicated on fig. 13, Paper IV[2]. The observations of 1896–97 do not, however, give such a distribution of the mean components, for they show that there is a very small average drift northward in all strata, increasing slightly with the height above the surface. That is to say, the atmosphere in the eastern and central United States drifts northward a very little and thus supplies part of the air that descends into the anticyclones through the upper strata. We have indicated how the leakage in the lower strata from the Tropics in part replaces the air which descends in the anticyclonic areas, and it is assumed that the small residual northward drift complements the amount that is required to fill the anticyclonic areas. The downward vortex, therefore, draws in a portion of the air passing through it from the upper strata, as a consequence of the gyration induced through the countercurrent action, and therefore the feeble northward component of the circulation of the higher strata seeks the surface practically in the middle latitudes, before arriving at the polar zone, through the mechanism of the local vortices. Hence, it follows that there is little cause for the formation of a general anticyclone close to the pole itself, which Ferrel assumed to exist; the result of Kimball's discussion in the MONTHLY WEATHER REVIEW, September, 1901, goes to show that this movement only feebly exists, and is in conformity with this exposition of the general circulation. Therefore, the air that descends in a local anticyclone comes from two sources, the leakage currents from the Tropics in the lower and middle strata and the feeble northward drift in all strata, especially the higher.

There is one feature of the anticyclonic vortex which may be mentioned, though it belongs more properly to an analytic treatment of that circulation. The anticyclonic components of fig. 6, Paper III[3], show that we are not dealing with a pure form of vortex. The two possible laws are typically the parabolic $\frac{v}{\varpi} =$ constant, and the hyperbolic $v\varpi =$ constant.

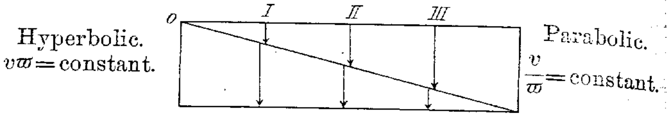

FIG. 19.—Mixed system of hyperbolic and parabolic components.

According to the parabolic law $\frac{v}{\varpi} =$ constant, the circulation causes simply a depression in the center of the gyrating fluid; according to the hyperbolic law $v\varpi =$ constant, there is a vertical component of circulation as in ordinary vortices. In the observed anticyclonic components the velocities v are about equal to each other on the I, II, III, circles, and it seems to me that this can only happen if there is a mixture of these two laws of motion. Thus we may divide the observed components v_{I}, v_{II}, v_{III}, into two parts by a diagonal as shown on fig. 19. The upper components represent the parabolic, and the lower the hyperbolic portions. This is physically necessary for the following reason. The anticyclone is formed by large currents of air moving in more or less independent streams on its outer portions, while only curling offshoots reach its central parts; this would produce the pure parabolic components only. But, through imperfect pressure gradients there is also near the center a true downward component of circulation, and this can be supplied only by the action of hyperbolic components, that is to say, of a simple vortex motion. Hence, the general anticlockwise movement of the anticyclone, strongest on the outer circles, has accompanying it a true downward or vortex component which lengthens the components u in the central portions. If this is correct, one sees an additional reason for holding that Ferrel's explanation of the anticyclone is impracticable, and also that the reversing of the cyclone to make an anticyclone, as proposed by Oberbeck and Pockels is not warranted. We need very accurate observations to settle so difficult a point of pure theory, but I can not at present see any other satisfactory explanation of the gyratory components derived from the Weather Bureau observations.

STRUCTURE OF THE CYCLONE.

In Table 18 we give the results of cloud observations in the United States.

In the following example the relations which should exist in a pure vortex are deduced for comparison with the data given under low areas in Table 18.

Taking the inward radial velocity $u = -1.25$, at the distance 1,250 kilometers, assuming $\lambda = .000100$ for $\theta = 46° 17'$, and $c = .000002$, also $k = .000050$, their introduction into the several formulæ gives the values found in Table 19.

An account will be given of the derivation of the formulæ in Paper VI.

It is seen that the rotational velocity v is about the same as that given by the observations up to the circle whose radius is 150 km = 93 miles from the center, as seen under v_2 of Table 18. The values of the radial velocity agree fairly well, if we admit that the observations may be somewhat imperfect for this component up to the region of the inner circle I, whose radius is 250 km. There the component u is much larger than expected in the upper strata, and this indicates some opposition to the free development of the vortex near the core. It is presumed that this implies a struggle to intrude into the rapid eastward drift, accompanied by a broadening of the vor-

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[2] MONTHLY WEATHER REVIEW, April, 1902, Vol. XXX, p. 166.

[3] MONTHLY WEATHER REVIEW, March, 1902, Vol. XXX, p. 117.

TABLE 18.—*Anticyclonic and cyclonic velocities at each 1,000-meter level. (Copy of Table 126 International Cloud Report.)*

Cloud forms.	Height in meters.	High areas.						Low areas.						Velocities in the general cyclone.	
		I		II		III		I		II		III			
		u_2	v_2	u_2	v_2	u_2	v_2	u_2	v_2	u_2	v_2	u_2	v_2	u_1	v_1
Ci. and Ci. St.	10000	−3.5	−4.0	+ 4.5	−5.5	+2.5	− 3.0	−6.5	+ 3.0	−1.0	+ 3.5	0.0	−3.0	−2.8	+35.4
	9000	−3.5	−5.0	+ 4.5	−5.0	+2.5	− 8.0	−3.0	+ 8.0	−3.0	+11.0	−1.5	−1.0	−2.6	+35.0
Ci. Cu	8000	−3.0	−6.0	+ 3.5	−4.5	+2.0	− 9.0	−2.5	+11.0	−5.5	+13.5	−2.0	+1.0	−2.4	+34.6
	7000	−1.5	−6.5	0.0	−4.5	−1.5	− 8.0	−6.5	+13.5	−5.0	+15.0	−2.0	+2.0	−2.2	+30.0
A. St	6000	0.0	−7.0	− 2.5	−4.5	−4.0	− 7.5	−9.0	+15.0	−2.0	+15.5	−0.5	+3.0	−2.0	+25.0
	5000	0.0	−7.5	+ 2.5	−6.0	−2.5	− 8.0	−9.0	+17.0	0.0	+15.5	+1.5	+5.0	−1.8	+23.6
A. Cu	4000	0.0	−7.5	+ 8.5	−8.0	0.0	−10.0	−7.5	+20.0	0.0	+14.5	+2.5	+7.0	−1.6	+22.6
	3000	0.0	−7.0	+10.0	−9.5	+2.0	−12.0	−3.5	+23.0	0.0	+13.0	+2.0	+7.5	−1.3	+21.0
S. Cu	2000	+1.0	−6.0	+ 7.5	−9.5	+2.0	−11.0	−1.0	+20.0	0.0	+11.0	0.0	+5.0	−1.0	+14.0
Cu. and St.	1000	+3.5	−4.5	+ 5.0	−7.5	+1.5	− 7.0	0.0	+ 8.0	−1.5	+ 8.0	−1.5	+4.0	−0.8	+ 6.4
Wind	0000	+4.0	−2.5	+ 2.5	−4.0	+1.0	0.0	0.0	+ 6.0	−3.5	+ 3.0	−2.0	+3.0	−0.5	+ 1.3
Means of the velocities, u_2, v_2		−0.3	−5.8	+ 4.2	−6.2	+0.5	− 7.6	−4.4	+13.1	−2.0	+11.2	−0.3	+3.8		
Radius, ϖ			1		3		5		1		3		5		
Product, ϖv_2			−5.8		−18.6		−38.0		+13.1		+33.6		+19.0		

$+u_2$ = outward on radius.
$+v_2$ = anticlockwise about center.

I. ϖ_{I} = 250,000.
II. ϖ_{II} = 750,000.
III. ϖ_{III} = 1,250,000.

$+u_1$ = south.
$+v_1$ = east.

TABLE 19.—*Application of the formulæ for a cyclone. (Copy of Table 127, International Cloud Report.)*

CONSTANTS AND FORMULÆ.

$\theta = 46° \ 17'$ $\qquad \lambda = 2n \cos\theta = .000100$

$u = -1.25$ $\qquad c = -\frac{2u}{\varpi} = .000002$ $\qquad \frac{\lambda}{k-c}\frac{c}{2} = .000002$

$\varpi = 1{,}250{,}000$ meters $\qquad k = .000050$

$$\psi = -\frac{c}{2}\varpi^2 z. \qquad u = -\frac{c}{2}\varpi. \qquad v = +\frac{\lambda}{k-c}\frac{c}{2}\varpi z. \qquad w = +cz. \qquad \cot i = \frac{\lambda}{k-c}z.$$

DERIVED DISTANCES, VELOCITIES, CHECKS, AND INCLINATIONS.

ϖ		z	ϖz	u	v	w	ϖv	$\varpi w = -2uz$	cot i	i	G
meters.	*miles.*									°	*mm.*
1250000	777	1.00	1250000	−1.25	2.50	.0000020	3125000	2.5	2.00	26.6	0.34
1000000	621	1.56	1560000	−1.0	3.125	31	3125000	3.1	3.12	17.6	0.40
750000	466	2.78	2085000	−0.8	4.17	56	3125000	4.2	5.56	10.2	0.55
500000	311	6.25	3125000	−0.5	6.25	125	3125000	6.3	12.50	4.6	0.78
250000	155	25.00	6250000	−0.25	12.50	500	3125000	12.5	50.00	1.2	2.03
200000	124	39.06	7812000	−0.20	15.61	781	3125000	15.6	78.12	0.7	3.00
150000	93	69.40	10412000	−0.15	20.82	1388	8125000	20.8	138.80	0.4	5.37
100000	62	156.30	15630000	−0.10	31.26	3126	3125000	31.3	312.60	0.2	13.90
50000	31	625.00	31250000	−0.05	62.50	12500	3125000	62.5			
40000	25	977.00	39080000	−0.04	78.16	19540	3125000	78.2			
30000	19	1736.00	52080000	−0.03	104.16	34720	3125000	104.2			
20000	12	3906.00	78120000	−0.02	156.24	78120	3125000	156.2			
10000	6	15625.00	156250000	−0.005	312.50	.0312500	3125000	312.5			

tex tube through the resistance, v being smaller than it should be and u greater, making the angle of the inclination i much greater than it ought to be in pure vortex motion. As the computation of the vortex goes to the height of 10 miles, far beyond the altitude to which the ordinary cyclonic motion penetrates, this being only 3 or 4 miles in the moderate movements of the air, it is seen that v should theoretically attain enormous velocities very near the center. The difference between these and such as are actually observed may be regarded as measuring the energy expended in breaking up the cyclone in the higher levels, which can be balanced only by retarding the movements of the general cyclone. The vertical velocity w is extraordinarily small from the bottom to the top, and in a measure justifies the method of discussing the motions of the air in the cyclone as a case of horizontal movement. It is impossible theoretically, however, that such a cyclonic motion without a vertical component should exist at all. The fact is that a slow vertical movement is acting over the very large area covered by the cyclone, and is sufficient to carry off all the air which flows into it through a thin disk at the outside in horizontal directions toward the center. The checks ϖv and $\varpi w = -2 u z$ hold throughout the cyclone, thus proving that our data are at least approximately correct. The angle i, between the tangential direction and the current, theoretically becomes very small within 300 miles of the center. It was shown by the results of the observations that the movement at the cumulus level is much more rounded than in the lower strata, the difference being caused by the retardation of the air operating upon the surface irregularities of the ground. A congested or irregular inflow near the center will similarly increase the angle i, since the component u is increased and v diminished by it. The observations given on the weather maps do not record the conditions within 60 miles of the center with any definiteness. In hurricanes the core is about 30 miles broad and its boundary is quite sharp, which shows that the component v is highly developed, while u is small. But the hurricane also penetrates to much greater altitudes, as already mentioned. Finally, the gradient shows that there is a slow change near the outer limit, but that it increases very rapidly on approaching close to the center.

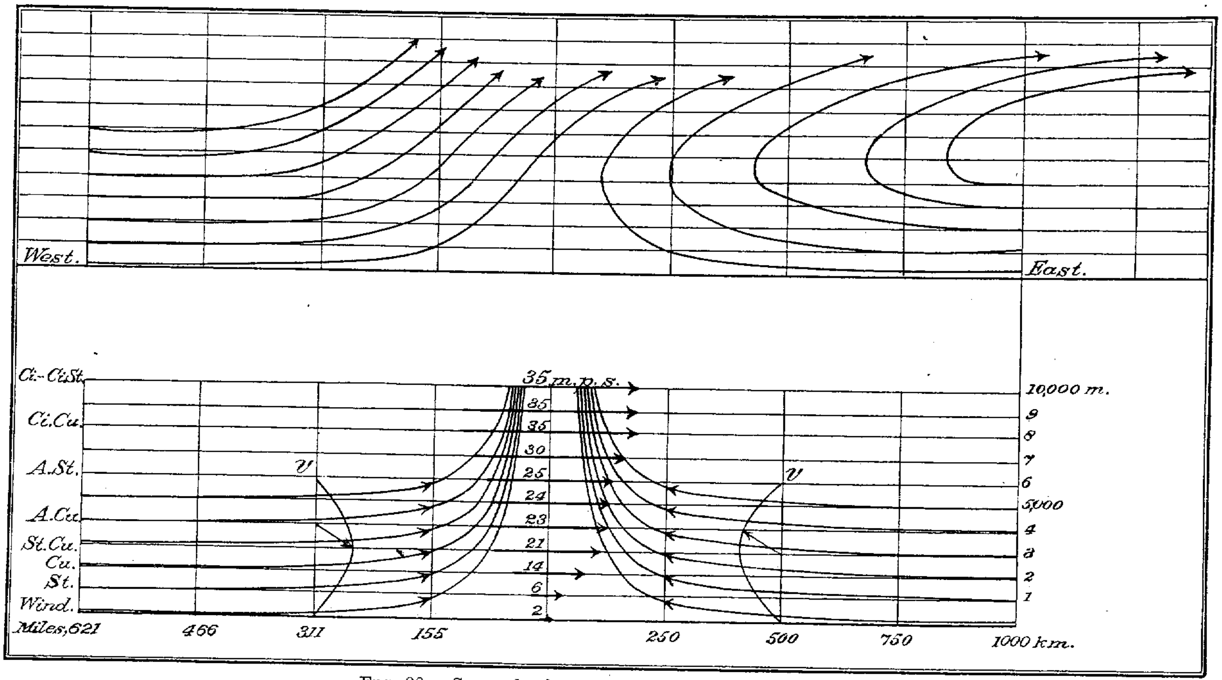

FIG. 20.—General scheme of the structure of cyclones.

THE SPECIAL FEATURES OF THE CIRCULATION.

The special features of the circulation indicated on fig. 20, which may be mentioned are as follows: (1) It is evident that this scheme avoids entirely the primary difficulties attending the Ferrel and also the German types of circulation. Each of these divided the cyclone into two parts having special properties. Ferrel divided his cyclone at a vanishing rotational velocity, $v = 0$, which involved a circulation in opposite directions on either side of it; the German type consists of two parts, separated by a discontinuous movement at the circle of the maximum velocity for v. The pure vortex, on the other hand, has only one law to deal with, and that, too, the simplest of all, in accordance with which the motion is generated. (2) Neither of the other types provides for a true calm region at the center of the cyclone, commonly observed in hurricanes as the eye of the storm. Ferrel's formula 402b shows that v increases from the circle $R = 0.707$ to the very center; the formulæ 450 and 471 show that the velocity v decreases gradually from the circle bounding the inner region toward the center, but does not vanish till reaching it. The construction here proposed indicates that since v is a function of the height z as well as of the radial distance ϖ, the air in streaming toward the center is gradually lifted above the ground by purely dynamic action and leaves a core without gyratory circulation. (3) The approach of a moving particle to the axis of the cyclone is attended by an increase of the velocity of rotation, which accelerates rapidly as it passes into the upper strata. There it accomplishes the work of deflecting the eastward drift, and it expends some of its energy in that way. The result of this opposition to free motion is to spread out the top of the vortex, reduce its gyratory velocity, and change the relations of v_2 and u_2. In the undisturbed gyratory motion the component v_2 becomes very great in comparison with u_2, and u_2 is always a small quantity. An inspection of Table 18 shows, however, that in the strata, between 5,000 and 10,000 meters the radial velocity u_2 is relatively large, and the angle of the inclination instead of being nearly 0° is from 25° to 35° on the inner circle II. This means that the original coefficient upon which the dimensions of the cyclone depend, namely, $c = -\frac{2u}{\varpi}$, does not remain a constant, but increases from the boundary toward the axis of the gyration. Thus we obtain,

$$c_{\mathrm{I}} = .0000352 = \frac{0.6}{1250000}$$

$$c_{\mathrm{II}} = .0000053 = \frac{4.0}{750000}$$

$$c_{\mathrm{III}} = .0000005 = \frac{8.8}{250000}$$

This involves an expenditure of energy in the struggle attendant upon intruding into the swiftly moving upper stratum. Furthermore, as was previously pointed out, Ferrel's theory of the slowing down of the excessive eastward velocities which would arise from the pure vortex law of the

TABLE 20.—*Dimensions and velocities in the waterspout off Cottage City, Vineyard Sound, Mass., August 19, 1896. (Copy of Table 128 International Cloud Report.)*

Working formulæ: $v^2=2gz$; $u=\frac{v}{z}$; $w=\frac{2v}{\varpi}$; $\varpi v=$ Const.; $2zu=\varpi w$; $\frac{c}{z}=\frac{4g}{v\varpi}$.

