ON THE DYNAMICS OF GEOSTROPHIC WINDS.*

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[Received June 27—Read December 16, 1925.]

1. *Introduction.*

In an earlier paper[1] I defined a geostrophic wind as one such that the pressure terms in the equations of horizontal motion are balanced mainly by the terms that involve the earth's speed of rotation. All terrestrial winds of great horizontal extent belong to this class. The present paper is an attempt to determine what winds would be expected to arise on dynamical principles from certain distributions of temperature. If the level surfaces within the atmosphere were isothermal no winds could exist; but actually the air is heated in such a way as to produce appreciable differences of temperature over the level surfaces, and accordingly winds are maintained. The actual distribution of temperature is here regarded as given, so that considerations as to how the heat supplied from the sun is redistributed in the atmosphere by radiation, convection, and turbulence do not enter. It is found that such problems have an intimate relation to the theory of the tides in a homogeneous incompressible ocean of uniform depth (different from that given by Margules). The depth of such an ocean must be rather less than the height of the homogeneous atmosphere. The disturbing potential of the tidal theory is replaced by a function of the mean conditions over the level surfaces and of the departures of the temperatures from mean values over level surfaces. The solution can be found by known methods when friction is ignored. Fair quantitative agreement with observation is obtained when the theory is applied to the monsoons; application to the diurnal and semi-diurnal motions would probably be possible, but has not been carried out here. The nature of the general circulation, in the absence of friction, has been determined, but is such that friction would introduce serious modifications. A discussion of the nature of these modifications is given, and indicates the dynamical necessity for a permanent region of low pressure around each pole, with a continual interchange of air between high and low latitudes by means of cyclones. These must apparently, if they are to produce the required effect, have a structure not unlike that described by Bjerknes. It appears, however, that friction must play the most important part in producing them in the first place.

2. *The analogy between tides and large-scale motions of the atmosphere.*

It was shown in the paper already mentioned that the equations of motion of the atmosphere can be written, with much greater accuracy than is ever required, in the form

$$\rho\left(\frac{du}{dt} - 2\omega v \cos\theta\right) = -\frac{1}{R}\frac{\partial p}{\partial \theta} + F \qquad . \qquad . \qquad (1)$$

* See note in reference to this paper on p. 126.

[1] "On the Dynamics of Wind," *Q.J.R.Meteor.Soc.*, **48**, 1922, pp. 29-47.

$$\rho\left(\frac{dv}{dt}+2\omega u\cos\theta\right)=-\frac{1}{\varpi}\frac{\partial p}{\partial\phi}+G \quad . \quad . \quad (2)$$

$$0=-g\rho-\frac{\partial p}{\partial z} \quad . \quad . \quad . \quad . \quad (3)$$

In these equations ρ is the density, θ the co-latitude, ϕ the east longitude, z the height above sea-level and t the time; p denotes the pressure, u the velocity southwards, v eastwards and w upwards; R is the radius of curvature of the meridian at sea-level, and ϖ the distance of the particle considered from the axis of the earth; g is the acceleration due to gravity, and ω the earth's angular velocity of rotation; F and G are the southward and eastward components of frictional force per unit volume.

In a state of equilibrium the pressure and density must be constant over a set of surfaces which, with sufficient accuracy, may be identified in a shallow atmosphere such as ours with the surfaces of uniform height above sea-level. Let the suffix zero attached to any quantity indicate that the value of that quantity at the same position in the equilibrium state is meant, and let an accent indicate the departure from the equilibrium value; thus, for instance,

$$p=p_0+p'\,;\ \rho=\rho_0+\rho' \quad . \quad . \quad . \quad . \quad (4)$$

Evidently p_0 and ρ_0 are functions of z alone, while (3) gives

$$g\rho_0+\frac{\partial p_0}{\partial z}=0 \quad . \quad . \quad . \quad . \quad (5)$$

The equation of continuity is

$$\frac{\partial\rho}{\partial t}+\frac{1}{R\varpi}\frac{\partial}{\partial\theta}(\rho\varpi u)+\frac{1}{\varpi}\frac{\partial}{\partial\phi}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad . \quad . \quad (6)$$

The pressure and the density tend to zero at a great height; hence (3) gives

$$p=\int_z^\infty g\rho dz \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In this equation, and in others with infinity as the upper limit, it is legitimate to ignore the variation of gravity with height; for the density falls off with height so rapidly that where the value of gravity differs appreciably from its value at sea level the density is so small that only a negligible contribution is made to the integral.

If now we consider only small deviations from rest relative to the rotating earth, u, v, w and ρ' are all small, and we may neglect their squares and products. Thus d/dt can be replaced everywhere by $\partial/\partial t$, and ρu, ρv, ρw by $\rho_0 u$, $\rho_0 v$, $\rho_0 w$.

Let us now integrate (6) from sea-level up to a great height. Subject to the approximations just made, the first term gives, by (7), $\frac{1}{g}\frac{\partial p_s}{\partial t}$ where p_s is the pressure at sea-level. The last term gives simply the difference between the values of ρw at sea-level and a great height. Both of these are zero; that at sea level because w is there zero, and that at a great height because if it were finite it would imply a loss of matter from the earth at the top of the atmosphere. If then we denote $\int_0^\infty \rho_0 u dz$ by U, and $\int_0^\infty \rho_0 v dz$ by V, the equation becomes

$$\frac{1}{g}\frac{\partial p_s}{\partial t}+\frac{1}{R\varpi}\frac{\partial}{\partial \theta}(\varpi U)+\frac{1}{\varpi}\frac{\partial V}{\partial \phi}=0 \quad . \quad . \quad . \quad (8)$$

Similarly we may integrate (1) and (2) with regard to the height. If we write P for $\int_0^\infty p dz$, they become

$$\frac{\partial U}{\partial t}-2\omega V\cos\theta=-\frac{1}{R}\frac{\partial P}{\partial \theta}+\int_0^\infty F dz \quad . \quad . \quad (9)$$

$$\frac{\partial V}{\partial t}+2\omega U\cos\theta=-\frac{1}{\varpi}\frac{\partial P}{\partial \phi}+\int_0^\infty G dz \quad . \quad . \quad (10)$$

Evidently the integrals that constitute the last terms of these equations represent the total frictional force acting on a column of air of unit horizontal section. There being no friction at the top of the atmosphere, these integrals depend only on the skin friction over the ground and on the shearing stresses across vertical planes.

Suppose now that

$$p=R'\rho T \quad . \quad . \quad . \quad . \quad (11)$$

where R' is a constant. In an atmosphere consisting of a perfect gas of uniform composition T would be the absolute temperature; but in the actual atmosphere T will differ somewhat from the absolute temperature, on account of variations from place to place in composition, and especially in humidity. It will be called the virtual temperature. We have from (3)

$$\frac{\partial p}{\partial z}=-\frac{gp}{R'T} \quad . \quad . \quad . \quad . \quad (12)$$

whence

$$p=p_s\exp\left(-\int_0^\infty\frac{g dz}{R'T}\right). \quad . \quad . \quad . \quad (13)$$

We can now obtain a relation between the time-variations of P and p_s. Any change of the distribution of pressure in the column of air over a place can be supposed carried out in two stages. First, new air can be added to the column at every height in such a way as to change p_s to the appropriate new value, without change of temperature at any height. We see from (13) that this will change the pressure at all heights in the same ratio. Thus, at a definite height z, p will be increased by $p'_s\, p_0/p_{s0}$, to the first order. Next, let the air at all heights be heated or cooled to the new temperatures, the changes of temperature by hypothesis being small, without any horizontal outflow taking place, so that p_s is not altered. Then, since the absence of outflow ensures that the column of air retains the original cross section, the height of any element dz becomes Tdz/T_0, and therefore the air at height z is lifted a distance $\int_0^z \frac{T'dz}{T_0}$. Thus a layer of air of this thickness, which was formerly below the level z, is now above that level, and its weight, by (3), must now be balanced by the pressure at z. Thus the pressure at z is increased by $g\rho_0 \int_0^z \frac{T'dz}{T_0}$. In all, therefore,

$$p' = \frac{p_0}{p_{s0}} p'_s + g\rho_0 \int_0^z \frac{T'dz}{T_0} \quad . \quad . \quad . \quad (14)$$

Integrating with regard to z, we obtain

$$P' = \frac{P_0}{p_{s0}} p'_s + \int_0^\infty g\rho_0 \int_0^z \frac{T'dz}{T_0} dz \quad . \quad . \quad (15)$$