Dimensions in meters.			Velocities in meters per second.				Dimensions in feet.			Velocity in miles per hour.		
h	z	ϖ	u	v	w	$2zu=\varpi w$	h	z	ϖ	u	v	w
1280	0	∞	—	0	0	0	4200	0	∞	—	0	0
1278	2	518.4	3.13	6.26	0.02	12.53	4193	7	1701	7.0	14.1	0.04
1189	91	76.9	0.46	42.24	1.10	84.49	3901	299	253	1.0	94.4	2.5
1097	183	60.8	0.29	53.39	1.76	106.79	3599	601	200	0.6	119.5	3.9
1006	274	44.3	0.27	73.31	3.31	146.61	3301	899	145	0.6	164.	7.4
914	366	38.3	0.23	84.72	4.42	169.45	2999	1201	125	0.5	189.	9.9
731	549	31.3	0.19	103.77	6.63	207.53	2398	1802	102	0.4	233.	14.9
549	731	27.1	0.16	119.73	8.83	239.47	1802	2308	89	0.4	268.	19.8
457	823	25.5	0.15	127.04	9.94	254.10	1409	2701	84	0.3	284.	22.2
366	914	24.3	0.15	133.89	11.04	267.78	1201	2999	79	0.3	300.	24.7
183	1097	22.1	0.13	146.68	13.25	293.36	601	3599	72	0.3	328.	29.6
146	1134	21.8	0.13	149.13	13.70	298.26	479	3721	72	0.3	333.	29.7
0	1280	20.5	0.13	158.44	15.46	316.89	0	4200	67	0.3	354.	34.6

$c=0.01208$

conservation of areas applied to the general cyclone, is that friction is largely concerned with the operation. It seems, however, that a much more efficient cause of retardation is the interaction between these two types of motion, namely, the linear and the rotary, by which the lower strata thrust themselves into the higher. The effect is to enlarge the size of the vortex tube at the top by the resistance, deflect the eastward drift into sinuous curves, slow down the eastward velocity, and thus restrain the general cyclonic movement from excessive values. (4) The resultant of these component forces and velocities is to produce a circulation along the parallels of latitude which may be represented by the upper part of fig. 20. The upper clouds of the cirrus region precede the cyclone proper as forerunners of this type of circulation; the lower clouds follow in succession, till precipitation is produced at the elevation of 1,000 to 3,000 meters; in very rapid circulations the eye of the storm is fully developed; the clearing up is more abrupt on the westward than on the eastward side of the cyclone. (5) The progressive movement of storms is partly an effect of the cyclone covering an area of sufficient extent to be in different latitudes, so that variations in cos θ amount to something. But other circumstances are more important. By Table 33, Section IV, International Cloud Report, it is seen that the northward components for the group of areas which are covered by the currents of air from the south are greater than those from the north. That is to say, the movement in the streams from the Tropics is more rapid than that from the polar regions, and the result is to roll up the eastward side of the cyclone to the north more than the westward to the south. The cyclone tends to rotate along the front of a high area toward the north. At the same time its top is fastened by means of the circulation into the eastward drift at 4,000 meters elevation, and these two components make the storm move northeastward in the central and eastern portions of the United States. If the general cyclone has other directions, as when hurricanes form in the Caribbean Sea, the same principles hold, and the storm there first moves westward, then northward and northeastward, because the general circulation is controlled in that region by the anticyclone in the southern portions of the north Atlantic Ocean. (6) It has been shown that the currents which feed the cyclone have different velocities at different altitudes, being greatest from 2,000 to 5,000 meters above the ground. Each stratum forms a stream for itself conforming to the general law, but modifies its dimensions according to the constants pertaining to the special locality. Since these different strata have thus distinct local movements, especially considering the variable temperatures and densities of the currents from the north and south, respectively, it follows that the conditions are favorable for the formation of turbulent minor circulations of all kinds. The movement of the air is therefore partly congested, and partly runs in free whirls, the difference of equilibrium in temperature being gradually reduced to the proper normal value for the latitude and altitude by this forced intermingling of the subordinate parts of the cyclone. This process of restoring to an equilibrium the temperature of masses, bearing with them that of the region from which they came, is generally completed by the time the 5,000-meter level is reached, judging from the records of the balloon observations. In summer, when the eastward drift is relatively slow, the pure vertical convective ascension may extend up to 10,000 meters. This is much more likely to happen on the eastward than on the westward side of a cyclone, because the vortex components throw back the eastward movement upon itself, and thus make the strata more stagnant in vertical directions. It has also been found practically very difficult to make the kites fly on the east of the low center, the best ascensions being made on the southerly and westerly quadrants. (7) The entire problem of analyzing the movements of the air in their details is so exceedingly complicated that only slow improvements in dynamic meteorology can be expected. A clearer idea of the fundamental conditions may, however, enable us to advance more rapidly than is now expected. It will be a very important gain if meteorology can free itself from some of the theories which have so long prevailed, but which now are seen to be quite untenable, and have seriously retarded its advancement.

THE VELOCITIES IN TORNADOES.

The motions in tornadoes are similar to those in cyclones, yet the tube is not only inverted in position, but the stream lines occupy only the central portions, and the lines of ψ become tangent to the plane whose height is H_0 at certain distances from the center. The fundamental equations for the tornado are 303–308. $\psi = +\frac{c}{2}\varpi^2 z$ holds for the tornado with vertical axis positive downward, and the single bounding plane at the distance H_0 above the ground. $\psi = -\frac{c}{2}\varpi^2 z$ is the equation for the cyclone with the vertical axis positive upward. A multitude of minor relations and comparisons can be drawn from the two sets of equations 303–308 and 488–490. If the lower part of fig. 20 be looked at as if the horizontal axis were the vertical, we have a picture of the tornado tube. The diagram is not good because not drawn to scale, but the idea is easily understood. Hence, one law serves for all types of local storms, cyclones, hurricanes, and tornadoes, which thus theoretically differ from each other only in their dimensions and in the details by which they are formed. Cyclones are generated chiefly by horizontal convection currents; hurricanes have a stronger vertical convection current and also horizontal convection currents; tornadoes arise chiefly from vertical convection, assisted by some horizontal currents which counter flow in the cumulus level.

It may be remarked that the stream lines indicated by Ferrel, page 300, Recent Advances, are conjectural only, and do not conform to the theory of stream lines in a vertical vortex tube, nor to observation, which shows that the air is quiet close up to the boundary of the tornado tube.

THE WATERSPOUT OFF COTTAGE CITY, MASS., AUGUST 19, 1896.

The result of the computation on this interesting waterspout is added, and it shows the dimensions and velocities in metric and English measures which were derived from the observed distances and the formulæ. The most important feature is the value of the vertical velocity of 35 miles per hour at the sea level. See Table 20.

ANNALS OF THE ASTROPHYSICAL OBSERVATORY OF THE SMITHSONIAN INSTITUTION, VOLUME I.[1]

By S. P. Langley, Smithsonian Institution, dated April 29, 1902.

The work upon the infra-red solar spectrum, described in this volume, is the latest outcome of investigations with the bolometer, begun with the invention of that instrument by the writer about the year 1880. In the use of the bolometer (in principle so well known now as to need no description) the practise at Allegheny during the nearly ten years that studies of the infra-red spectrum of the sun were in progress there, required at least two and preferably three observers. One made and read the settings of the spectroscope, a second read as often as he could the indications of the galvanometer, while the third recorded all the observations and operated the shutter in front of the slit of the spectroscope. In those days the galvanometer, though far less sensitive than that now employed, was rarely free from "drift," or wandering of the spot of light upon the scale and this "drift" was usually so rapid that the spot of light would cross the whole scale within a few minutes.

It had been my intention at Allegheny to replace the tedious and incomplete system of eye observations at the galvanometer by an automatic photographic recording apparatus, but the "drift" stood as a seemingly insuperable obstacle in the way. The difficulty was attacked immediately after the installation of apparatus at the Astrophysical Observatory at Washington in 1891. Without stopping to mention details, it need only be said that by persistent efforts carried on through more than ten years as described in the early chapters of the Annals, and with the aid of several devices which the reader may find there illustrated, these obstacles have been removed and it has become possible to use the galvanometer with perfect success to produce a purely automatic record, in connection with the bolometer, and with a photographic recording apparatus.[2]

As thus perfected, the procedure of mapping a portion of the spectrum is this:

A prismatic spectrum is formed by a spectroscope of the so-called "fixed arm type," in which, as the reader will recall, the ray which is in minimum deviation is given a certain fixed direction by an optical method, so that the observing instrument remains fixed also. This observing instrument is here the bolometer, whose essential part is a Wheatstone's bridge, chiefly formed of two excessively narrow and thin strips of platinum, only one of which is exposed to the radiation to be studied. On looking into the eyepiece of the bolometer, this appears like a single spider web wire of a micrometer, seen against the background of the spectrum, for instance just below the *A* line, whose position might be read by it on the large azimuth circle on which the bolometer is mounted. But this thread is not alone a fiducial index, for since it conveys an electric current it is also sensitive and comparable to a nerve which will recognize the heat or cold which falls on it, and not only recognize but accurately measure it by the changes set up in the current in question.

The whole forms an electrical thermometer which actually measures to very much less than the millionth of a degree.

As the bolometer strip is warmed or cooled the galvanometer mirror turns to and fro, and the spot of light which it reflects passes to and fro on its scale, which is here a photographic plate. A clockwork of extreme accuracy moves the rock salt prism so that the spectrum marches uniformly across the bolometer thread at a rate, let us say, of one minute of arc in one minute of time, while the photographic plate moves as regularly in a vertical direction at right angles to the movements of the galvanometer spot at a rate, for example, of a centimeter in one minute of time. There is thus produced a curve called a bolograph, in which ordinates accurately correspond to relative amounts of energy, and abscissae to deviations in the prismatic spectrum. Depressions in this curve correspond to cooler lines or bands in the spectrum; the visible Fraunhofer lines, which are cool to the bolometer, appear as such deflections in the bolographs; the invisible lines, which are wholly insensible to the eye and chiefly insensible to the photographic plate, but which are recognized everywhere by the bolometer and discovered to fill the whole infra-red, are recorded also.

The first object of the principal research described in the Annals was to map out in detail the hitherto scarcely explored region of the solar spectrum between the limit of the visible, just below *A* at about 0.8μ and a wave length of 5.3μ, beyond which little energy reaches the earth from the sun. In the accompanying illustration, Plate I (similar to Plate XX of the Annals), two bolographs taken on different days are superposed to show the coincidence of their indications, which directly represent the solar energy in this (invisible) part of the spectrum, and beneath is a corresponding line spectrum drawn to show the region mapped in a more conventional aspect. At the left we see the comparative space occupied by the Newtonian or visible spectrum, on the scale of the average dispersion of the prism in the infra-red. Above is a curve obtained by Lamansky about thirty years ago, which gives the sum of the knowledge of the infra-red spectrum at that time, and was then justly regarded as a great triumph.

In the work of the Astrophysical Observatory over 700 lines

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[1] Washington, D. C., Government Printing Office, 1900.

[2] The "drift," long the great enemy to bolometric research, has become so much reduced that often it amounts to less than a single centimeter an hour, whereas in the old days a centimeter a minute would have been regarded as relative perfection.

STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.

By Prof. FRANK H. BIGELOW.

VI. CERTAIN MATHEMATICAL FORMULÆ USEFUL IN METEOROLOGICAL DISCUSSIONS.

THE NEED OF A STANDARD SYSTEM OF FORMULÆ.

There is a large number of mathematical papers that have been written by meteorologists in the exposition of various theories, which must be thoroughly considered by students who seek to go beyond a descriptive statement of the problems into a close examination of the principles upon which the solutions rest. The question arose at an early stage in my study of comparative meteorology as to the form in which such mathematical discussions should be presented to the public. To traverse the entire range of treatises and explain them in detail was clearly impracticable; to adopt an abstract mathematical synopsis, such as is found in Carr's or Laska's synopsis of pure mathematics, was to put too great a strain upon readers who are not specialists in mathematical meteorology. Finally it seemed to me to be a fair compromise to take the following course: (1) reduce the important papers to one common standard notation, and (2) make an analysis of the result in a sufficiently expanded form to enable a good reader to follow the series of equations without difficulty. The only step required to transform the contents of the mathematical compendium as given in chapters 10 and 11 of the International Cloud Report into a complete treatise on analytic meteorology is to supply such transition precepts as are usually placed between the formulæ to aid the thought. It is, however, a distinct advantage for a working use of the formulæ, to one who has once become familiar with such problems, to dispense with these explanatory sentences, which only take up space. A ready reference to the standard equations under each subject is quickly appreciated by any one who uses these formulæ in a practical way, just as one would use a mathematical table in computing. It is my purpose to complete such a collection of formulæ, in addition to the tables contained in my report on Eclipse Meteorology and Allied Problems, Weather Bureau Bulletin I, 1902, by appropriate tables covering the subjects, spherical harmonics, thermodynamics, and the kinetic theory of gases, because these are indispensable in meteorological studies. I have taken the opportunity in this connection to present several original sets of formulæ, which have an advantage in their applications to meteorological problems, and it is my purpose to call attention to some of them in this paper.

THE GENERAL EQUATIONS OF MOTION.

The methods of deriving the general equations of motion on the rotating earth, as presented in Ferrel's paper, "The motions of fluids and solids on the earth's surface," or in the standard treatises of hydrodynamics, are so complicated as to discourage all who are not expert mathematicians from an examination of the solution. The fact that Ferrel did not evaluate the total differential of inertia $\frac{d(u, v, w)}{dt}$, introduced an error into the equations contained in his "Mechanics and general motions of the atmosphere," United States Coast Survey Report, 1875, Appendix 20; this was eliminated in his "Recent advances in meteorology," Annual Report of the Chief Signal Officer, 1885, Appendix 71. There are no doubt many ways of solving this problem, but the following is original, as expanded from Table 75, International Cloud Report, and it leaves little to be desired in respect of simplicity and completeness.

(1) THE POLAR EQUATIONS OF MOTION ON THE ROTATING EARTH.

Using the notation already adopted in Paper II of this series,[1] we write the primary equations of acceleration of motion referred to axes which have their origin at the center of a nonrotating earth, as follows:

The accelerations due to motion and to external forces are:

155.
$$-\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\partial V}{\partial x} = \frac{du}{dt} - v\omega_3 + w\omega_2$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial y} - \frac{\partial V}{\partial y} = \frac{dv}{dt} - w\omega_1 + u\omega_3$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial V}{\partial z} = \frac{dw}{dt} - u\omega_2 + v\omega_1$$

where the angular velocities of motion for a point are

166.
$$\omega_1 = -\frac{v}{r} \qquad \omega_2 = +\frac{u}{r} \qquad \omega_3 = +\frac{v}{r \tan \theta}.$$

Compare diagram in my Report, page 498, or Basset, pages 13 and 14, noting the transformations of notation.

In case the earth rotates with the constant angular velocity n, carrying the fixed axes with it, the linear velocities (u, v, w) and the angular velocities $(\omega_1, \omega_2, \omega_3)$ are changed as follows, denoting these terms on the rotating earth with primes:

177.
$$u' = u \qquad v' = v + n\, r \sin\theta \qquad w' = w$$

178.
$$\omega'_1 = -\frac{v + n\, r \sin\theta}{r} \qquad \omega'_2 = +\frac{u}{r} \qquad \omega'_3 = +\frac{v + n\, r\sin\theta}{r\tan\theta}.$$

This is due to the fact that the rotation of the earth adds the velocity $n\, r \sin\theta = n\, \varpi$ to the eastward linear velocity, because ϖ is the perpendicular distance from the axis of rotation.

The differentials $\frac{du'}{dt}$, $\frac{dv'}{dt}$, $\frac{dw'}{dt}$ evaluate into,

179.
$$\frac{du'}{dt} = \frac{du}{dt}$$
$$\frac{dv'}{dt} = \frac{dv}{dt} + \frac{d\,(n\, r \sin\theta)}{dt} = \frac{dv}{dt} + u\, n \cos\theta + w\, n \sin\theta$$
$$\frac{dw'}{dt} = \frac{dw}{dt}$$

since $u = \frac{rd\theta}{dt}$ and $w = \frac{dr}{dt}$ by formulæ 153, page 497, of the International Cloud Report.

Substituting these values in the equations of motion for the rotating earth, which are the same as those of 155 with the letters all primed, and taking the equivalents of dx, dy, dz in polar cordinates from 153, we have:

180.
$$-\frac{1}{\rho}\frac{\partial P}{r\partial\theta} = \frac{du}{dt} - (v + n r \sin\theta)\frac{(v + n r \sin\theta)}{r\tan\theta} + w\frac{u}{r},$$
$$-\frac{1}{\rho}\frac{\partial P}{r\sin\theta\,\partial\lambda} = \frac{dv}{dt} + w\frac{(v + n\, r\sin\theta)}{r} + u\frac{(v + n\, r\sin\theta)}{r\tan\theta}$$
$$+ u\, n\cos\theta + w\, n\sin\theta,$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial r} - g = \frac{dw}{dt} - u\frac{u}{r} - (v + n r\sin\theta)\frac{(v + n r \sin\theta)}{r}.$$

The external forces derived from the potential V are:

$$-\frac{\partial V}{\partial x} = 0, \qquad -\frac{\partial V}{\partial y} = 0, \qquad -\frac{\partial V}{\partial z} = -g.$$

Performing the algebraic work, these equations reduce to

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[1] See Monthly Weather Review for February, 1902, Vol. XXX, p. 81.

181.
$$-\frac{1}{\rho}\frac{\partial P}{r\partial\theta} = \frac{du}{dt} - \frac{v^2\cot\theta + uw}{r} - 2n\cos\theta\,.\,v - r\,n^2\sin\theta\cos\theta\,,$$
$$-\frac{1}{\rho}\frac{\partial P}{r\sin\theta\partial\lambda} = \frac{dv}{dt} + \frac{uv\cot\theta + vw}{r} + 2n\cos\theta\,.\,u + 2n\sin\theta\,.\,w\,,$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial r} - g = \frac{dw}{dt} - \frac{u^2 + v^2}{r} - 2n\sin\theta\,.\,v - rn^2\sin^2\theta.$$

The successive terms are the inertia, the centrifugal forces, the deflecting force, and the forces which change the figure of the earth from a sphere into an elipsoid of revolution.

(2) THE CYLINDRICAL EQUATIONS OF MOTION ON THE ROTATING EARTH.