This may now be used to eliminate $\partial p_s/\partial t$ from (8). We find

$$-\frac{1}{g}\frac{\partial P'}{\partial t} - \int_0^\infty \rho_0 \int_0^z \frac{1}{T_0}\frac{\partial T}{\partial t} dz dz + + \frac{P_0}{p_{s0}} \left[\frac{1}{R\varpi}\frac{\partial}{\partial \theta}(\varpi U) + \frac{1}{\varpi}\frac{\partial V}{\partial \phi} \right] = 0 \quad (16)$$

Now let us compare (9), (10) and (16) with the equations occurring in the theory of slow tidal motions in an incompressible ocean of uniform depth. The latter are

$$\frac{\partial u}{\partial t} - 2\omega v \cos\theta = -\frac{g}{R}\frac{\partial}{\partial \theta}(\zeta - \bar{\zeta}) + F_1 \quad . \quad . \quad (17)$$

$$\frac{\partial v}{\partial t} + 2\omega u \cos\theta = -\frac{g}{\varpi}\frac{\partial}{\partial \phi}(\zeta - \bar{\zeta}) + G_1 \quad . \quad . \quad (18)$$

$$\frac{\partial \zeta}{\partial t} + \frac{h}{R\varpi}\frac{\partial}{\partial \theta}(\varpi u) + \frac{h}{\varpi}\frac{\partial v}{\partial \phi} = 0 \quad . \quad . \quad (19)$$

Here ζ is the elevation of the free surface, $\bar{\zeta}$ is the height of the equilibrium tide, F_1 and G_1 are the components of the frictional force per unit volume, averaged through the depth, h is the depth of the ocean, and the other symbols have meanings analogous to those they bear in the meteorological problem.

We see at once that, if U and V are to correspond to u and v respectively, P' must correspond to $g(\zeta - \bar{\zeta})$, F_1 and G_1 with $\int_0^\infty F dz$ and $\int_0^\infty G dz$, P_0/p_{s0} with h, and $P' - \int_0^\infty \rho_0 \int_0^z g \frac{T'dz}{T_0} dz$ with $g\zeta$. With these substitutions the analogy between the two problems is complete. If in the tidal problem we write ζ' for $\zeta - \bar{\zeta}$, P' corresponds to ζ', and $\int_0^\infty \rho_0 \int_0^z \frac{T'dzdz}{T_0}$ with $-\bar{\zeta}$.

If then the tidal problem is solved for the appropriate values of $\bar{\zeta}$ and the frictional forces, the results can be adapted immediately to give the values of P, U and V in the meteorological problem, and then from (8) or (15) we can find the variation of p_s. Thus for every problem of the motion of the atmosphere produced by changes of temperature or composition over large regions of the earth's surface there is a corresponding tidal problem, whose solution, if known, will enable us to infer that of the meteorological problem.

In this discussion the actual distribution of virtual temperature is supposed to be among the data of the problem. The same methods could be applied if we were given the rate of supply of heat to all parts of the atmosphere from outside, but then an extra differential equation would have to be included, expressing the conditions of heat transference. Since the distribution of temperature is better known than the theory of the transfer of heat in the atmosphere, it seems better in the present state of knowledge to use the former as the starting point of the dynamical theory. It must be noticed, however, that in some of the free oscillations of the atmosphere temperature changes will be produced by the pressure changes that occur, and these must be allowed for if the theory is applied to such oscillations.

The depth of the equivalent ocean is P_0/p_{s0}, which can be found roughly from the results of W. H. Dines,[2] who has tabulated the pressure and density at intervals of 1 km. up to 20 km. above sea level at numerous stations. The value of $\int p dz$ from sea level up to 20 km. has been found by means of the Gregory formula for single integration.[3] The integral from 20 km. up to an indefinitely great height can be easily found; for T is practically constant at these heights, except where the height is so great that the density is too small to affect the result, so that this integral can be calculated from the pressure and density at 20 km. as for an isothermal atmosphere. Its value is $p^2/g\rho$ evaluated for 20 km.

The results are as follows. All are annual means.

	p_s	$\int_0^{20} p dz$	$\int_{20}^{\infty} p dz$	$\int_0^{\infty} p dz$	P/p_s	H
England S.E. ...	1014	7033	353	7386	7.28	8.24
Equator	1012	7287	314	7601	7.51	8.78

All pressures are in millibars and all heights in kilometres. The probable error of P, given p_s, does not exceed 4 units. The last column gives the height of the homogeneous atmosphere, calculated as $p_s/g\rho_s$. It is seen that the depth of the equivalent ocean is less variable than that of the homogeneous atmosphere and distinctly smaller. In an atmosphere at rest both would of course be constant. In investigating the theory of atmospheric motions mean values over the surface of the earth are really required, but since these are not available, and since the error will in any case be of the second order, it will be assumed in what follows that the depth of the equivalent ocean is 7.30 km.

3. *Periodic and steady motions without friction.*

The above theorem finds an immediate application when the temperature disturbances are periodic functions of the time or expressible as sums of such functions and friction is ignored. The

[2] "The Characteristics of the Free Atmosphere," *Meteor. Office, Geophys. Mem.* No. 13, 1919, p. 63.

[3] Whittaker and Robinson, *Calculus of Observations*, 1924, p. 143.

analogous tidal problem can in any such case be solved completely by known methods; either the method of Laplace or that of Hough may be used.[4] Thus every problem of this type may be regarded as soluble.

Certain periodic motions of a gas covering a rotating spheroid have been considered by Margules,[5] who showed that they could be reduced to analogous tidal problems; but Margules's solutions are inapplicable to the atmosphere, an important boundary condition being incorrect in his work. He assumes that there is no vertical motion in the atmosphere, which amounts to supposing the atmosphere enclosed within a rigid external boundary. To prevent vertical motion, variations of pressure over this boundary are necessary; whereas the actual atmosphere is not in contact at the top with anything capable of exerting pressure. The correct boundary condition is that the atmosphere is under no stress from outside, which is what has been assumed in the present paper. The theorem just proved differs from Margules's result in two respects: I find that, if U and V are eliminated, $\int_0^\infty p\,dz$ satisfies the same differential equation as is satisfied by ζ' in the theory of the tides, whereas Margules finds that this equation is satisfied by p; and in my theory the depth of the equivalent ocean is P_0/p_{s0}, whereas in that of Margules it is the height of the homogeneous atmosphere $p_{s0}/g\rho_{s0}$. It may be noted that P/p_s is the height of the centre of mass of a vertical column.

3·1. *The monsoons and similar winds.*

A problem readily soluble by methods akin to those used in tidal theory is that of a periodic wind produced by local heating, so that the pressure changes within a certain region, small in area in comparison with an octant of the earth's surface, are great in comparison with the changes elsewhere. In such a case we may ignore variations of θ within the region affected and replace $\omega\cos\theta$ by a constant Ω. Let us suppose that all changes are proportional to $e^{i\gamma t}$, where γ is a constant, and take polar coordinates r and ϕ in the earth's surface,[6] the origin being near the centre of the region considered. Then the equations 2 (9) and (10) transform in the usual way to

$$i\gamma U - 2\Omega V = -\frac{\partial P}{\partial r} \quad . \quad . \quad . \quad . \quad (1)$$

$$2\Omega U + i\gamma V = -\frac{1}{r}\frac{\partial P}{\partial \phi} \quad . \quad . \quad . \quad . \quad (2)$$

where U and V are now the results of treating the radial and transverse velocities as in chapter 2. Solving these we have

$$U = -\frac{1}{4\Omega^2-\gamma^2}\left(i\gamma\frac{\partial P}{\partial r} + \frac{2\Omega}{r}\frac{\partial P}{\partial \phi}\right) \quad . \quad . \quad . \quad (3)$$

$$V = -\frac{1}{4\Omega^2-\gamma^2}\left(\frac{i\gamma}{r}\frac{\partial P}{\partial \phi} - 2\Omega\frac{\partial P}{\partial r}\right) \quad . \quad . \quad . \quad (4)$$

[4] Accounts of both methods, with references, are given in Lamb's *Hydrodynamics*.

[5] *Wien, Sitzber. Ak. Wiss.*, **99** (iia), 1890, pp. 204-227; **101** (iia), 1892, pp. 597-626, **102** (iia), 1893, pp. 11-56, 1369-1421.

[6] In previous sections ϕ has been used to denote the longitude, but no confusion can arise.