If the axis of rotation of the earth is taken as the axis of rotation in cylindrical coordinates, the tangential velocity $= v + n\varpi$; but if the axis of rotation is any radius of the earth extended above the surface, the tangential velocity becomes $= v + n\,\varpi\cos\theta$. Hence we have, in cylindrical coordinates,

182.
$$u' = u \qquad w'_1 = 0$$
$$v' = v + n\,\varpi\cos\theta \qquad w'_2 = 0$$
$$w' = w \qquad w'_3 = n\cos\theta + \frac{v}{\varpi}.$$

The differentials $\frac{du'}{dt}, \frac{dv'}{dt}, \frac{dw'}{dt}$ evaluate into,

183.
$$\frac{du'}{dt} = \frac{du}{dt}$$
$$\frac{dv'}{dt} = \frac{dv}{dt} + \frac{d\,(n\,\varpi\cos\theta)}{dt} = \frac{dv}{dt} + u\,n\cos\theta$$
$$\frac{dw'}{dt} = \frac{dw}{dt}$$

since $u = \frac{d\varpi}{dt}$, by formulæ 152, and $\cos\theta$ is a constant. Substituting these values in the equations of motion for the rotating earth, which are the same as those of 155, with the letters all primed, and taking the equivalents of *dx, dy, dz,* in cylindrical coordinates from 152, we have:

184.
$$-\frac{1}{\rho}\frac{\partial P}{\partial\varpi} = \frac{du}{dt} - (v + n\,\varpi\cos\theta)\left(n\cos\theta + \frac{v}{\varpi}\right)$$
$$-\frac{1}{\rho}\frac{\partial P}{\varpi\partial\varphi} = \frac{dv}{dt} + u\,n\cos\theta + u\left(n\cos\theta + \frac{v}{\varpi}\right)$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} - g = \frac{dw}{dt}.$$

The external forces derived from the potential *V* are:
$$-\frac{\partial V}{\partial x} = 0, \qquad -\frac{\partial V}{\partial y} = 0, \qquad -\frac{\partial V}{\partial z} = -g.$$

Performing the algebraic work, these equations reduce to

185.
$$-\frac{1}{\rho}\frac{\partial P}{\partial\varpi} = \frac{du}{dt} - 2n\cos\theta\,.\,v - \frac{v^2}{\varpi}$$
$$= \frac{du}{dt} - (2n\cos\theta + \nu_1)\,v$$
$$-\frac{1}{\rho}\frac{\partial P}{\varpi\partial\varphi} = \frac{dv}{dt} + 2n\cos\theta\,.\,u + \frac{uv}{\varpi}$$
$$= \frac{dv}{dt} + (2n\cos\theta + \nu_1)\,u$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial r} - g = \frac{dw}{dt}$$

where the term $+\,\varpi\,n^2\cos^2\theta$ is neglected in the first equation, and $\nu_1 = \frac{v}{\varpi}$ the relative angular velocity.

The successive terms are the inertia, the deflecting force, and the centrifugal forces.

REMARKS ON THE SEVERAL TERMS IN THE GENERAL EQUATIONS OF MOTION.

It is customary to add to the terms developed in a frictionless medium, a term expressing the retardation of acceleration due to friction, either in Ferrel's form $+\,k\,(u, v, w)$, which is proportional to the velocity and expresses a sliding friction, or in Oberbeck's form $\frac{k}{\rho}\,\Delta^2\,(u, v, w)$, which expresses a retardation proportional to the turbulent internal resistances of a mixing fluid. This function is hard to evaluate on account of the uncertainty which attaches to the invisible internal motions, and to the effect of discontinuous surfaces separating different velocities and temperatures. Near the ground turbulent motions and large coefficients of friction up to about 300–500 meters are required; above this level and especially in the higher strata the coefficient of friction is very small.

The inertia terms $\frac{d\,(u, v, w)}{dt}$ disappear in steady motion, and they are small in slow changes of velocities. There are, however, cases in which inertia may amount to a considerable quantity, as where a tornado, in passing along its path, sucks in new masses of air, and transforms them suddenly from rest into excessively rapid motion. Also, when the cyclonic vortex raises masses of air from strata having slow motion into strata of rapid velocities; but especially where countercurrents meet, and the stream lines are bent and reflexed in their direction.

These two terms, friction and inertia, act in the path of motion and they directly affect the quantity of kinetic energy possessed by the elementary masses. All forces which act at right angles to the path, such as the centrifugal and the deflecting forces, do not change the momentum, but they do alter the direction of the path. Hence, in integrating for the kinetic energy in an orbit, or in a circuit, the centrifugal and the deflecting forces drop out of the equations, but they must be retained when discussing the angle that the stream line makes with the isobars, which angle expresses the influence of the velocity potential function on the motion. The following integration of the general equations will establish these propositions.

INTEGRATION OF THE GENERAL EQUATIONS OF MOTION IN POLAR COORDINATES.

Make the following substitutions in 181:

197.
$$\frac{v}{r} = \nu\sin\theta$$
$$\frac{v^2\cot\theta}{r} = v\cos\theta\,.\,\nu$$
$$\frac{u\,v\cot\theta}{r} = u\cos\theta\,.\,\nu$$

and neglect the terms in n^2, which are very small, with the result that,

200.
$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{du}{dt} - \cos\theta\,(2n + \nu)\,v + \frac{uw}{r}$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{dv}{dt} + \cos\theta\,(2n + \nu)\,u + \sin\theta\,(2n + \nu)\,w$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{dw}{dt} - \sin\theta\,(2n + \nu)\,v - \frac{u^2}{r} + g.$$

Now multiply these equations respectively by *dx, dy, dz,* and remember that $v\partial x = u\partial y$, $w\partial x = u\partial z$, $w\partial y = v\partial z$; take the sum of the partial differentials, the result being,

203. $$-\frac{\partial P}{\rho} = u\partial u + v\partial v + w\partial w + g\partial z.$$

The integral of this is,

$$\int -\frac{\partial P}{\rho} = \tfrac{1}{2}(u^2 + v^2 + w^2) + gz + \text{const.} = \tfrac{1}{2}q^2 + gz + \text{const.}$$

This is the fundamental equation of steady motion found in all treatises on hydrodynamics; its discussion is carried on in Table 81, International Cloud Report. The centrifugal and the deflecting forces have disappeared, and the integral is equivalent to the kinetic energy, $\frac{1}{2}q^2$, plus the external force due to the acceleration of gravitation. An arbitrary term may be added to express the frictional retardation.

If the integration is between two points of a fluid that has the same density throughout, the term $\int -\frac{\partial P}{\rho} = -\frac{P}{\rho}$ simply. Such lines of homogeneous integration may be found by observing the surfaces of equal density in the atmosphere, or, a mean density between two points may be assumed in place of the existing variable density. If the velocity term $\frac{1}{2}q^2$ is neglected, we obtain $dP = -g\rho dz$, and this is the simple form from which the usual hypsometric formulæ for barometric reductions are derived. Compare formulæ 54, Table 66, p. 490.

It is noted, however, that the usual method employed in static barometric reductions is incomplete, and that the velocity term $\frac{1}{2}(q^2 - q_0^2)$, where q, q_0 are the observed velocities at the two points limiting the path of integration, has been omitted.

If the integration is continued in any closed circuit the gravity term disappears from the equation, and the velocity terms alone remain. This line integral (C) measures the work done in moving the unit mass once around the circuit, while $A = \frac{dC}{dt}$ is the rate of doing the work, or the activity. From this point of view the circulation of the atmosphere may be treated by the ordinary theory of the line integral. It is more convenient to observe the velocities than the pressures and densities around a circuit, in the present state of meteorology.

EXPRESSIONS FOR THE GRADIENTS OF PRESSURE.

If we take the formulæ for acceleration, Cloud Report, page 499,

155. $$\dot{u}_1 = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\partial V}{\partial x}$$

$$\dot{v}_1 = -\frac{1}{\rho}\frac{\partial P}{\partial y} - \frac{\partial V}{\partial y}$$

$$\dot{w}_1 = -\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial V}{\partial z}$$

we can write for the gradient,[1]

501. $$G_x = -\frac{\partial P}{\partial x} - \rho\frac{\partial V}{\partial x} = \rho\,\dot{u}_1$$

$$G_y = -\frac{\partial P}{\partial y} - \rho\frac{\partial V}{\partial y} = \rho\,\dot{v}_1$$

$$G_z = -\frac{\partial P}{\partial z} - \rho\frac{\partial V}{\partial z} = \rho\,\dot{w}_1.$$

The gradient terms, $-\rho\frac{\partial V}{\partial x}$ in latitude, and $-\rho\frac{\partial V}{\partial y}$ in longitude, are small terms, while $-\rho\frac{\partial V}{\partial z}$ is the principal term, and these are due to the attraction of the earth upon the atmosphere. The terms $-\frac{\partial P}{\partial x}$ in latitude, $-\frac{\partial P}{\partial y}$ in longitude, and $-\frac{\partial P}{\partial z}$ in altitude are the gradients due to the thermal disturbance of the isobaric surfaces, the first two being the gradient terms producing the horizontal flow of the atmosphere, and the last one the term which causes the up and down movement of the atmosphere by the variations of the normal buoyancy from that of stable equilibrium as controlled by the static terms in the potential function for external force V.

It is next important to evaluate the gradient terms for use in practical meteorology. There are many ways of doing this, as is indicated by the collection of formulæ in Table 65, page 489, of the International Cloud Report. There is a generally accepted convention which is adopted as the basis for the practical measures of gradients by the mercurial barometer.

Thus, the difference of barometric pressure, G, at two points which are 111 111 meters apart in a horizontal direction, is taken as the standard for reductions.

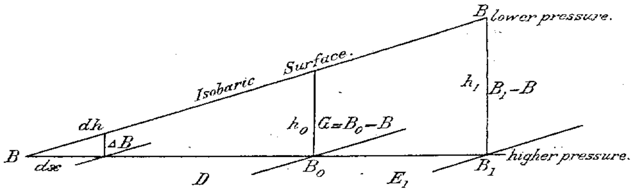

FIG. 21.—Vertical section through the atmosphere.

In fig. 21, which shows a vertical section through the atmosphere, let

D = 111 111 meters = 1° on surface of the earth,
dx = 1 meter,
E = any distance between given points of observation.

Then,

502. $$\frac{B_1 - B}{E_1} = \frac{B_0 - B}{D} = \frac{G}{D} = \frac{G}{111\,111} = \frac{\Delta B}{dx};$$

$$G = (B_1 - B)\frac{D}{E_1}.$$

It is necessary first to find G from the observed values of B at two stations at the distance E_1 from each other.

EVALUATION OF THE COEFFICIENT $\frac{dh}{dB}$ AND OTHER TERMS.

If the change in elevation of the isobaric surface is as follows: h_1 at distance E_1, h_o at distance D, dh at distance dx, then,

503. $g\frac{h_1}{E_1} = g\frac{h_o}{D} = g\frac{dh}{dx} = g\tan a = gb$ measures the acceleration.

Also by the law of falling bodies, $v = \sqrt{2gh}$ for the velocity.

I. We have,

504. $\frac{dh}{dx} = \frac{h_o}{D} = \frac{h_1}{E_1} = \frac{l}{E_l}$ for the top of the homogeneous atmosphere.

505. $\frac{dB}{dx} = \frac{B_o - B}{D} = \frac{G}{D} = \frac{G}{111\,111} = \frac{\partial P}{\partial x}\cdot\frac{1}{g_o\rho_m}$, from 24, p. 487 of the Cloud Report.

Hence,

506. $$\frac{dh}{dB} = \frac{h_o}{B_o - B} = \frac{l}{E_l}\frac{D}{G} = \frac{l}{E_l}\frac{E_1}{B_l - B} = \frac{l}{E_l}\frac{E_l}{B_l - B} = \frac{l}{B_l} = \frac{RT}{B_l}$$

$$\text{since } \frac{D}{G} = \frac{E_1}{B_1 - B} = \frac{E_l}{B_l - B} = \frac{E_l}{B_l},$$

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[1] The series of equations beginning with 501 may be considered as an extension of the system given in the International Cloud Report, which ends on page 603.

because B at the top of the homogeneous column l is negligible compared with B_l at the bottom of it. B_l is in this connection the barometric pressure at the surface, and $B_l = B_n = 0.760$ meter.

II. We have by 50, page 490, for the standard weight of the atmosphere,

507. $$p_o = \sigma h = \sigma_o l = \sigma_m B_n. \quad \text{Hence,}$$

508. $$h = \frac{\sigma_o}{\sigma} l = \frac{\sigma_m}{\sigma} B_n. \quad \text{That is, } h = l \text{ for } \sigma = \sigma_o. \quad \text{Hence,}$$

509. $$h = l = \frac{\sigma_m}{\sigma_o} B_n. \quad \text{Therefore,}$$

510. $$s = \frac{\sigma_m}{\sigma_o} = \frac{l}{B_n} = \frac{RT}{B_n} = \frac{13{,}595.8}{1.29305} = \frac{7{,}991.04}{0.760} = 10{,}514.5.$$

511. $$dh = \frac{\sigma_m}{\sigma_o} dB = \frac{RT}{B_n} dB = 10{,}514.5 \; dB = s \; dB.$$

512. $$\frac{dh}{dx} = 10{,}514.5 \frac{B_o - B}{D} = 10{,}514.5 \frac{G}{111\,111} = 0.09463 G.$$

III. Let $\Gamma =$ the gradient force per meter; that is, for $dx = 1$.

513. $\Gamma = \Delta P = g_o \rho_m \Delta B$ in terms of the units of force P.

514. $\Gamma = \Delta p = \sigma_m \Delta B$ in terms of the units of weight p.

The gradient force changes with the temperature.

Let $\Gamma_o =$ the gradient force for $T_o = 273°$ C. and $B_n = 0.760$ meter.

$\Gamma =$ the gradient force for T and B.

515. $$\Gamma = \Gamma_o \frac{T}{T_o} \frac{B_n}{B}.$$

IV. To evaluate $-\frac{1}{\rho}\frac{\partial P}{\partial x}$ and $-\frac{1}{\rho}\frac{\partial p}{\partial x}$:

516. We have $P_o = g_o \rho_m B_n$; and hence,

517. $$-\frac{1}{\rho}\frac{\partial P}{\partial x} = -\frac{1}{\rho} g_o \rho_m \frac{\partial B}{\partial x} = -\frac{1}{\rho}\frac{g_o \rho_m}{111\,111} G = -\frac{0.0012 G}{\rho}.$$

(G is in meters.)

518. Also, we have $p_o = \sigma_m B_n$; and hence,

519. $$-\frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{1}{\rho}\sigma_m \frac{\partial B}{\partial x} = -\frac{1}{\rho}\frac{\sigma_m}{111\,111} G = 0.12237 G.$$

(G is in meters.)

Numerous other evaluations of $-\frac{1}{\rho}\frac{\partial P}{\partial x}$ are given in Table 65, p. 489, of the Cloud Report.

EVALUATION OF THE GRADIENTS IN POLAR COORDINATES.

Since the angular velocity of the rotating earth is $n \sin \theta = \frac{v'}{r}$, where v' is the absolute eastward velocity, and $r = 6{,}370{,}191 + h$ meters, we have $n = 0.00007292$, and also $n \cos \theta = \frac{v' \cot \theta}{r}$, in which r can be taken practically equal to R. The general polar equations of motion become, by substituting these values in 181,

194. $$-\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{du}{dt} - \frac{\cot \theta}{r}(2v' + v)v + \frac{uw}{r}$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{dv}{dt} + \frac{\cot \theta}{r}(2v' + v)u + (2v' + v)\frac{w}{r}$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{dw}{dt} - \frac{1}{r}(2v' + v)v - \frac{u^2}{r} + .g.$$

The terms in n^2 which give the figure to the rotating earth have been omitted, and the inertia terms become equal to zero for steady motions of the atmosphere; also, for all except computations of great precision the terms in w can be neglected.

To evaluate the acceleration $\frac{1}{\rho}\frac{\partial P}{\partial x}$, we have, first, from 47,

47a. $$\frac{1}{\rho} = \frac{1}{\rho_o}\frac{P_o}{P}\frac{T}{T_o} = \frac{1}{\rho}\frac{P_o}{P}(1 + \alpha t)$$

$$= \frac{1}{\rho_o}\frac{B_n}{B}\frac{T}{T_o}\frac{1}{n_1}, \text{ for variations of gravity, since } g = g_o n_1,$$

$$= \frac{1}{\rho_o}\frac{760}{273}\frac{T}{B} \quad \text{for constant gravity and } B \text{ in mm.}$$

From the formulae on page 489,

47b. $$\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{1}{\rho} g_o \rho_m \frac{dB}{dx}, \text{ and since } \frac{dB}{dx} = \frac{G_x}{D} = \frac{G_x}{111\,111} \text{ in meters}$$

$$= \frac{1}{\rho}\frac{g_o \rho_m}{111\,111} G_x \text{ for the gradient measured in meters}$$

$$= \frac{1}{\rho}\frac{g_o \rho_m}{111\,111\,111} G_x \text{ for the gradient } G_x = B_o - B \text{ in mm}$$

$$= \frac{1}{\rho_o}\frac{B_n}{T_o}\frac{T}{B}\frac{g_o \rho_m}{D} G_x$$

520. $$= \frac{13.5958}{0.00129305} \times \frac{760}{273} \times \frac{9.806}{111\,111\,111} \times \frac{T}{B} \times G_x$$

521. $$= 0.0025833 \frac{T}{B} G_x$$

522. $$= \frac{\cot \theta}{r}(2v' + v)v, \text{ by equation 194.} \quad \text{Hence,}$$

523. $$G_x = +\,387.102 \frac{B}{T}\frac{\cot \theta}{r}(2v' + v)v,$$

and similarly,

$$G_y = -\,387.102 \frac{B}{T}\frac{\cot \theta}{r}(2v' + v)u,$$

$$G_z = +\,387.102 \frac{B}{T}\left[\frac{1}{r}(2v' + v)v + \frac{u^2}{r} - g\right].$$

Since v' is a function of θ, that is, $v' = nr \sin \theta$, these terms can be computed by simple tables, such as those in Tables 104, 105, 106, of the International Cloud Report, where some of the terms are evaluated. By expressing the variation of $387.102 \times \frac{B}{T}$ in a table with B and T as the arguments, the several products can be quickly computed.

Examples:

I. For $B = 700$ mm. and $T = 260°$ C., $\frac{B}{T} = 2.6923$;

For $\theta = 30°$ north polar distance, $2v' = 464.5$ meters per second;

For $v = 40$ meters per second, $(2v' + v)\,v = 20{,}180$.

Hence, $G_x = 387.102 \frac{B}{T}\frac{\cot \theta}{R}(2v' + v)\,v = 5.71$ millimeters per 111 111 meters.

II. This latter has been computed from the tables as follows:

$$378.102 \frac{B}{T} = 1{,}042.2;$$

$$\frac{v'}{R} \cot \theta \,.\, 2v = 0.005052, \text{ by Table 104;}$$

$$\frac{\cot \theta}{R} \,.\, v \,.\, v = 0.000435, \text{ by Table 106.}$$

The sum of these is $\frac{\cot \theta}{R}(2v' + v)v = 0.005487$. Hence the product, $G_x = 1{,}042.2 \times 0.005487 = 5.71$ millimeters per 111 111 meters. Similarly, the gradients G_y and G_z can be computed.

For these values of B and T, we find in other examples,

$$\left.\begin{array}{l}\theta = 40°\\ v = 45°\end{array}\right\} G_x = 6.89 \quad \left.\begin{array}{l}\theta = 50°\\ v = 50°\end{array}\right\} G_x = 5.23 \quad \left.\begin{array}{l}\theta = 60°\\ v = 55°\end{array}\right\} G_x = 4.47.$$

203. $$-\frac{\partial P}{\rho} = u\partial u + v\partial v + w\partial w + g\partial z.$$

The integral of this is,

$$\int -\frac{\partial P}{\rho} = \tfrac{1}{2}(u^2 + v^2 + w^2) + gz + \text{const.} = \tfrac{1}{2}q^2 + gz + \text{const.}$$

This is the fundamental equation of steady motion found in all treatises on hydrodynamics; its discussion is carried on in Table 81, International Cloud Report. The centrifugal and the deflecting forces have disappeared, and the integral is equivalent to the kinetic energy, $\frac{1}{2}q^2$, plus the external force due to the acceleration of gravitation. An arbitrary term may be added to express the frictional retardation.

If the integration is between two points of a fluid that has the same density throughout, the term $\int -\frac{\partial P}{\rho} = -\frac{P}{\rho}$ simply. Such lines of homogeneous integration may be found by observing the surfaces of equal density in the atmosphere, or, a mean density between two points may be assumed in place of the existing variable density. If the velocity term $\frac{1}{2}q^2$ is neglected, we obtain $dP = -g\rho dz$, and this is the simple form from which the usual hypsometric formulæ for barometric reductions are derived. Compare formulæ 54, Table 66, p. 490.

It is noted, however, that the usual method employed in static barometric reductions is incomplete, and that the velocity term $\frac{1}{2}(q^2 - q_0^2)$, where q, q_0 are the observed velocities at the two points limiting the path of integration, has been omitted.

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we can write for the gradient,[1]

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The gradient terms, $-\rho\frac{\partial V}{\partial x}$ in latitude, and $-\rho\frac{\partial V}{\partial y}$ in longitude, are small terms, while $-\rho\frac{\partial V}{\partial z}$ is the principal term, and these are due to the attraction of the earth upon the atmosphere. The terms $-\frac{\partial P}{\partial x}$ in latitude, $-\frac{\partial P}{\partial y}$ in longitude, and $-\frac{\partial P}{\partial z}$ in altitude are the gradients due to the thermal disturbance of the isobaric surfaces, the first two being the gradient terms producing the horizontal flow of the atmosphere, and the last one the term which causes the up and down movement of the atmosphere by the variations of the normal buoyancy from that of stable equilibrium as controlled by the static terms in the potential function for external force V.

It is next important to evaluate the gradient terms for use in practical meteorology. There are many ways of doing this, as is indicated by the collection of formulæ in Table 65, page 489, of the International Cloud Report. There is a generally accepted convention which is adopted as the basis for the practical measures of gradients by the mercurial barometer.

Thus, the difference of barometric pressure, G, at two points which are 111 111 meters apart in a horizontal direction, is taken as the standard for reductions.

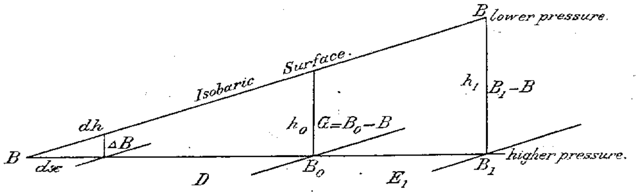

FIG. 21.—Vertical section through the atmosphere.

In fig. 21, which shows a vertical section through the atmosphere, let

D = 111 111 meters = 1° on surface of the earth,
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E = any distance between given points of observation.

Then,

502. $$\frac{B_1 - B}{E_1} = \frac{B_0 - B}{D} = \frac{G}{D} = \frac{G}{111\,111} = \frac{\Delta B}{dx};$$
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It is necessary first to find G from the observed values of B at two stations at the distance E_1 from each other.

EVALUATION OF THE COEFFICIENT $\frac{dh}{dB}$ AND OTHER TERMS.

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Also by the law of falling bodies, $v = \sqrt{2gh}$ for the velocity.

I. We have,

504. $$\frac{dh}{dx} = \frac{h_o}{D} = \frac{h_1}{E_1} = \frac{l}{E_l}$$ for the top of the homogeneous atmosphere.

505. $$\frac{dB}{dx} = \frac{B_o - B}{D} = \frac{G}{D} = \frac{G}{111\,111} = \frac{\partial P}{\partial x}\cdot\frac{1}{g_o\rho_m},$$ from 24, p. 487 of the Cloud Report.

Hence,

506. $$\frac{dh}{dB} = \frac{h_o}{B_o - B} = \frac{l}{E_l}\frac{D}{G} = \frac{l}{E_l}\frac{E_1}{B_l - B} = \frac{l}{E_l}\frac{E_l}{B_l - B} = \frac{l}{B_l} = \frac{RT}{B_l}$$

$$\text{since } \frac{D}{G} = \frac{E_1}{B_1 - B} = \frac{E_l}{B_l - B} = \frac{E_l}{B_l},$$

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[1] The series of equations beginning with 501 may be considered as an extension of the system given in the International Cloud Report, which ends on page 603.

We are at last in a position to examine the system of gradients in the United States on the 10,000-foot plane and on the 3,500-foot plane. For we have obtained by the nephoscope and theodolite observations as given in the Cloud Report a large number of corresponding values of u and v, which enter these equations. The values of B and T on these planes have been carefully determined for each month, and also the gradients by which such values can be determined at any time. This will enable us to discuss the effect of friction at these planes, by means of the residuals which occur between the values of G as found by these formulæ and those read off from the charts of isobars contained in the Barometry Report of 1900–1901.

Furthermore, our Weather Bureau stations will soon be provided with suitable tables for computing pressures on the 3,500-foot and the 10,000-foot planes, and this will give daily configurations of isobars on these two levels. If, in addition, we had measures of the velocity of the clouds, $q\ (u, v, w)$, above each station by means of nephoscopic observations, it would enable us to make complete dynamic computations of the forces acting in cyclones and anticyclones, as is seen by an inspection of the formulæ.

Since the tabular computations are constructed for average conditions, it is of the utmost importance that *check observations* be made on these two planes in order to control these dynamic discussions and make them more perfect. Such observations can be made by balloon ascensions up to 2 or 3 miles, or by kite ascensions up to 10,000 feet, or by certain computations on cloud observations.

It seems to me very clear that a series of suitable research explorations would soon result in placing our dynamic meteorology upon a satisfactory scientific basis, and put an end to the fruitless speculations which have done so little to advance our knowledge of the laws of the atmospheric motions.

THE EQUATION OF CONTINUITY, AND SOME DERIVED RELATIONS.

1. The equation of continuity can be found as follows:

Consider a cylinder of the height z and radius ϖ, into which air of the density ρ streams equally from all sides with the velocity $-u$, since the direction is negative.

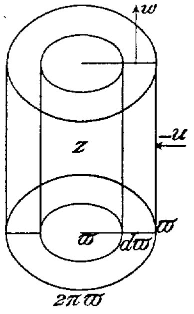


FIG. 22.—Illustrating the formation of the equation of continuity.

The amount of instreaming air in the unit of time is

$$-\,2\pi\omega\ z\ u\rho.$$

If the air is incompressible, then there will stream into a cylinder, whose radius is smaller by $d\varpi$, the amount $-\,2\pi\ (\varpi - d\varpi)\ zu\rho$, and at the same time there will escape upward between these two cylinders the amount $2\pi\varpi\ d\varpi\ w\rho$. Hence

524. $$-\,2\pi\varpi\, z\, u\rho + 2\pi\ (\varpi - d\varpi)\ z\ u\rho = -\,2\pi d\varpi\ zu\rho$$
$$= 2\pi\varpi d\varpi\ w\rho.$$

Integrating along the entire radius from 0 to ϖ, we have,

$$-\,2\pi\rho\int_0^{\varpi} zud\varpi = 2\pi\rho\int_0^{\varpi}\varpi d\varpi\ w.$$

Therefore, $-\,zu\varpi = \frac{1}{2}\varpi^2 w$, and the equation of continuity becomes

487. $$-\,2uz = \varpi w.$$

This applies to pure vortex motion, and it finds some examples in the atmosphere, such as in tornadoes, in many hurricanes, and in some highly developed cyclones.

It may be remarked that in treating of the general circulation of the atmosphere, the application of the pure vortex law $\varpi v =$ constant, has failed to give correct results, for example in the writings of Ferrel, von Helmholtz, Oberbeck, Sprung, and others. This leads to the theory of contracting rings on the earth with progressive motion towards the poles, or expanding rings with progression towards the equator. While the law of the sum of the momenta $\Sigma\, mv = 0$ must prevail, the rings are in nature broken up into such complex stream lines as to render integration by the simple vortex law too rough and ready a method. We must, therefore, study the theory of typical stream lines, before attempting any general integration for the entire circulation.

The following derived relations are convenient:

2. Since $-\,2uz = \varpi w$, we have $-\,u = \frac{w}{2z}\ \varpi$

$$= \frac{c}{2}\ \varpi, \text{ if } w = cz.$$

3. For z and $-\,u$ both constant, we have $\varpi zu = -$ const. $= \psi$.

Hence, $z = -\,\frac{\text{const.}}{u\varpi} = -\left(\frac{\text{const.}}{c/2}\right)\frac{1}{\varpi^2}$, and by differentiation,

$$dz = +\,\frac{\text{const.}}{c/2}\cdot\frac{2\varpi d\varpi}{\varpi^4} = -\ \varpi^2 z\,\frac{2d\varpi}{\varpi^3}$$

$$= -\,2z\,\frac{d\varpi}{\varpi}.\quad \text{Therefore,}$$

$$\frac{dz}{z} = -\ 2\,\frac{d\varpi}{\varpi};\ \text{also, } \varpi\,\frac{dz}{dt} = -\ 2z\,\frac{d\varpi}{dt}.$$

4. These give the form for the current function ψ, and the velocity potential φ, in two cases.

488. I. $\psi = u\varpi z = -\,\frac{c}{2}\,\varpi^2\, z = \varphi z.$ II. $\psi = -\,cz.$

489. $\varphi = -\,\frac{c}{2}\,\varpi^2.$ $\varphi = -\,c.$

$a = \frac{\lambda}{k-c}.$ $a = \frac{\lambda}{k}.$

5. If the current function is modified through a deflecting force and also through friction, then the equation of motion has two solutions, so that the roots of

$$u\ \frac{\partial v}{\partial\varpi} + \frac{uv}{\varpi} + \lambda u + kv = 0$$

are, by 438,

1. $u = -\ \frac{c}{2}\,\varpi.$

$$v = +\ \frac{\lambda}{k-c}\,\frac{c}{2}\,\varpi = -\,\frac{\lambda}{k-c}\,u.$$

2. $u = -\ \frac{c}{\varpi}.$

$$v = +\ \frac{\lambda}{k}\,\frac{c}{2} = -\,\frac{\lambda}{k}\,u.$$

In obtaining the velocities of the rotation, v, we can modify the current function, as follows, namely, multiply by

$$a = \frac{\lambda}{k-c},\ \text{and } a = \frac{\lambda}{k} \text{ in the two cases.}$$

6. Hence, by using Stokes's current functions, we find for the velocities u, v, w in the two cases,

490. Case I.
$$u = + \frac{1}{\varpi}\frac{\partial\psi}{\partial z} = \frac{\varphi}{\varpi} = -\frac{c}{2}\varpi.$$
$$v = + \frac{a\psi}{\varpi} = -\frac{\lambda z}{k-c}\frac{c}{2}\varpi = +\frac{\lambda}{k-c}z\,u.$$
$$w = -\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi} = +c\dot{z} = -\frac{2u}{\varpi}z = -\frac{2v}{\varpi}\frac{k-c}{\lambda}.$$

Case II.
$$u = + \frac{1}{\varpi}\frac{\partial\psi}{\partial z} = \frac{\varphi}{\varpi} = -\frac{c}{\varpi}.$$
$$v = + \frac{a\psi}{\varpi} = -\frac{\lambda}{k}\frac{c}{\varpi}z.$$
$$w = -\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi} = 0.$$

7. In unconstrained motion the vortex law of preservation of areas is

$$v\varpi = \frac{c}{2}\varpi^2 z = 2g\frac{\varpi z}{v} = \varpi u z = \text{constant, by 307.}$$
$$w\pi\varpi^2 = \text{const., by 308, introducing the value } v^2 = 2gz.$$

This vortex law when not modified by deflection and friction becomes,

492. Case I.
$$v\varpi = \frac{\lambda}{k-c}\frac{c}{2}\varpi^2 z = -\text{const.}$$

Case II.
$$v\varpi = \frac{\lambda}{k}cz = -\text{const.}$$

8. The inclination of the stream line to the isobars is,

491. Case I.
$$\cot i = + \frac{\lambda}{k-c}z.$$

Case II.
$$\cot i = \frac{\lambda}{k}z.$$

9. The equation of continuity (163) is satisfied by these following values:

493.
$$\frac{\partial u}{\partial\varpi} + \frac{u}{\varpi} + \frac{\partial w}{\partial z} = -\frac{c}{2} - \frac{c}{2} + c = 0.$$

10. The equation for gradient has a term to express the unevaluated variation due to temperature effects, $f(t_x)$, and it becomes, for the radial component,

494.
$$-\frac{\mu}{\rho}G_x = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$
$$= -\frac{c}{2}\varpi\left[k-c+\lambda\frac{\lambda z}{k-c} + \frac{c}{2}\left(\frac{\lambda z}{k-c}\right)^2\right] + f(t_x)$$
$$= -\frac{c}{2}\varpi\left[k-c+\lambda\cot i + \frac{c}{2}\cot^2 i\right] + f(t_x).$$

11. The total velocity is

495.
$$q^2 = u^2 + v^2 + w^2 = \frac{u^2}{\sin^2 i} + w^2 = u^2\left(\frac{1}{\sin^2 i} + \frac{4z^2}{\varpi^2}\right) + f(t_x).$$

12. The variation of pressure can be expressed by

496.
$$\log P_o - \log P = \frac{(q^2 - q_o^2) + 2g(z - z_o)}{360862\ (1 + at)} + f_1(k) + f_2(t).$$

These formulæ are all collected in Table 121, page 602, of my International Cloud Report.

This system of formula applies directly only to the pure vortex motions that satisfy the assumed current function and velocity potential. The components u, v, w, are so simply interrelated that it is usually possible to make enough observations of some sort from which to derive all the other vortex relations. Applications of them were made in the International Cloud Report to two cases; (1) The waterspout observed off Cottage City, Marthas Vineyard, Mass., August 19, 1896, on page 633; upon this important formation, a fuller report will be published. (2) The average velocities in a cyclone from the data in Table 126, as given on page 629 of the International Cloud Report. The outcome of these computations is to show that the natural stream lines of the atmosphere conform on the average to these formulæ. There are, however, wide divergences of such a type as to indicate that the pure vortex motion is seriously modified by several conflicting forces, and that the true problems for the meteorologist consist in discovering the nature and amount of these deviations of the currents of the atmosphere from the simple laws. This is in fact a task of great difficulty, but it has now become evident what should be the course of scientific development for meteorology. There is little use in a further discussion of the general theorems at the present time, but there is great need of procuring the right kind of observations for use in such problems. The Weather Bureau has accordingly been engaged in such a reconstruction of its data as will contribute to the solution of these problems for the United States. We have already published a large number of nephoscope velocities for the eastern half of the country; the velocities of the upper currents for the West Indies have been determined for about three years, July 1899–July 1902, and their computation will be commenced at once; similar nephoscope observations will be undertaken for the Rocky Mountain and Pacific districts, beginning about July 1902. Our barometric observations have been thoroughly reduced for the years 1873 to the present time, and the tables necessary for reductions to the three reference planes are in hand for the construction of daily maps at three levels, containing the system of isobars corresponding with them. It will be necessary to revise the temperature and vapor tension observations and reduce them to homogeneous systems before our data will be complete for the application of the theoretical equations to the observational data. It is desirable to put an end to general mathematical speculation in meteorology, and to substitute for it definite comparisons between observations and computations together with dependent solutions for the outstanding unknown quantities.

THE PROBLEMS OF THE AQUEOUS VAPOR CONTENTS OF THE ATMOSPHERE.

I shall allow myself only a few remarks regarding the methods which were used in my report for the discussion of the various complicated problems that concern the aqueous vapor contents of the atmosphere, because the details are too complex for a brief summary like this, and also because the work was given in such an extended form as to enable students to follow it without difficulty. There are, however, a few leading ideas to which attention may be especially directed, as they serve for an introduction to the subject in general.

There is collected in the International Cloud Report, Table 64, "Fundamental constants," a series of elementary constants in the English and metric systems, with the logarithms of the constants, and also a set of elementary formulæ which are most useful in meteorological studies. They cover nearly all the simple relations which constantly recur in manifold forms in the treatises and papers on meteorological subjects, and by transformation and combination a multitude of different relations can be readily obtained. Tables 63, 64, and 65 supply the basis for much descriptive matter commonly found in treatises, in so compact and accurate a form as to quite supersede the lengthy statements with which the same laws are usually presented, and this is a great convenience for the student and computer. Those who will take the trouble to become familiar with these tables will find much saving of time in general work, and also they will be guarded from such errors of thought and statement as are likely to occur from not having these formulæ in mind, or accessible for convenient reference.

In treating the vapor problems I have referred all the formulæ to the ratio $\frac{e}{B}$, vapor tension divided by barometric pressure, as the most convenient and accurate argument for combination with another argument, as the height h, the temperature T, or the pressure B. The Table 67 summarizes the formulæ for the hypsometric reductions, and they are more fully explained in the forthcoming Barometry Report. The general idea is that having found the ratio $\frac{e_o}{B_o}$ at the base of a column, the application of Hann's law for the diminution of the vapor pressure with the height gives the most accurate average law for computing the integral of the vapor tension throughout the entire column. A small secondary term can be added whenever our knowledge of the facts justifies such an increased degree of accuracy, though it is usually of little importance, especially for a series of observations where mean results are required.