At the same time 2 (16) transforms into

$$i\gamma P' + gh\left[\frac{1}{r}\frac{\partial}{\partial r}(rU) + \frac{\partial V}{r\partial\phi}\right] = i\gamma g\int_0^\infty \rho_0 \int_0^z \frac{T'}{T_0}\,dz\,dz \quad . \quad (5)$$

$$= i\gamma Q \quad . \quad . \quad . \quad . \quad . \quad (6)$$

say. Here h is the depth of the equivalent ocean. Substituting in this from (3) and (4), and cancelling the factor $i\gamma$, we have

$$-\frac{gh}{4\Omega^2-\gamma^2}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P'}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 P'}{\partial\phi^2}\right] + P' = Q \quad . \quad (7)$$

Now Q can always be expressed as the sum of terms of the form

$$Q_n = AJ_n(\lambda r)\exp(i\gamma t \pm in\phi) \quad . \quad . \quad . \quad (8)$$

where A and λ are constants and n is a positive integer. Then (7) is satisfied by making P' the sum of terms of the form $BJ_n(\lambda r)\exp(i\gamma t \pm in\phi)$. For if P' is equal to a term of this form,

$$r\frac{\partial}{\partial r}\left(r\frac{\partial P'}{\partial r}\right) + (\lambda^2 r^2 - n^2)P' = O \quad . \quad . \quad . \quad (9)$$

and (7) is equivalent to

$$\left(\frac{gh\lambda^2}{4\Omega^2-\gamma^2} + 1\right)B = A \quad . \quad . \quad . \quad (10)$$

Thus the general solution is obtained, except in the special case when the coefficient of B in (10) vanishes; if such a case were to arise special treatment would be necessary. It is enough to notice at present that the coefficient is essentially positive when γ is less than 2Ω; thus the solution will succeed for all periods much exceeding a day.

Returning now to 2 (15) we have

$$hp'_s = P' - Q$$

$$= -Q\Big/\left(1 + \frac{4\Omega^2-\gamma^2}{gh\lambda^2}\right) \quad . \quad . \quad . \quad (11)$$

Two extreme cases evidently arise, according as the second term in the denominator is large or small. If the horizontal extent of the disturbance is very great, so that λ is very small, we see that hp'_s/Q is very small. Thus temperature disturbances over a very large region give rise to small changes of surface pressure. The horizontal extent being great, the pressure gradients are small for two reasons, and therefore there are only slight winds at sea level. If, however, the horizontal extent is small, and thus λ great, P'/Q is small, and we have approximately

$$p'_s = -Q/h \quad . \quad . \quad . \quad . \quad (12)$$

Consider now the critical value of λ that makes $(4\Omega^2-\gamma^2)/gh\lambda^2$ equal to unity. If the latitude is 45°, we have approximately

$\Omega = 5\times10^{-5}/1$ sec.; $g = 981$ cm.²/sec.; $h = 7.3\times10^6$ cm.,

while for all motions of long period γ^2 is very small compared with $4\Omega^2$. Thus

$$\frac{gh}{4\Omega^2-\gamma^2} = 7.1\times10^{16}\ \text{cm.}^2 \quad . \quad . \quad . \quad . \quad (13)$$

and the critical value of $1/\lambda$ is 2.7×10^8 cm. or 2700 km.

A direct comparison of the theory with observation is difficult, on account of the permanent general circulation over the earth, which in most places overwhelms the changes due to periodic local heating. The only exceptions are Asia and, possibly, Antarctica. The extent of Asia along a central meridian, omitting the projections of southern India and Indo-China, amounts to 55° of latitude, or about 6000 km. Thus it is comparable with the size above found to be critical; further, it is so large that the variations of $\cos\theta$ within the region affected will produce appreciable effects. As a first approximation let us give θ its mean value, about 45°, and suppose that λ has just its critical value. Then p'_s should be equal to $-Q/2h$.

The annual variation of temperature at great heights in Central Asia is unfortunately unknown. If we make the simple assumption that it varies in the same ratio at all heights, we can write

$$Q=\frac{gT'_s}{T_{os}}\int_0^\infty \rho_0 z\,dz \qquad (14)$$

But

$$\int_0^\infty g\rho z\,dz=-\int z\,dp=-[pz]_0^\infty+\int_0^\infty p\,dz=P \qquad (15)$$