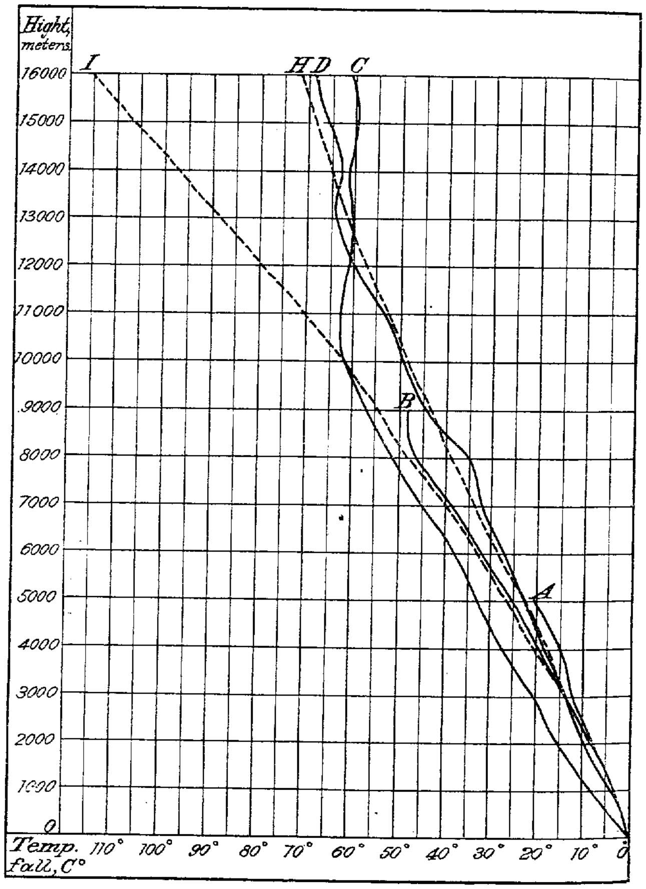
In the development of the α, β, γ, δ stages of the adiabatic thermodynamic formulae, the ratio $\frac{e}{B}$ is made the primary argument by the series of transformations given on Table 72. These formulæ are reduced to numerical tables, 94–102, and their accuracy is tested by comparing directly with the Hertzian logarithmic formulæ, as given in the examples of Table 108. Their use involves a series of solutions by trials, which though laborious, yet lead to perfectly rigorous results, and after a little practise it becomes quite easy to obtain the true trial values without much difficulty. The graphical diagrams of Hertz give only approximate values, because they throw out the vapor tension term in the critical places and thus render inaccurate the very problems they were designed to discuss. Special applications were made to finding the gradients of pressure, temperature, and vapor tension in the α, β, γ, δ stages, and the results are found in Tables 147 for metric measures, and in Tables 153 for English measures.

Table 21.—*Comparison of several determinations of the total temperature change from the surface to high levels.*

	A.	*B.*	*C.*	*D.*	*E.*	*F.*	*G.*	*H.*	*I.*
16000			-60.4	-68.3				-71.1	-115.0*
15000			-59.1	-66.0				-68.0	-106.0*
14000			-60.9	-62.5				-64.7	- 97.0*
13000			-60.1	-63.5				-61.0	- 88.0*
12000			-61.0	-60.3				-57.0	- 79.0*
11000			-62.8	-52.7				-52.6	- 70.0*
10000			-60.6	-48.5		-60	-62	-48.1	- 61.0†
9000		-48.0	-57.0	-44.6	-56.8	-51	-56	-43.4	- 54.5†
8000		-47.4	-51.0	-34.9	-48.7	-47	-48	-38.5	- 47.9†
7000		-38.4	-44.8	-31.7	-39.8	-38	-41	-33.3	- 39.6†
6000		-32.0	-37.5	-26.9	-34.6	-30	-34	-28.1	- 32.9†
5000	-20.8	-25.5	-32.3	-23.1	-27.0	-25	-26	-22.8	- 26.0†
4000	-15.0	-19.6	-28.0	-19.0	-20.7	-18	-21	-17.9	- 19.9†
3000	-12.9	-14.3	-19.5	-13.0	-15.4	-13	-15	-13.1	- 14.5†
2000	- 7.9	- 8.5	-15.8	- 9.6	- 9.9	- 9	- 8	- 7.8	- 9.0†
1000	- 3.2	- 3.7	- 8.3	- 3.8	5.0	- 4	- 4	- 3.9	- 4.3†
0000	0.0	0.0	0.0	0.0	0.0	0	0	0.0	0.0†

A = 49 ascensions not above 5,000 meters in manned balloons.
B = 12 trips upward and 5 downward, not above 10,000 meters, in manned balloons.
C = 9 ascensions of unmanned balloons above 10,000 meters.
D = Bigelow's compiled data, Tables 156, I. II., International Cloud Report.
E = Berson's mean results, Meteorologische Zeitschrift, Oct. 1901, p. 449.
F = Teisserenc de Bort's mean results, Meteorologische Zeitschrift, Oct. 1901, p. 449.
G = Hergesell's mean results. Meteorologische Zeitschrift. Oct. 1901, p. 449.
H = Bigelow's mean results, Tables 157, I. II., International Cloud Report.
I = The mean of *E*, *F*, *G* up to 10,000 meters, and a gradient of 9° per 1,000 meters from 11,000 to 16,000 meters.
* Hergesell's assumed gradient 9° per 1,000 meters.
† Mean of *E*, *F*, *G*.

Finally the same tables were employed to discuss the important problem of the difference between an adiabatic atmosphere and the one given by the upper strata observations, whereby a new method was illustrated, with results in Table 162. The value of this computation depends, of course, upon the data B, T, e, adopted for the upper atmosphere, as measured by the balloon and kite ascensions. It was especially necessary to have the temperatures at high levels, and for this purpose I collected such material as was available up to the end of the year 1896, when I began this compilation, and for that purpose employed the 102 balloon ascensions enumerated in Table 155, embracing all those then available for the United States, England, France, Germany, and Russia. I expressed myself cautiously regarding the result, page 750, holding the computation as preliminary to a fuller one which would become possible when accurate observations had been accumulated for the upper air temperatures, and I have therefore had an interest in examining the Berlin report of the German balloon ascensions.[2] In the first volume of this work is contained the data for each ascension, and in the Meteorologische Zeitschrift, October, 1901, page 449, H. Hergesell gives a summary of the resulting free air temperatures. I have extracted the observed temperatures from this report, interpolated them to each round 1,000-meter level, and computed the total temperature fall from the surface to the respective strata, with the result given in Table 21 and fig. 23. If the ascensions are divided into three sets, *A*, those reaching heights between the surface and 5,000 meters, *B*, those between the surface and 10,000 meters, and *C*, those between the surface and 16,000



A. and *B*. Berlin observations with manned balloons.
C. Berlin observations with unmanned balloons.
D. Bigelow's summary from all countries.
H. Bigelow's adopted mean result.
I. Berlin adopted mean result.

Fig. 23.—Total temperature fall from the surface to high levels by several systems.

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[2] Wissenschaftliche Luftfahrten. Assmann und Berson. 3 Bänden. Berlin, 1899.

meters, we have the following remarkable data. Class *A* contains 49 ascensions of manned balloons, and gives a temperature fall of 20.8° at the 5,000-meter level; class *B* contains 12 upward and 5 downward trips of manned balloons and gives a fall of 25.5° at the 5,000-meter level, or 5° more than class *A*; class *C* contains 12 ascensions of unmanned balloons, with a fall of 32.3° at 5,000 meters, or 11.5° more than in class *A*, and 57° at 9,000 meters, or 9° more than in class *B*. This class shows also a fall of 60.6° at 10,000 meters and 60.4° at 16,000 meters. These widely different temperature falls by classes *A*, *B*, *C* may possibly be explained by those who are familiar with the circumstances, but the fact deserves attention; also the other fact that there is no temperature fall between 10,000 and 16,000 meters as observed in the Berlin unmanned balloon ascensions. In column *D* is given the result of my own compilation found by taking the mean of all the figures as they stand in Tables 156, I, II; and on fig. 21 the line *D* is seen to fall between *A* and *B* and to cross *C* at the height of 12,000 meters.

In his review of the Berlin ascensions H. Hergesell gave the Berson results as shown as in column *E*, the Teisserenc de Bort results as in column *F*, and his own results as in column *G*. He also stated the conclusion that above 10,000 meters the adiabatic rate of temperature fall in free air prevails, and this may be considered as 9.0° per 1,000 meters, as suggested by him. Column *I* is the mean value of *E*, *F*, *G*, up to 10,000 meters, and from that level to 16,000 the fall is calculated at 9.0° per 1,000 meters, these values being plotted on fig. 21. Finally, by taking the means of the data given in Tables 157, I, II, which was derived from Charts 78, 79, as constructed to determine the gradients for each month in the year, we have the data of column *H*, also plotted on fig. 21. It is seen that my adopted result, *H*, lies midway between *A* and *B*, and is a fair average of all the ascensions taken in the unmanned balloons, while the adopted Berlin result, *I*, is 45° lower at 16,000 meters, giving at that level a temperature of —115° approximately. There is a further consideration of importance to be noted in this connection. E. Rogovsky in his paper on the "Temperature and composition of the atmospheres of planets and the sun," Astrophysics, November, 1901, discusses the temperature of the interplanetary medium (according to Pouillet —142° C., Froelich —131° to —127°), and assumes it to be —142° C. A fair assumption regarding the efficient depth of the atmosphere makes it 64,000 meters or about 40 miles, and hence we have the following data:

Height of atmosphere.	Bigelow.		Berlin.	
	Temperature.	Necessary gradients.	Temperature.	Necessary gradients.
Meters.	° *C.*	° *C.*	° *C.*	° *C.*
64,000	—142		—142	
		—1.8		—0.9
16,000	— 55		—100	
		—4.4		—7.2
Surface	15		15	

If the temperature falls from 15° at the surface to —55° at 16,000 meters with a gradient of about —4.4° per 1,000 meters, then to reach —142° at 64,000 meters the gradient should on the average be —1.8°. It will be seen by my Charts 78 and 79, International Cloud Report, that I adopted an increasingly slower temperature fall with the height in the strata above 10,000 meters, in accordance with this general view. If the Berlin theory is assumed that a fall of 9.0° per 1,000 meters prevails above the 10,000-foot level, then it must somewhere rapidly decrease to a very small gradient in order not to diminish the exterpolated temperatures far below that value assigned by certain astrophysicists to the celestial medium at the earth's distance from the sun. In fact the gradient becomes one-tenth of the adiabatic rate, which was actually assumed. If the temperature —260° C. is that of the interplanetary medium, as supposed by other writers, these inferences must be modified accordingly.

From these two considerations, (1) that my temperature system includes the data of the highest balloon ascensions, and (2) that my gradients are in harmony with the requirements of astrophysics, I shall let my computations on the heat difference between the adiabatic and the actual atmosphere stand as they were given in my report. The accurate measurement of the temperatures in the highest strata is a very difficult process, and all efforts to secure reliable results deserve the hearty support of meteorological physicists. There are several problems whose solution depends upon the possession of such data in a satisfactory form.

THE FIRST NATIONAL METEOROLOGICAL CONGRESS OF MEXICO.[1]

By Prof. FRANK H. BIGELOW.

The report of the proceedings of the first Meteorological Congress of Mexico has been published and contains the acts and resolutions and papers presented during the sessions of November 1, 2, 3, 1900, held under the auspices of the Scientific Society "Antonio Alzate." The president was Señor D. Manuel Fernandez Leal, and there were about thirty members present at the sessions in an official capacity. The proceedings opened at 9:20 a. m., Thursday, November 1, 1900, with an address by the President, after which the papers to be read were presented. In the afternoon the session opened at 3:35, C. A. Gonzalez presiding, at which a discussion and the adoption of resolutions occurred, the purpose being to indicate the necessary steps in the organization of a national meteorological service for weather forecasts and climatology along recent modern lines, as laid down by the International Meteorological Congresses. Also a report was approved on the formation of a survey of the atmosphere by cloud observations, in three classes: (1) direction and motion of clouds by eye, (2) by nephoscopes, (3) by theodolites and photogrammeters.

On Friday, November 2, at 9:20 a. m., F. R. Rey presiding, papers were read by S. Diaz, L. G. Léo, M. Moreno y Anda, Señorita R. Sánchez Suárez, and J. M. Romero. At 4:30 p. m., D. M. Leal presiding, resolutions were passed as to the hours of observation, reduction of temperatures to the mean of 24 hourly observations, computation of the vapor tension, reduction of the barometer to zero temperature and to sea level, classification of clouds, the computation of the mean direction of the wind, and as to various special observations.

On Saturday, November 3, at 9 a. m., G. B. y Puga presiding, the reading of papers was continued by A. Prieto, Leal, and Olmedo. A discussion took place with the adoption of the following resolutions:

> The first National Meteorological Congress expresses its desire that the Federal Government should provide for the organization of a meteorological service upon a basis analogous to that which exists in the United States; especially, will it be desirable to secure a modification of the existing services, taking account of the elements which actually exist, in conformity with the following principles: (1) That the Central Meteorological Observatory of Mexico be recognized as the central office of the national service; (2) that it be the center of all the scientific relations; (3) that the Federal Government equip this office for that work; (4) that the government establish and equip other observatories in suitable localities for cooperation with it; (5) that the state governments organize a network of stations in their own districts; (6) that a suitable telegraphic service be developed for meteorological messages; and (7) that a commission be organized to further the development of these plans.

At 4:30 p. m., J. de M. Tamborrel presiding, the discussion was continued, and resolutions were adopted concerning the

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[1] Actas, resoluciones y memorias del primer Congreso Meteorologico Nacional, iniciado por la Sociedad Cientifica "Antonio Alzate," y celebrado en la ciudad de México los dias 1, 2 y 3 de Novimbre de 1900. Mexico. 1901. 272 pp.

improvement was noticed in the condition of June corn, although this crop suffered along river bottom lands. Rice, sugar cane, and all minor crops made satisfactory advancement.—*Edward H. Bowie.*

Utah.—A remarkably cool spell prevailed during the first decade. Freezing temperatures and heavy frosts occurred in the elevated valleys of the State on several mornings between the 2d and 9th, badly damaging potatoes and other tender vines, and slightly damaged corn, wheat, and alfalfa in places. Irrigation water became very short before the close of the month, but on the whole growing crops did fairly well.—*L. H. Murdoch.*

Virginia.—During the first half of the month fairly favorable weather prevailed over the State and crop growth was, in the main, satisfactory, but the latter half was quite droughty and vegetation suffered considerably, especially early corn, pastures, and tobacco. Opportunities for field work were almost uninterrupted, and at the close of the month all crops were clean. Hay making, thrashing and housing of wheat, and cutting and stacking of oats were vigorously prosecuted.—*Edward A. Evans.*

Washington.—Long drought broken by heavy rains beginning on 1st, and lasting three to six days, averting danger to and causing great improvement in crops. In parts of the eastern section it was the heaviest July rainfall known. With the exception of the first week it was a splendid month for haying. Fall wheat harvest progressed during the last week; the yield was fair to good. Oats were in need of rain. The potato crop promises to be excellent.—*S. N. Salisbury.*

West Virginia.—The weather conditions during the month were generally favorable for crop growth and also for harvesting. By the end of the first week wheat was mostly in shock and some was being thrashed, with light yield. Meadows continued to thicken and improve, so that haying was not in full progress until the last week, when hay was secured in good condition, with about half a crop. Oat harvest was in progress during the last week, with a good yield, and early potatoes were made, with a good crop. Corn made excellent growth during the month, and the prospects were for a fairly good crop. Some fall plowing was being done during the latter part of the month. Apples continued to fall during the month, and the prospects were for about half a crop; peaches and plums were scarce, but grapes were quite promising.—*E. C. Vose.*

Wisconsin.—Crops generally in a very promising condition, although frequent and heavy rains damaged grains and hay in most sections and delayed the harvest. Killing frosts occurred the first of the month in the extreme northern counties and considerable damage resulted; light frost occurred in the middle section, but no damage resulted. The first of the month was exceptionally cool and retarded the advancement of corn, but later it showed a gradual and thrifty growth. Rain was materially deficient the greater portion of the month in the northeast counties. Hail did material damage in some localities, especially in St. Croix County on the 22d.—*J. W. Schaeffer.*

Wyoming.—The weather was unseasonably cool during the first half of the month and retarded crop growth some, but was warm and favorable for good crop growth during the latter half. Precipitation was sufficient in most sections of the State, except parts of Big Horn County, where some crop failures resulted. At the end of the month haying was in general progress, and the second crop of alfalfa was being secured in several sections. Gardens and grain were backward, but doing well. Ranges had become generally dry, but stock was in good condition. Prospects for winter feed were poor in sections. As a whole the month was favorable for agricultural interests.—*W. S. Palmer.*

SPECIAL CONTRIBUTIONS.

STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.

NO. VII. A CONTRIBUTION TO COSMICAL METEOROLOGY.

By Prof. FRANK H. BIGELOW, dated August 12, 1902.

GENERAL REMARKS.

I have already published the results of certain computations and discussions on the subject of the direct connections between the variations of the solar output of energy, and the corresponding synchronisms in the meteorological elements of the earth's atmosphere. These are in particular, Solar and Terrestrial Magnetism, Weather Bureau Bulletin No. 21, 1898, and Eclipse Meteorology and Allied Problems, Weather Bureau Bulletin I, 1902, which include the substance of other minor papers related to this subject. The purpose of these studies has been, (1) to establish the fact that a synchronous connection does exist between the solar and the terrestrial forces, and (2) to derive the operation of these periodic movements so as to ultimately lead meteorology to a scientific understanding of the terrestrial seasonal climatic changes, and to a true basis for forecasts of weather conditions, at least one year in advance.

The difficulty of reaching a correct solution of this problem is well understood by those who have worked upon it, to reside in the unsteadiness of the solar output itself, and the numerous subordinate transformations of the energy, through the radiation, the general and local cyclonic circulations, till it culminates in a season having certain characteristics. The material for the study consists in the variations of the pressures, temperatures, and vapor tensions at many stations in different portions of the earth, in the fluctuations of the terrestrial magnetic field, in the changes of the spectrum energy of the solar and the aqueous vapor curves, and in the variations of the sun spots, the prominences, and the solar faculæ. The magnitude of the task involved in handling this material is such as to limit the attempt to deal with it to a few institutions having these subjects specially in charge. Among them the United States Weather Bureau has been able to make some contributions from time to time.

SUMMARY OF THE DISCUSSION OF 1898.