since the integrated portion vanishes at both limits. Thus

$$Q=\frac{T'_s}{T_{os}}P_0 \qquad (16)$$

But

$$h=P_0/p_0 \qquad (17)$$

and we have simply

$$p'_s=-\frac{Q}{2h}=-\tfrac{1}{2}\frac{T'_s}{T_{s0}}p_{-0} \qquad (18)$$

Recent values of the annual variations of temperature and pressure are given in Sir Napier Shaw's "*The Air and Its Ways*," Plates IV., V., XX., XXI. The range of temperature at sea-level in Mongolia, about midway between Lakes Baikal and Balkash, is about 75°F. from January to July, the mean being about 40°F. The absolute zero of temperature being −459°F., we can take T_{s0} as 499°, and then by (18) the annual range of pressure should be practically 75 mb. The actual range is about 30 mb. Thus the theory implies an annual variation of pressure decidedly greater than is observed. The difference is probably due to the temperature at great heights varying in the course of the year to a smaller relative extent than the temperature at the surface. In the paper already quoted, Dines finds from Patterson's observations that in the stratosphere over Canada, where the conditions resemble those of Central Asia more closely than in any other place that we have data for, the annual temperature range is only about a quarter of what it is at the surface, and the phase is reversed. By a rough numerical calculation from Dines's figures, I find that if the annual variations at various heights in Canada and Central Asia were in proportion, the value of Q would have to be multiplied by 0.55, and the annual pressure variation then becomes 41 millibars. This is near enough to the truth to warrant the hope that the principal factors of the problem have been taken into account.

The formula 2 (14) may be used to find the height where the annual pressure variation vanishes. Assuming 41 mb. for the annual range of pressure at sea level, and 0.15 for T'/T_0 at all heights, the vanishing of p' gives an equation for z, which is satisfied if z is 2.1 km. Thus the monsoon should be reversed in direction above this height. The layer where the annual variations of pressure and temperature are systematically opposite in sign is therefore comparatively shallow. This result is not merely a result of the present theory, for it would follow equally if the pressures at various heights were calculated from the observed surface pressures. I know of no relevant data for upper air winds in Central Asia. The reversal of the monsoon, if it occurs at any height in India, does so at a much higher level; but the monsoon in India is subsidiary to the pressure changes in Central Asia, and cannot play a large part in producing them, on account of the small size of India compared with Asia as a whole. Further, Indian conditions are probably much affected by the mechanical obstruction to movement afforded by the Himalayas. The height found here is comparable with an estimate by G. C. Simpson[7] on the Antarctic Continent, which was 1.7 km. by calculation from the surface temperature and pressure, and less than 4 km. by observation of the smoke from Mt. Erebus.

3.11. The results of this section may be compared with those of an earlier paper,[8] in which the motions due to periodic temperature changes in a homogeneous incompressible fluid layer were investigated. If we ignore the effect of friction, which was considered in the paper mentioned but found to be small, the pressure variation on the ground in such an atmosphere is nearly zero for a large enough region, but for a small region it is $-\frac{1}{2}g\rho\alpha T'H$, where α is the coefficient of volume expansion, T' the variation of temperature, supposed the same at all heights, and H is the depth. Comparing this with the present discussion, we see that in a homogeneous atmosphere the undisturbed pressure is $g\rho_0(H-z)$, so that P_0 is $\frac{1}{2}g\rho_0 H^2$ and the depth of the equivalent ocean is $\frac{1}{2}H$. Further, by 3.1 (6),

$$Q=\frac{1}{2}g\rho_0\alpha T'H^2 \quad . \quad . \quad . \quad . \quad (1)$$

in the present conditions, so that in a small region we should have by 3.1 (12)

$$p'_s=-g\rho_0\alpha T'H \quad . \quad . \quad . \quad . \quad (2)$$

just double what was found in the earlier paper. The discrepancy arises from the fact that the results of the present paper have been derived essentially for an atmosphere of indefinite height, obeying Boyle's law, and cannot be applied directly to a homogeneous atmosphere of finite height. In a homogeneous atmosphere, indeed, 2 (14) is replaced by

$$p'=p'_s+g\rho_0\alpha\int_0^z T'dz \quad . \quad . \quad . \quad . \quad (3)$$

and on integrating with regard to the depth we have

$$P'=Hp'_s+g\rho_0\alpha\int_0^H\int_0^z T'dzdz \quad . \quad . \quad . \quad (4)$$

[7] *Brit. Ant. Exp. Rept.*, 1910-13, **1**, p. 136.
[8] *Phil. Mag.*, **34**, 1917, 449-458.

The treatment of the equations of motion is as before, but we see from this equation that the depth of the equivalent ocean is not P_0/p_{s0} as formerly defined, but H. The difference arises from the presence in 2 (14), for the actual atmosphere, of the factor p_0/p_{s0}, which is a function of z, but which is replaced by unity for the homogeneous atmosphere.