On pages 121–130, Bulletin No. 21, is given a brief account of an extensive discussion of the data then at hand, and the result was such as to show that there is a marked synchronism between the solar and terrestrial variations of energy. Fig. 24 serves to recall this fact and it shows that in the sun-spot period, 1878–1893, there was a true synchronism in the variation of the sun-spot areas, the European magnetic force, which is the resultant of the two components measured on a horizontal plane, and the American meteorological system. The latter includes a variation of temperature at 25 stations in the northwestern portions of the United States, the pressure at 10 stations, the variable mean movements of the storms in latitude and longitude, and the movement of the tracks of the cold waves in latitude. Each of the two latter elements was derived from an exhaustive compilation of the coordinate positions of the cyclonic centers for the interval of fifteen years ending with 1893. It led me to the following summary:

> The increase of solar magnetic intensity is synchronous with a diminution of temperature but with an increase of pressure, and this function persists throughout every phase of the research.
>
> In spite of some irregularity, there is a distinct conformity in the general sweep of these curves, and also in the tendency to describe crests during the same years. Indeed, the occurrence of four subordinate crests in the 11-year period suggests strongly that a $2\frac{3}{4}$-year period is superposed upon the long sweep of that periodic curve. Apparently this minor period is the basis of these seasonal variations of the weather conditions of the United States more than anything else, so that in long-range forecasting this period must be very carefully considered.

It was for the purpose of carrying this subject one step further forward that the discussion of the data summarized in this present paper was undertaken. There has been considerable delay in completing the work on account of many other important duties.

It is evident that the terrestrial magnetic field affords the data which is most available for studying the fundamental periods in this solar-terrestrial synchronism. An exact quantitative computation for the several elements involves a very large amount of labor, and therefore it is important as an alternative to derive the periods by *methods which shall give reliable proportional variations* of the elements. The ideal treatment is to compute the total deflecting force of the magnetic field, by using the means of 24-hourly observations of the horizontal force, the declination, and the vertical force, taking out their daily component variations in rectangular coordinates

(dx, dy, dz,) and combining them into polar coördinates s, α, β.

The next simpler method is to omit the vertical component, as one is tempted to do in consequence of the unreliable action of the Lloyd's balance, and turn the horizontal components, dx, dy, into polar coordinates σ, β, on the horizontal plane. Since it has been proved by computation that the east-west component, dy, derived from variations of the declination, practically disappears, as it should do by theory, we may adopt the variations of the horizontal component, dx, along the magnetic meridians, as the best single component for computation. This is all the more satisfactory because the bifilar horizontal magnet is the most efficient instrument in use in the magnetic observatories, and is generally free from objectionable features in its operations. I have, therefore, in this discussion, adopted the variations of the horizontal force, as shown by the 24-hourly means, on the ground that they are proportional to the total variations of the magnetic field and quite free from instrumental errors.

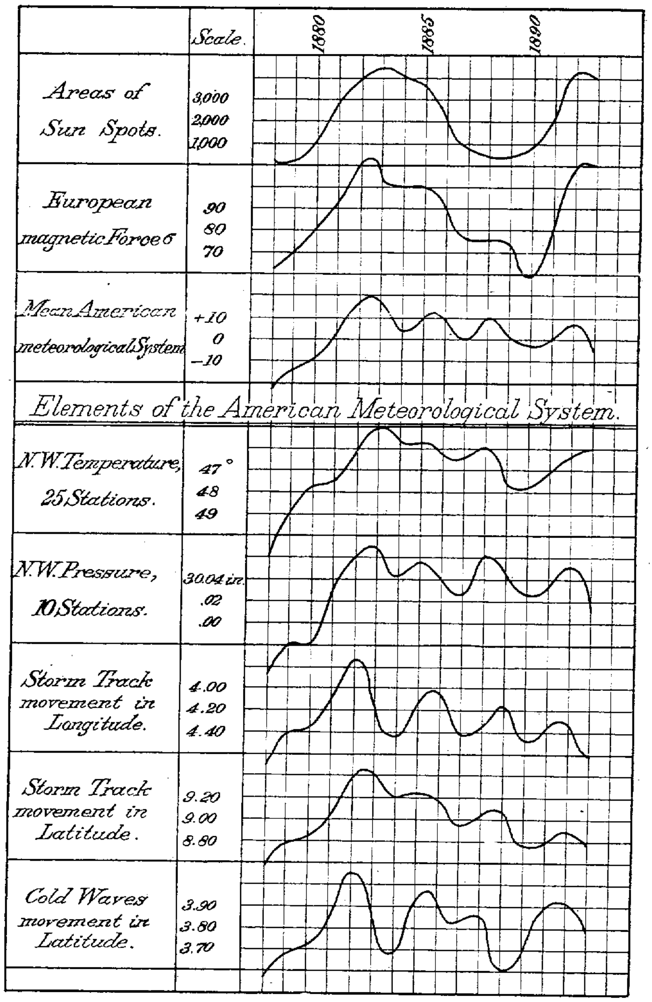

FIG. 24.

THE MAGNETIC OBSERVATIONS 1841–1899.

Accordingly, the magnetic horizontal force for the interval 1841–1899 has been submitted to a discussion, the result of which is summarized in this section. The synchronous action of the solar energy, as exhibited in the variation of the sun spots, the terrestrial aurora, the magnetic field, and several other phenomena, has been frequently developed, so that the general fact is admitted by all students, but it is now important to trace out this sympathetic movement in these cosmical forces more in detail, especially as they relate to the annual and seasonal variations in the earth's atmosphere. In the Proceedings of the Royal Society, volume 63, Mr. William Ellis, F. R. S., has exhibited this synchronism between Wolf's sun-spot numbers and the declination and horizontal force at the Greenwich Observatory for the interval 1841–1896. (Compare Bulletin I, 1902, page 105.) In his diagram not only do the curves present the same large sweeps, but also the minor variations appear simultaneously in the three curves. It is for the purpose of developing yet more distinctly these minor fluctuations that the compilation of the following magnetic observations was executed.

Instead of confining the study to a single observatory, it has been extended so as to practically include the entire earth, at least sufficiently to demonstrate that the variations are common to the whole terrestrial magnetic field. Thus, for different years we studied the records at the following stations:

1841–44. Toronto, St. Helena, Hobarton.[1]
1845. Greenwich, Toronto, Singapore, St. Helena, Cape of Good Hope, Hobarton.
1846–47. Toronto, St. Helena, Hobarton.
1848. Toronto, Greenwich, Hobarton.
1849–1870. Whatever was available, as Greenwich, Toronto, Madras, Batavia, Pavlosk, some of the data being unsatisfactory.
1871–77. Greenwich, Pavlosk.
1878–1885. Greenwich, Pavlosk, Vienna, Prague, Tiflis.
1886–1887. Los Angeles, Toronto, Greenwich, Paris, Pola, Prague, Pavlosk, Tiflis, Zi-ka-wei, Batavia.
1888. Greenwich, Prague, Pavlosk.
1889–90. Greenwich, Washington, Pavlosk.
1891. Greenwich, Prague, Pavlosk.
1892–99. Paris, Pola, Pavlosk.

By thus changing the stations it becomes impossible that the peculiar action of any set of instruments, should such exist, can impose a bias upon the final result. It seems to me that it makes no difference what three stations are chosen to represent the cosmical variation of the magnetic field, as indicated by the horizontal force which is proportional to the total force. Three stations are desirable in order to eliminate the local impulses of the field, and if they had been available I should have used the same three stations throughout, all reduced to the C. G. S. system of units, for the sake of having rigorous quantitative results. The data of this paper limit it to showing relative synchronisms, but these are quite sufficient for our purposes in the present stage of meteorology.

The horizontal force, as given by the means of twenty-four successive hourly ordinates, or by a smaller number of selected hours in some cases, was considered, and the *daily variation from the normal horizontal force* was computed either numerically or graphically. (Compare the methods of Bulletins Nos. 2 and 21.) In some of the years the normal force was found by drawing a mean line through the monthly trace of the curve, as plotted from the daily means; in other years the daily variation was computed from the numerical data contained in the published reports. In all years from 1841–99 the horizontal trace was graphically transferred to curves and distributed in the period of 26.68 days, whose epoch is June 13.72, 1887; the exact adopted period was 26.67928 days, as given in the January Ephemeris. (See Bulletin No. 21, page 120.) Therefore throughout this interval the several curves, generally three in number, are plotted on the sheets, so that the irregularities as well as the agreements are open to inspection. On examining this long series of curves in succession, it is evident that a decided change occurs in the amplitudes of the variable curve with regard to the normal base line which was superposed upon each of them. In some years the curves are flat and

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[1] Now called Hobart Town, or Hobart, Tasmania.

quiet, so that the ordinates are small, and the curves are free from violent or spasmodic impulses. In other years, on the contrary, the curves sway about roughly and are much disturbed, great irregularities being superposed upon them. My procedure was to measure, at least proportionally, the area which is inclosed between the variable curve and the average base line and to integrate these areas in the 26.68-day period, the 11-year period, and throughout the interval 1841–99. This was accomplished by measuring these ordinates from day to day, taking the mean of those stations selected for the given day, usually three in number, and transferring these mean ordinates with their plus and minus signs to the Ephemeris Tables. For by taking the ordinates from day to day, that is to say, at frequent intervals along the curve, their *sum is proportional to the area developed between the bounding curve and the base line.* If the actual area had been read off by means of a planimeter, the relative variation between different epochs would have been the same. These relative numbers were transferred to tables, showing the direct type curves in black ink and the inverse type curves in red ink, which can not be reproduced in this connection. These figures were now summed together by the year, the direct type and the inverse type separately, but no account is given here of this summation. Also the figures were summed across each period, without regard to their signs, so that the total divergence of amplitude might appear as a sort of integral. In each year there are thirteen or fourteen periods, but when the successive years are placed in the 11-year period, and five or six of these in the final summary, then the mean dates of the total ephemeris can be restored by simply applying 26.68 days to the mean of the January dates, that is to January 15.44, as on page 106, of Bulletin No. 21.

Table 22, "Total variations of the horizontal force for the earth generally, arranged in 26.68-day periods," contains an exhibit of these integral sums of the included areas for each 26-day period within the limits of the computation; the sums are also given for each year for five successive 11-year cycles, and for the general mean; and, finally, the differences between the annual sums for each successive year and also the average

TABLE 22.—*Total variations of the horizontal magnetic force for the earth generally, arranged in 26.68-day periods.*

11-YEAR PERIOD, 1841-1851.

26-day period.	1841.	1842.	1843.	1844.	1845.	1846.	1847.	1848.	1849.	1850.	1851.	Means.
1.	171	142	130	155	177	101	193	282	152	204	141	168
2.	170	181	159	153	139	100	200	272	160	225	112	169
3.	204	168	130	151	111	238	289	277	170	137	104	180
4.	254	165	126	134	102	231	220	223	362	147	153	193
5.	226	209	178	135	122	229	196	161	170	200	164	181
6.	134	175	109	96	142	155	137	199	114	261	155	153
7.	134	260	75	132	82	110	155	105	130	317	140	149
8.	169	93	138	118	79	127	130	216	144	217	162	145
9.	143	89	119	113	85	155	164	196	129	178	279	150
10.	145	160	96	182	99	220	296	120	161	80	250	164
11.	219	136	84	242	108	208	231	85	188	160	187	168
12.	197	176	58	147	97	139	257	152	121	183	159	153
13.	211	240	68	142	62	131	320	106	182	111	191	160
14.	239	107	130	......	126	95	......	76	206	184	......	160
Sums	2,516	2,301	1,600	1,900	1,631	2,239	2,788	2,470	2,389	2,604	2,197	2,293
Differences.	......	—215	—701	+300	—269	+608	+549	—318	—81	+215	—407	333

11-YEAR PERIOD, 1852-1862.

26-day period.	1852.	1853.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	Means.
1.	225	118	175	86	183	105	281	162	174	281	231	183
2.	248	138	134	124	134	117	186	239	224	169	182	172
3.	146	138	179	126	157	125	241	150	269	208	199	176
4.	258	132	170	146	142	171	262	206	273	207	194	196
5.	189	168	116	107	151	188	324	225	161	131	144	173
6.	153	162	100	81	128	90	190	139	168	233	171	147
7.	128	139	126	90	95	125	177	225	272	202	194	161
8.	133	118	91	141	111	114	163	150	305	176	208	155
9.	107	163	125	112	163	192	134	271	230	212	169	171
10.	154	189	127	100	165	143	194	221	168	267	253	180
11.	137	122	79	131	124	213	199	379	115	187	281	179
12.	152	161	104	106	92	231	205	168	148	289	166	166
13.	167	148	72	93	124	282	196	330	256	259	187	192
14.	129	117	......	151	168	......	174	151	......	240	170	179
Sums	2,315	2,015	1,558	1,594	1,937	2,096	2,926	3,016	2,763	3,061	2,749	2,430
Differences.	+118	—300	—457	+36	+343	+159	+830	+90	—253	+298	—312	291

TABLE NO. 22.—Continued.

11-YEAR PERIOD, 1863-1873.

26-day period.	1863.	1864.	1865.	1866.	1867.	1868.	1869.	1870.	1871.	1872.	1873.	Means.
1.	163	120	89	135	85	80	183	166	120	149	137	130
2.	140	99	121	195	80	117	82	204	196	232	181	150
3.	159	117	94	96	80	159	91	184	165	167	118	130
4.	128	102	85	105	93	129	143	201	161	210	144	136
5.	116	103	78	68	115	139	217	272	137	175	126	141
6.	113	121	147	84	73	111	150	209	222	202	139	143
7.	99	105	98	149	125	94	111	173	151	227	95	130
8.	122	151	156	140	67	90	129	250	144	196	101	141
9.	133	138	113	135	69	90	159	230	160	215	125	142
10.	131	133	89	150	118	159	150	290	141	193	107	151
11.	115	176	126	124	58	178	99	319	111	225	130	151
12.	123	146	122	66	65	128	131	263	163	228	142	143
13.	66	93	69	98	71	101	189	251	100	144	120	118
14.	....	95	171	98	......	98	281	......	186	199	....	177
Sums	1,608	1,699	1,558	1,643	1,099	1,673	2,123	3,012	2,157	2,762	1,665	1,983
Differences.	—1,141	+91	—141	+85	—544	+574	+450	+889	—855	+605	—1,097	616

11-YEAR PERIOD, 1874-1884.

26-day period.	1874.	1875.	1876.	1877.	1878.	1879.	1880.	1881.	1882.	1883.	1884.	Means.
1	120	108	114	112	172	160	134	302	224	230	235	174
2.	239	212	146	82	146	97	194	251	165	241	223	181
3.	199	125	109	95	151	127	135	171	144	283	196	158
4.	202	132	83	64	141	120	171	182	421	223	233	179
5.	152	114	93	108	152	162	215	145	215	210	251	163
6.	159	128	57	107	191	187	173	235	264	192	203	172
7.	106	97	85	148	168	161	160	220	256	224	296	175
8.	159	82	73	113	135	177	310	187	295	197	184	174
9.	115	75	79	118	103	156	250	157	183	356	255	168
10.	112	139	92	90	141	204	221	264	344	194	242	186
11.	184	100	105	92	116	159	278	149	281	229	218	174
12.	132	105	110	98	187	182	176	238	486	287	264	206
13.	115	91	120	128	176	194	219	219	290	186	267	182
14.	97	79	......	70	180	172	......	160	141	......	244	157
Sums	2,091	1,587	1,266	1,425	2,165	2,257	2,636	2,880	3,809	3,052	3,311	2,449
Differences	+426	—504	—321	+159	+740	+92	+379	+244	+929	—757	+259	437

11-YEAR PERIOD, 1885-1895.

26-day period.	1885.	1886.	1887.	1888.	1889.	1890.	1891.	1892.	1893.	1894.	1895.	Means.
1.	265	136	161	177	128	101	158	261	279	143	182	181
2.	223	224	162	160	153	116	174	286	200	389	156	204
3.	194	276	136	222	143	126	164	298	220	205	186	197
4.	197	225	172	218	147	102	182	409	207	254	191	210
5.	324	198	163	223	131	94	287	278	209	210	200	211
6.	183	156	130	161	130	102	139	301	223	189	156	170
7.	273	174	172	140	137	115	140	330	230	147	181	185
8.	141	207	178	149	138	123	157	245	302	250	232	193
9.	177	263	190	157	189	115	178	211	212	295	191	198
10.	247	272	208	171	184	131	302	174	229	269	170	214
11.	171	242	179	184	225	164	162	255	312	128	247	206
12.	201	209	149	205	251	140	191	179	191	295	256	209
13.	221	127	197	139	136	117	214	231	247	176	164	179
14.	257	......	194	127	......	110	178	150	......	135	216	188
Sums	3,074	2,709	2,391	2,533	2,092	1,656	2,626	3,608	3,061	3,135	2,728	2,745
Differences	—237	—365	—318	+142	—441	—436	+970	+982	—547	+74	—407	447

1896-1899.

26-day period.	1896.	1897.	1898.	1899.	Means.	Average for the whole interval, 1841-1899.
1.	152	165	152	172	161	166
2.	220	169	179	151	180	176
3.	177	169	260	249	214	176
4.	254	188	178	190	203	186
5.	273	203	153	157	197	178
6.	194	195	190	166	186	162
7.	205	203	121	134	191	165
8.	185	144	128	109	142	158
9.	209	100	208	103	155	164
10.	229	115	169	132	161	176
11.	227	124	190	145	172	175
12.	248	113	148	109	155	172
13.	261	178	166	102	177	168
14.	....	218	127	....	173	172
Sums	2,834	2,285	2,369	1,920	2,467	....
Differences	+106	—549	+84	—449	....	....

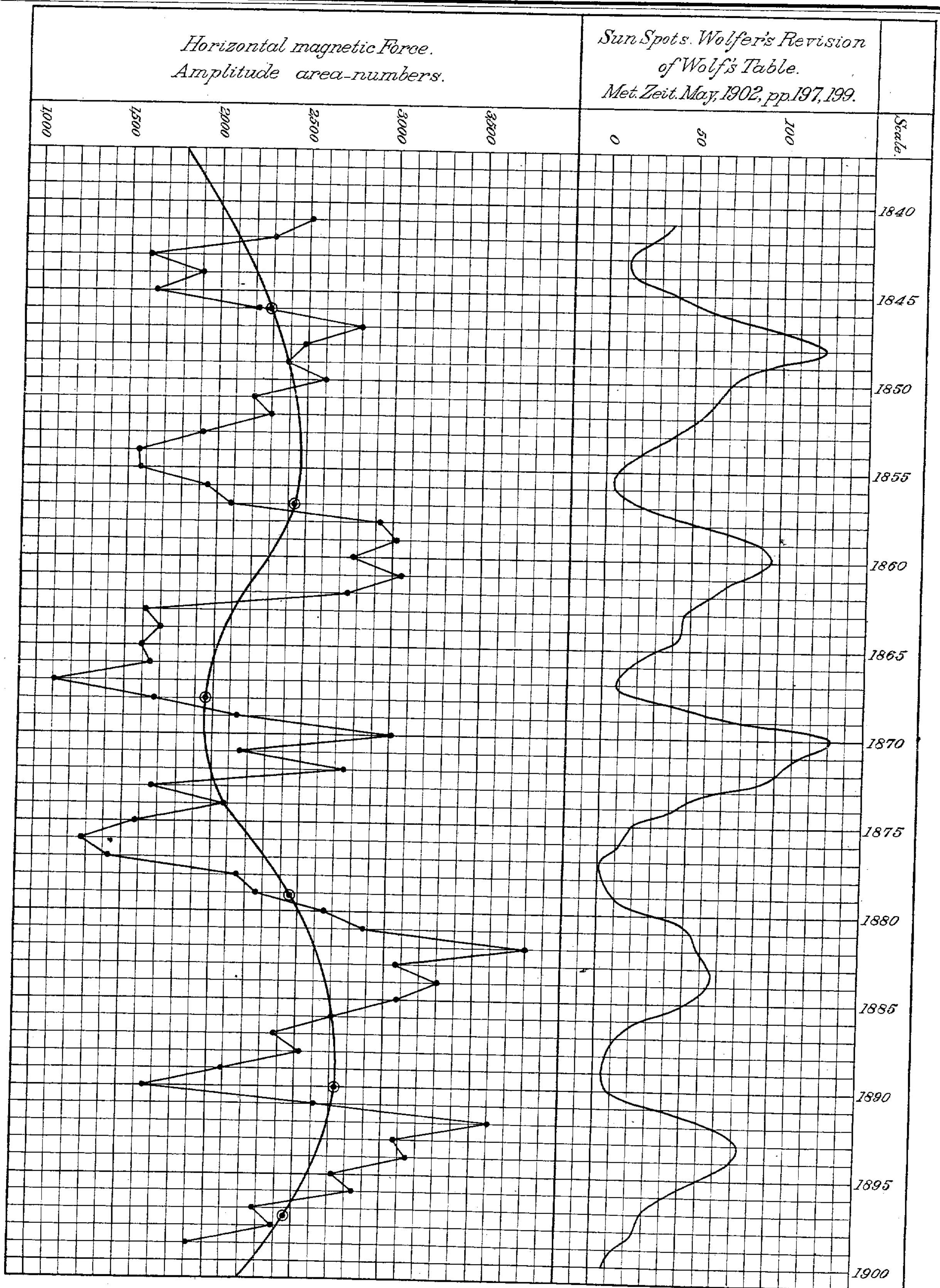

FIG. 25.—Variation of the sun-spot numbers and the amplitude area numbers.

annual variation. In taking the mean of each bottom row of periods, the mean of the numbers visible, usually 7 or 8, was increased proportionally to 11, in order to make it comparable with the other 13 periods; the action of the ephemeris causes the periodic skip in the 14th period. There was one other place in which an arbitrary change was introduced, namely, wherever the disturbance in the negative direction, at a few of the largest perturbations, exceeded —0.00040 C. G. S. units, the disturbance ordinate was computed at this value. There are only a few excessive disturbances of the horizontal force above this limit, and I did not wish to distort the average annual numbers with these great abnormalities.

The revised Wolf's table of the sun-spot numbers, by Prof. A. Wolfer, Meteorologische Zeitschrift, May 1902, page 197,[2] has been used to give the curve of the sun-spot variations, it being unimportant for this discussion whether the observed or smoothed numbers are employed.

The result of this computation, "Variation of the sun-spot numbers and the amplitude area numbers," is shown in fig. 25, the figures of Table 22 being transferred thereto. The annual sums were plotted so as to give the horizontal force curve, and the mean sums for the successive 11-year periods were plotted for the mean curve. This curve brings out three variations with extraordinary clearness: (1) The 35-year period, with maxima in 1855 and 1890, a minimum at about 1868, another probable minimum at about 1833, and one more at about 1903. After this exhibit there can be little doubt of the existence of this long period variation, discussed by Lockyer and others, and it is certain that a continuation of this method of computation will eventually fix the characteristics of this period with exactness. The fall from maximum to minimum seems to occupy thirteen years, and the rise from minimum to maximum requires a longer time, probably twenty-two years. (2) The 11-year period is seen to be in exact synchronism throughout the interval 1841–1899 with the sun spots and the horizontal force taking the curve as a whole, but there are superposed upon it a series of abrupt minor variations, which, as stated above, it is chiefly desirable to obtain for comparison with our meteorological data. (3) These subordinate crests of energy indicate that in the rise and fall of the 11-year period there is a series of spasmodic impulses, generally one in ascending the curve and two in descending, which, added to the maximum crest itself, makes four minor crests to be superposed upon the mean 11-year curve, as mentioned in the opening paragraphs, and shown in fig. 24. In the ascending branch the successive annual changes are not equal to the mean value, and this branch must evidently be considered as produced by a secondary system of crests, even though the 11-year line is not deeply indented. The discussion of this $2\frac{3}{4}$-year period will be resumed in a later section of this paper.

If the mean values of the fourteen periods as collected in the 11-year periods and indicated in Table 22 be plotted successively, the result is as shown in fig. 26. We find that there is a distinct semiannual period in the horizontal force areas, with maxima at March 22 and September 22, and minima at June 22 and December 22, thus indicating that it depends upon the orbital relations of the earth to the sun. But, furthermore, it is noted that the same 35-year period is indicated within this semiannual period, since there is a distinct minimum in the period 1874–1884, and maxima in the 1852–1862 and 1896–1900 periods, as measured by their amplitudes. Also, there is, apparently, a tendency for the spring maximum to surpass the autumn maximum, whenever they are strongest within the 35-year period. We have here indicated a field of research of importance in mechanical astronomy, since it implies that another force besides simple Newtonion gravitation is binding the sun and the earth together. It becomes an interesting problem to discover whether these magnetic forces are capable of fulfilling the outstanding theoretical requirements involved in the orbital perturbations of the earth and the other planets. It becomes, also, a further argument, in addition to those presented in my bulletins, Solar and Terres-

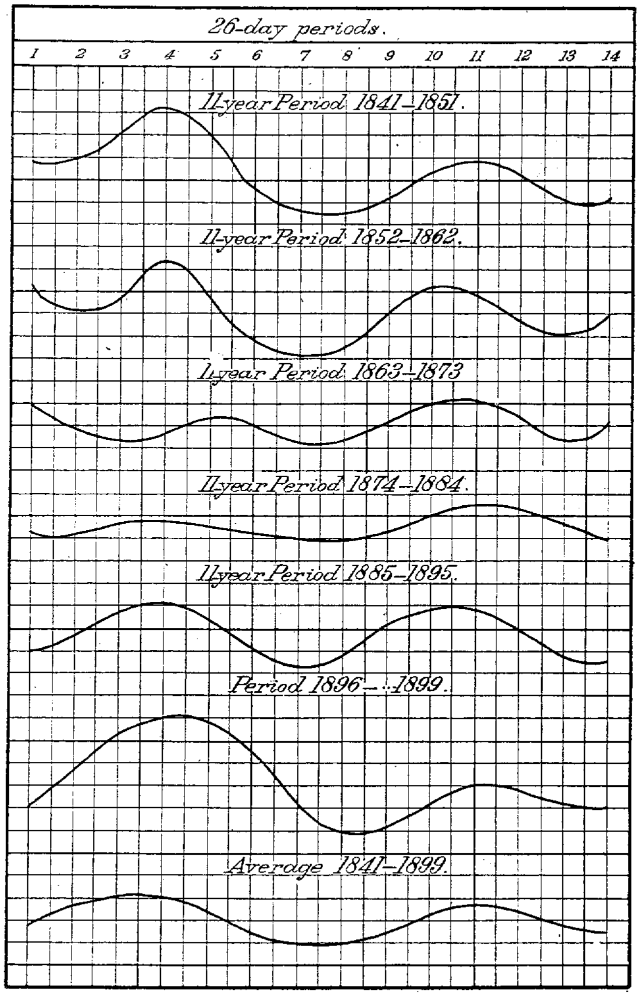

FIG. 26.—Semiannual period in the horizontal force of the terrestrial magnetic field, arranged for six successive 11-year periods.

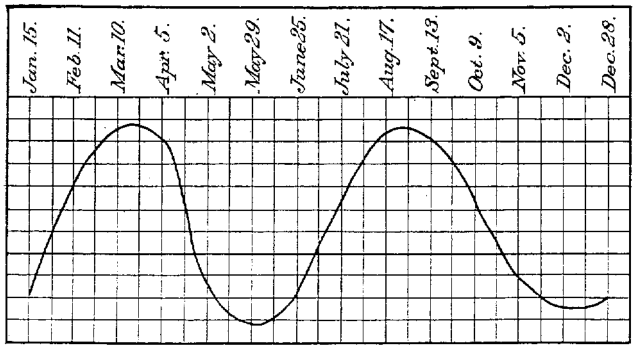

FIG. 27.

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[2] This table had been originally communicated to the Monthly Weather Review and the proof sheets sent to Professor Wolfer for revision, so that as published in the Monthly Weather Review, April, 1902, page 175, the figures have the full authority of Professor Wolfer, and it is believed no typographical error exists therein.—ED.

trial Magnetism, and Eclipse Meteorology and Allied Problems, to show that the sun is a great magnetised sphere, in whose external field the earth is immersed. On fig. 27 I have copied Chart No. 19, page 106, of Bulletin No. 21, which shows the curve of the frequency of the direct type in the 26.68-day period. Its crests regularly precede those in the semiannual orbital period by a small interval, and there must be a physical reason for this divergence, such as explained in my other papers.

COMPARISON OF THE VARIATIONS OF THE SOLAR PROMINENCES WITH THOSE OF THE TERRESTRIAL HORIZONTAL MAGNETIC FORCE FOR THE INTERVAL 1874–1900.

It is well understood that the variations of the sun-spot frequency constitute only one of the manifestations of the changes in the output of the solar energy; the frequency of the hydrogen prominences, or of the faculæ, and of the extensions of the solar corona are other forms of the display of this variable force. Indeed, there is reason to believe that the sun spots are a somewhat sluggish type of the variable impulses, although the first to be studied, on account of the ease with which the spots are observed. Since scientific processes of observation have improved of late years, it has become possible to measure the frequency of the prominences and of the faculæ with precision, so that a continuous record is now being made of these types of solar energy. The prominences have been observed by Tacchini since 1873 and the faculæ by Hale and others for several years, so that it is now possible to add to the sun-spot record that of each of these two phenomena. The prominences are distributed all over the surface of the sun, and the relative frequency has been determined in 10-degree zones between latitudes ± 90° annually since 1873, so that we possess a prominence curve of relative frequency extending through more than two 11-year cycles. Through the courtesy of Sir Norman Lockyer, of the Solar Physics Observatory, South Kensington, London, I have had an opportunity to see some advance copies of different sets of curves of a very valuable character prepared by him for a paper published by the Royal Society, in which this subject is discussed. It is gratifying to note that his work confirms my curves of 1898 and is in agreement with those presented in this paper. I reproduce the Lockyer-Tacchini prominence curve, which represents the mean frequency ni all latitudes for the years 1874–1900. It is found at the head of fig. 28. It shows a large curvature synchronous with the sun-spot frequency in the 11-year cycle, and also a series of minor crests of a characteristic nature. Underneath this curve is placed the series of minor variations which were found in the horizontal magnetic force, as shown in fig. 25, after the 11-year cycle curve has been eliminated. The remarkable synchronism between these curves can not escape recognition, except after the year 1894, when an extra minor crest is developed in the horizontal force. If these two curves are compared with the 15-year systems exhibited on fig. 24, it is evident that my paper of 1898 had detected the same synchronism, not only throughout the curve of sun-spot frequency, but also throughout the whole European magnetic field and the entire American meteorological system.

THE VARIATIONS OF ATMOSPHERIC PRESSURE OVER THE ENTIRE EARTH.

In the course of my studies into this set of phenomena, including the solar and terrestrial magnetic fields and the meteorological elements, it became evident that in cosmical problems we should be compelled to deal with the variations of small quantities in meteorology, such as a few hundredths of an inch of pressure and a few degrees of temperature. It was, therefore, necessary to carefully exclude all possible sources of error due to the imperfect methods of observation and reduction, in order that variations arising from such causes might not be falsely attributed to cosmical forces. The result of such a discussion of the barometric pressures

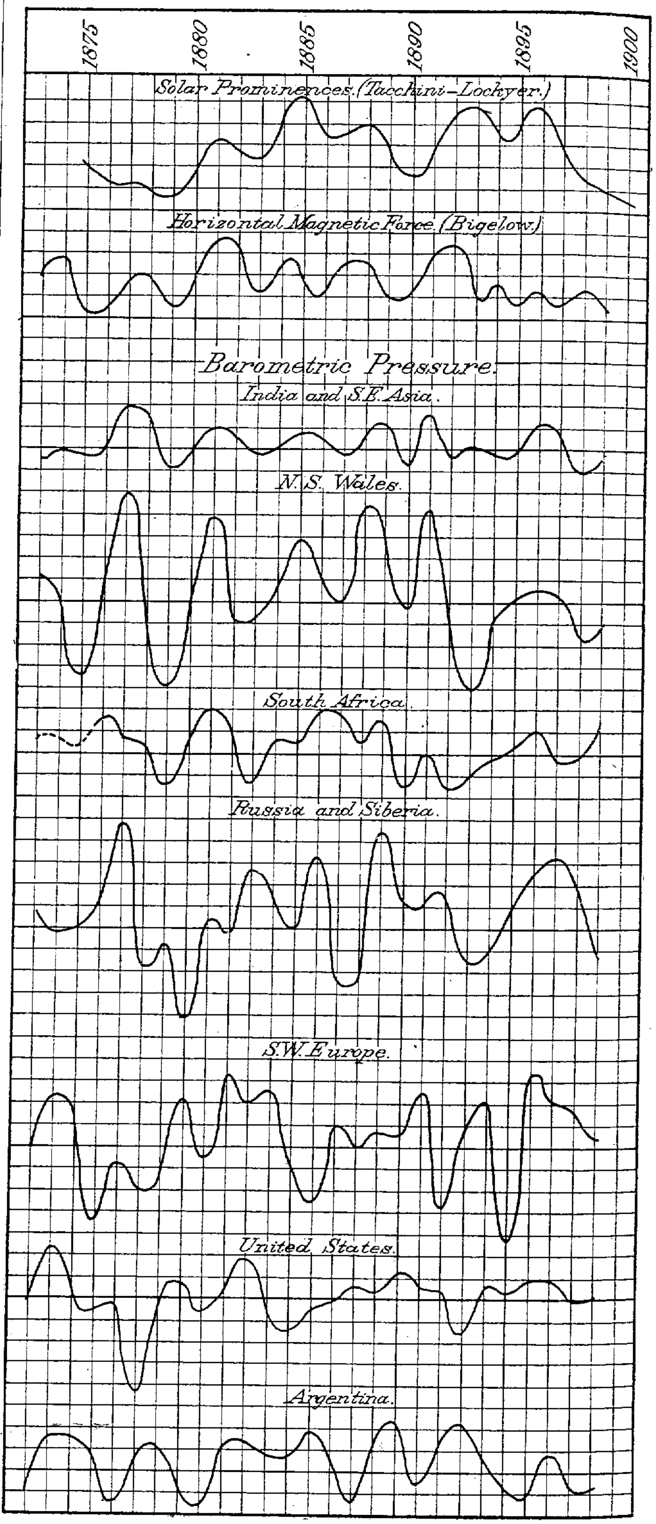

FIG. 28.—Comparison of the solar prominence variations with those of the terrestrial horizontal magnetic force and the atmospheric pressures over the entire earth.

for the United States will be found in Report of the Chief of the United States Weather Bureau for 1901, Volume II. A similar rediscussion of the temperature and the vapor tension will be executed as soon as practicable. It is important that comparable rigorously homogeneous systems should be prepared by other weather services which possess continuous long series of records of the meteorological elements. Pending the preparation of such revised systems, I have collected together a considerable number of sets or series of barometric pressures, taken in different parts of the earth, and have reduced them to a homogeneous basis, as well as I could, from a study of the published data. There are still annoying discontinuities at many stations, due to changes in the elevations of the barometers. It is also probable that the instrumental errors and the methods of reduction employed still need to be thoroughly examined.

Table 23, "The variations of the annual atmospheric pressure in many districts of the earth," serves to indicate, at least approximately, the relations of the annual barometric pressure variations to the changes in the solar output. It contains a summary of the results in the several countries where long series of barometric observations exist. It is arranged in an order which will bring out a remarkable feature of the pressure variations, as will be briefly indicated. The table gives the mean data for comparatively large districts; it is divided into groups and the mean pressures for these groups are transferred to fig. 28, which has been already mentioned. The following catalogue shows the stations that were employed in the discussion:

Northeast China.—Zi-ka-wei, Pekin, Vladivostok.

Japan.—Tokio, Nagasaki, Hieroshima, Osaka, Kioto.

North India.—Leh, Murree, Simla.

Central India.—Darjeeling, Lahore, Lucknow, Calcutta.

South India.—Pachmari, Bangalore, Nagpur, Bombay, Madras.

Batavia and Mauritius.

North New South Wales.—Albury, Bathurst, Deniliquin.

South New South Wales.—Goulburn, Newcastle, Sidney.

Kimberley.—Kimberley, Bloemfontein.

Inland Cape Colony.—Grahamtown, Lovedale, Aliwal North.

Coast Cape Colony.—Cape Town, Port Elizabeth, East London, Mossel Bay.

North Russia.—Archangel, St. Petersburg.

East Russia.—Moscow, Katharinenburg.

Russia and Southwestern Siberia.—Odessa, Tiflis, Baku.

Central Siberia.—Tomsk, Barnaul, Irkutsk.

France.—Paris.

Spain.—Madrid, Lisbon, San Fernando, Coimbra.

South Europe.—Pola, Budapest, Kalocsa, O'Gyalla, Vienna.

United States.—Pacific coast States, 20 stations; northern Plateau, 33 stations; southern Plateau, 19 stations; Lake region, 31 stations; west Gulf States, 41 stations; North Atlantic States; 26 stations; South Atlantic States, 32 stations; total number of United States stations, 202.

North Argentina.—Villa Formosa, Corrientes, Salta, Tucuman, Santiago, Goya, Hernandarias.

Central Argentina.—Cordoba, San Juan, Parana, Rosario, Carcaraña, Estancia San Juan, Buenos Ayres, Chacra de Matanzas, Bahia Blanca, Colonia Chabut.

In all cases the mean annual pressures were extracted from the observatory reports; these were reduced to the same elevation of the barometer, usually that for 1899, and all known corrections were applied. The mean for the homogeneous series was computed and then the variation of each year from this mean, the result being always changed, if necessary, into units of 0.001 inch in the English system. These annual variations were plotted as curves on sheets for the several countries, so that the several districts could be studied for their characteristic types. The stations were finally grouped as indicated in the the catalogue, and the larger district means, including about all the region having the same type of curve, were transferred to fig 28. It was very interesting to study these local curves, and to note that the same pressure variations in fact prevail over very large districts of the earth, though varying from one region to another. The variations were also transferred to charts of the earth, one for each year, and it was found that while there is an irregularity from year to year, it was possible to discover some very suggestive features. I regret that these charts can not be reproduced in this connection. Some years

Table No. 23.—*The variations of the annual mean atmospheric pressures in many districts of the earth, in units of 0.001 inch.*

Stations.	1873.	1874.	1875.	1876.	1877.	1878.	1879.	1880.	1881.	1882.	1883.	1884.	1885.	1886.	1887.	1888.	1889.	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.
Northeast China	− 8	− 8	+ 4	−16	+12	+32	−20	−32	− 4	+24	+24	+12	−24	+20	− 4	+ 4	+24	− 8	+ 4	− 4	+ 4	+12	−12	− 4	+24	−12	−12
Japan	0	+ 8	+ 4	0	+ 4	+12	−12	+ 4	+ 8	+12	+ 8	− 4	+ 8	+16	− 8	−12	0	−24	+ 8	− 8	+ 8	+16	− 8	+12	+16	− 8	− 8
North India	.....	.....	+ 6	− 1	+ 7	+18	− 9	− 3	+11	−13	−16	+ 1	+ 3	− 5	− 3	+ 7	+13	− 9	+20	+ 6	−14	−10	+13	+18	+ 7	− 1	+14
Central India	.....	.....	− 8	− 2	+35	−16	− 6	0	+ 3	− 9	− 4	+12	+16	+ 7	− 3	+ 8	− 9	−11	−16	+24	+ 6	− 8	− 3	+ 5	− 1	−16	− 3
South India	.....	.....	−10	− 8	+31	− 6	−14	0	+ 7	− 9	− 5	+11	+20	− 1	+ 2	+19	+ 8	− 3	+17	− 9	− 6	−10	+ 2	+ 5	− 7	−15	+12
Batavia	− 4	+ 4	− 8	+ 8	+43	+ 4	−20	− 8	−16	0	−16	−24	−20	− 8	− 8	−20	+12	+ 4	−24	−12	− 4	+ 4	0	+20	+ 8	−12	+12
Mauritius	0	− 6	− 3	+ 8	− 7	+ 3	+16	+24	+22	+16	− 7	−12	0	+ 8	+12	−12	+17	+ 2	−14	−16	+10	−22	−13	+14	0	−13	.....
Means	− 3	0	− 2	− 2	+20	+11	− 9	0	+ 9	+ 3	− 2	+ 1	+ 7	+ 5	− 2	+ 3	+12	− 7	+15	− 4	+ 1	− 3	− 1	+10	− 7	−11	− 2
North New South Wales	+17	+ 9	−32	− 1	+55	−13	−39	−21	+46	− 3	−12	− 4	+12	+15	0	+42	+26	− 1	+54	− 6	−36	− 9	+ 7	+ 7	+ 5	−21	.....
South New South Wales	+ 7	− 2	−38	− 7	+40	−35	−37	− 4	+31	−18	− 5	+ 5	−43	+11	− 2	+44	+30	− 7	+27	−23	−47	− 3	− 3	− 2	+ 1	−13	.....
Means	+12	+ 4	−35	− 4	+50	−24	−38	−13	+39	−11	− 9	+ 1	+28	+13	− 1	+43	+28	− 4	+41	−15	−42	− 6	+ 2	+ 3	+ 3	−17	.....
Kimberly	.....	.....	.....	+ 5	− 2	+ 1	−25	+ 2	+16	+10	−14	− 4	+11	+38	+12	+34	+ 7	−23	−14	−15	− 3	− 5	+ 3	+12	− 4	− 9	+ 4
Inland Cape Colony	.....	.....	.....	+14	+ 9	+ 5	−16	− 7	−25	+17	− 5	+ 5	− 8	+11	+29	+ 4	+27	− 5	+ 3	−11	−11	+ 2	− 3	+ 3	−11	− 2	+19
Coast Cape Colony	.....	.....	.....	+30	+ 8	+ 4	− 3	+17	+18	+ 4	−26	+25	+17	+11	+14	−18	+ 2	−15	+ 5	−24	−13	− 2	+ 3	+15	+ 5	+ 2	+18
Means	.....	.....	.....	+16	+ 5	+ 3	−15	+ 4	+20	+10	−15	+ 9	+ 7	+20	+18	+ 7	+12	−14	− 2	−17	− 9	− 2	+ 1	+10	− 3	− 3	+14
North Russia	.....	−24	+55	+39	+35	−75	+20	−71	+16	+24	+43	+55	+ 4	+55	−35	− 8	+79	−47	+20	− 8	−47	−16	+24	+59	+63	+20	−43
East Russia	.....	−20	−12	+ 8	+63	−16	−12	−67	− 4	−32	+32	−12	−20	+24	−47	−55	+51	+20	0	+36	−16	− 8	−12	+36	+39	+16	−24
Southwest Siberia	.....	+28	−20	+ 8	+32	−24	−12	− 8	+ 8	+16	+20	+20	+ 4	+16	− 8	−16	+ 8	+28	+36	−12	−16	+12	−24	+12	+ 4	+24	−20
Central Siberia	.....	0	−12	−24	+67	+40	−32	−16	− 4	−12	+ 8	0	+ 8	+28	−32	−24	+43	−39	−20	+43	+16	−20	+20	− 8	+24	− 4	+24
Means	.....	− 3	+ 3	+ 8	+49	−19	− 9	−41	+ 4	− 1	+26	+16	− 1	+31	−27	−26	+45	+14	+ 9	+15	−16	− 8	+ 2	+25	+33	+14	−16
Paris	0	+39	+24	−43	−28	−24	−12	+20	0	+ 8	+20	+35	+12	−43	+39	0	+12	+16	−24	−32	+20	+16	−43	+51	+20	+32	.....
Spain	−12	+20	+12	−28	+16	0	−12	+16	−32	+55	+32	+ 4	−32	−20	−28	− 8	+20	−16	−12	−39	− 4	+20	−47	+35	+24	0	− 8
South Europe	+12	−12	+ 4	−24	−12	−35	−28	+32	+16	+32	+12	+39	−12	−16	+20	+12	−12	+16	−35	−16	+12	+28	−43	+16	+16	+12	+16
Means	0	+24	+13	−32	− 8	−20	−17	+23	− 5	+32	+21	+26	−11	−26	+10	+ 1	+ 7	+ 5	−25	−29	+ 9	+21	−44	+34	+20	+15	+ 4
Pacific coast	+15	+ 3	+22	0	−12	−31	− 4	+19	− 1	+ 7	+16	−27	−15	− 2	+ 1	−16	−15	+16	+ 3	+ 6	+ 9	+ 8	+ 8	− 4	+ 7	− 1	+ 1
North Plateau	.....	.....	.....	−20	− 8	−28	−24	− 7	− 4	− 4	+18	− 7	− 5	0	− 6	+11	+16	+14	− 7	+ 8	−18	− 5	+10	− 5	+12	+12	− 4
South Plateau	.....	.....	.....	+ 3	− 9	−25	− 9	− 8	−13	− 1	+11	− 5	+ 8	+ 5	− 1	+ 4	−22	+20	− 5	− 2	−12	+14	+12	+ 8	+ 6	+ 1	− 3
Lake region	−15	+28	0	−12	+11	−51	+10	− 6	+ 2	+ 7	+15	− 2	−20	0	+ 1	+24	+ 8	+12	+ 7	+20	−32	−12	−10	+ 6	+11	−11	+ 2
West Gulf States	− 1	+19	+ 8	+ 7	+ 2	−49	+ 7	+13	−15	+ 1	+15	− 6	−12	− 9	− 5	+ 8	+ 6	+11	+ 1	+ 4	−21	+12	+ 6	+ 9	0	− 3	0
North Atlantic States	−10	+34	+ 4	−20	+ 5	−67	+10	−21	− 8	+20	−31	− 1	−33	− 6	− 1	− 1	−11	− 1	+15	− 8	−16	+17	− 8	+10	+ 8	− 3	+11
South Atlantic States	− 1	+31	+ 9	− 2	−10	−58	+17	−25	−14	+13	−18	− 8	−31	−13	− 2	+ 4	−12	+12	+ 6	+ 6	−19	+ 9	− 7	+15	+ 2	− 1	+ 4
Means	− 2	+23	+ 9	− 6	− 3	−44	+ 1	+ 8	− 8	+ 6	+18	− 7	−15	− 4	− 2	+ 5	+ 2	+12	+ 3	+ 5	−16	+ 6	+ 2	+ 7	+ 7	− 1	+ 2
North Argentina	− 4	+12	+ 8	+ 8	−20	+ 4	+ 8	− 4	−12	+20	+16	+16	+16	+16	− 8	−20	+ 8	+20	− 8	+ 8	+24	0	− 4	.....	.....	.....	.....
Central Argentina	−16	+20	+20	+12	−12	+16	+16	−12	−20	+ 4	+ 4	0	− 4	+20	+12	− 8	+12	+28	− 4	+ 8	+24	+16	− 8	−12	+ 8	− 8	.....
Means	−10	+16	+14	+10	−16	+10	+12	− 8	−16	+12	+10	+ 8	+ 6	+18	+ 2	−14	+10	+24	− 6	+ 8	+24	+ 8	− 6	−12	+ 8	− 8	.....

show that in North America and South America the annual pressure prevails in excess, or that the variation is positive, as 1874, 1875, 1883, 1890, 1892, 1897. Others show that the entire Northern Hemisphere is in defect as a whole, as 1876, 1878, 1879, 1885, 1887, 1893. Others show the Northern Hemisphere to be in excess, as 1883, 1896, 1897. Other years are more irregular. I have the impression that there is a westward movement of the defect in pressure, or of the negative residuals; and that there are similar groups separated by intervals of seven or eight years. This subject will require an exhaustive study by meteorologists in the future, and much valuable information will be extracted from it.

If the positive values of the pressure variations be added together for each year, and also the negative values by themselves, the result may be indicated as it is plotted in the curves of fig. 29' The upper curve is for the positive and the lower for the negative summation, but these curves show, since they rise and fall together, that these values do not cancel each other. The curves match fairly well with the prominence curve, and I take it to mean that *some external force is at work to raise and lower the total atmospheric pressure by a small amount from year to year.* It is probable that a more rigorous discussion would eliminate certain distortions of this curve, and show that it synchronizes very closely with the curve of the variations of solar energy. If this proves to be so, it raises some exceedingly interesting questions in cosmical meteorology.

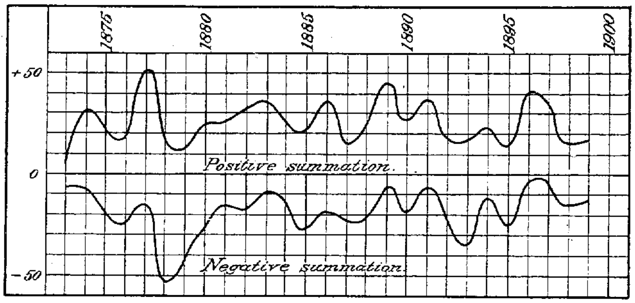

FIG. 29.—Positive and negative pressure variations over the earth as a whole for successive years, on a scale of relative numbers.

It is interesting to compare the results of this series of annual variations, 1873–1899, with those of the series, 1874–1884, studied by H. H. Hildebrandsson,[3] the latter, however, extending the details to the monthly values. The data of the Barometry Report make it possible to do this readily for the United States with little additional labor.

Returning to fig. 28, if we compare the successive pressure groups with the prominence curve, it will be seen that India and southeastern Asia are in very close synchronous agreement. This synchronism extends also to New South Wales, the Indian Ocean, and even to south Africa. In Siberia and Russia the synchronism begins to break a little and seems to be transferred somewhat toward the right, although this may be due in part to defective data. In Europe and in the United States, while the same curve is developed as to the number of the maxima and minima, the synchronism becomes more irregular. In South America, on the other hand, the synchronism is resumed very distinctly, but the *entire curve is reversed as referred to India and the Eastern Hemisphere.* Thus we perceive that around the Indian Ocean the synchronism is clearly developed; it weakens in Europe and North America, and it becomes a distinct reversal in South America. I presume that this remarkable phenomenon is due to the fact that the Pacific-Indian Ocean is quite free from frequent cyclonic disturbances, as is also South America, and that the atmospheric pressure surges back and forth between these two central or southern hemispheres, or else slowly rotates about the entire earth, probably from east to west. In North America and Europe, while the type curve reappears less perfectly, it still exists, and the disturbance may be due to the turbulent cyclonic circulation, which prevails over this region of the earth in marked contrast with the quiescent circulation of the other regions. It is, however, of much importance to have shown that changes in the annual atmospheric pressure of the earth synchronize approximately with the typical output of solar energy.

From this rapid survey of the cosmical meteorological problem, it is obvious that meteorology has large interests in solar and terrestrial magnetism. The annual reports of magnetic observatories are usually published several years after the records are made, hence, if meteorology is to insure any progress in seasonal forecasting, it evidently must possess its own magnetic apparatus, so that the state of the solar-terrestrial field may be known in connection with current meteorological phenomena. It must be conceded that considerable scientific skill will be required to bring this system of cosmical forces into control for the benefit of mankind, but I do not see how it can be doubted that the true pathway of research is already open before us. It is to be hoped that meteorologists generally will take up these cosmical problems, and compute the necessary homogeneous systems, so that it may become possible to advance promptly to practical results.

* * *

HAWAIIAN CLIMATOLOGICAL DATA.

By CURTIS J. LYONS, Territorial Meteorologist.

Rainfall data for July, 1902.

Stations.	Elevation.	Amount.
HAWAII.		
HILO, e. and ne.	*Feet.*	*Inches.*
Waiakea	50	12.82
Kaumana	1,250	13.80
Pepeekeo	100	11.75
Hakalau	200	11.88
Honohina	300	9.03
Puuohua	1,050	13.73
Laupahoehoe	500	9.72
Ookala	400	4.48
HAMAKUA, ne.		
Kukaiau	250	2.69
Paauhau (Mill)	300	1.49
Honokaa (Muir)	425	1.22
Kukuihaele	700	2.22
KOHALA, n.		
Niulii	200	3.44
Kohala (Mission)	521	3.13
Kohala (Sugar Co.)	235	2.42
Hawi Mill		3.70
Puuhue Ranch	1,847	1.65
Waimea,	2,720	1.04
KONA, w.		
Holualoa	1,350	11.97
Kealakekua	1,580	13.23
Napoopoo	25	5.16
KAU, se.		
Kabuku Ranch	1,680	3.09
Pahala	850	0.49
PUNA, e.		
Volcano House	4,000	3.79
Olaa, Mountain View	1,700	16.80
MAUI.		
Waiopae Ranch, s	700	0.00
Kaupo (Mokulau), s	285	5.42
Kipahulu, s	300	8.41
Nahiku, ne	1,600	22.75
Nahiku, ne	850	14.36
Haiku, n	700	2.98
Kula (Erehwon), n	4,500	5.02
Puuomalei, n	1,400	2.40
Kula (Waiakoa), n	2,700	3.49
Paia, n	180	1.32
Haleakala Ranch, n	2,000	1.84
MAUI—*Continued.*	*Feet.*	*Inches.*
Wailuku, ne	200	0.04
OAHU.		
Punahou (W. B.), sw	47	2.87
Kulaokahua, sw	50	1.76
Makiki Reservoir	120	2.99
U. S. Naval Station, sw	6	0.88
Kapiolani Park, sw	10	0.33
Manoa (Woodlawn Dairy), c.	285	10.49
School street (Bishop), sw	50	2.27
Insane Asylum, sw	30	2.32
Kalihi-Uka, sw	260	8.76
Nuuanu (W. W. Hall), sw	50	2.45
Nuuanu (Wyllie street), sw	250	5.15
Nuuanu (Elec. Station), sw	405	4.05
Nuuanu (Luakaha), c	850	12.66
Waimanalo, ne	25	1.68
Maunawili, ne	300	6.13
Ahuimanu, ne	350	6.56
Ewa Plantation, s	60	0.25
Waipahu, s	200	0.00
Moanalua, sw	15	2.02
Rhodes gardens (Manoa)	300	13.05
Nahuina (Castle)	1,150	10.21
Tantalus Heights (Frear)	1,360	10.95
KAUAI.		
Lihue (Grove Farm), e	200	2.38
Lihue (Molokoa), e	300	2.29
Lihue (Kukaua), e	1,000	6.68
Kealia, e	15	1.14
Kilauea, ne	325	4.85
Hanalei, n	10	6.55
Eleele, s	200	0.25
Wahiawa Mountain, s	2,100	12.85
McBryde (Residence)	850	4.15
Lawai	450	4.96
East Lawai	800	3.49
West Lawai	200	1.68
Delayed June reports.		
Waimanalo		7.45
Kailua (Hawaii)		5.78
Nuuanu (Wyllie street), sw		1.87
Wahiawa (Oahu)		3.07

NOTE.—The letters n, s, e, w, and c show the exposure of the station relative to the winds.

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[3] Quelques recherches sur les centres d'action de l'atmosphère, par H. H. Hildebrandsson, Stockholm, 1897.