3.12. Reference may be made at this stage to a property of the extreme case of slow periodic motion, namely, steady motion. It is easy to prove that the inflow of air into a vertical column contains the factor γ, and therefore is zero at all heights in a steady motion. Further, if the temperature distribution is steady no expansion is taking place within the column. Hence there is no vertical motion anywhere. Yet in this case P' will still satisfy 3.1 (7), and there will be steady horizontal winds. When the region affected is so large that differences of latitude within it must be taken into account, this result requires some modification; but it remains exactly true (as is physically almost obvious) when the distribution of temperature and pressure is symmetrical about the axis of rotation. It appears desirable to point out this fact, since meteorologists habitually speak of circulation in vertical planes as essential to the maintenance of atmospheric motion. Vertical motion is essential to the initiation of a pressure disturbance by heating, but when the appropriate redistribution of pressure has taken place it can in many cases maintain itself permanently without vertical motion; and indeed since the system here contemplated is a non-dissipative one and steady, the vertical motion must in those cases where it occurs absorb just as much energy as it renders available. In the actual atmosphere friction produces a loss of energy and a drift of air across the isobars, which must lead to a slow vertical motion; but friction and vertical motion alike are only secondary phenomena of atmospheric motion, appearing only after the primary agencies, namely rotation, density changes and acceleration have done their work.

3.2. *The General Circulation.*

Let us now apply the same methods to the theory of the general circulation. The whole distribution will be supposed symmetrical about the polar axis and not changing with the time. The notation of §2 is reintroduced. The indeterminateness of such a problem of slow steady motion[9] will be met by regarding the steady motion as the limit of a periodic motion when the period becomes very long. In these conditions V is independent of ϕ, while

$$U = -\frac{i\gamma}{4\omega^2\cos^2\theta - \gamma^2}\frac{\partial P}{R\partial\theta} \quad . \quad . \quad . \quad (1)$$

and 2 (16) gives on cancelling a factor $i\gamma$

$$-\frac{gh}{R\varpi}\frac{\partial}{\partial\theta}\left(\frac{\varpi}{4\omega^2\cos^2\theta}\frac{\partial P}{R\partial\theta}\right) + P = Q \quad . \quad . \quad (2)$$

where

$$Q = \int_0^\infty g\rho_0 \int_0^z \frac{T'}{T}\,dz dz \quad . \quad . \quad (3)$$

[9] *Cf.* Lamb, *Hydrodynamics*, §214.

In (2) we may now neglect the ellipticity of the earth, putting R equal to a, the mean radius of the earth, and ϖ equal to $a \sin \theta$. Then

$$-\frac{gh}{4\omega^2 a^2} \frac{\partial}{\sin \theta \partial \theta}\left(\frac{\sin \theta}{\cos^2 \theta} \frac{\partial P}{\partial \theta}\right) + P = Q; \quad . \quad . \quad (4)$$

or, with the notations

$$\frac{4\omega^2 a^2}{gh} = \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\cos \theta = \mu, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{d}{d\mu}\left(\frac{1-\mu^2}{\mu^2} \frac{dP}{d\mu}\right) - \beta P = -\beta Q. \quad . \quad . \quad . \quad (7)$$

For a given form of Q this equation can be solved by the methods applicable in tidal theory.[10] In the present problem the mean annual temperature at sea-level increases fairly regularly from both poles to the equator, and thus a rough representation of the conditions will be obtained by taking

$$Q = -k\mu^2 \quad . \quad . \quad . \quad . \quad (8)$$

where k is a positive constant. Then P will be an even function of μ. Further, with h equal to 7.3 km., and known values for the other quantities involved, we have

$$\beta = 12.1 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In what follows the adopted value of this important number will be 12. To solve the equation, then, we follow the classical method and assume

$$\frac{1}{\mu^2} \frac{dP'}{d\mu} = A_1\mu + A_3\mu^3 + A_5\mu^5 + \ldots \quad . \quad . \quad (10)$$

whence

$$P' = B + \tfrac{1}{4}A_1\mu^4 + \tfrac{1}{6}A_3\mu^6 + \ldots \frac{A_{2n+1}}{2n+4}\mu^{2n+4} + \ldots \quad . \quad (11)$$

$$\frac{d}{d\mu} \frac{1-\mu^2}{\mu^2} \frac{dP'}{d\mu} = A_1 + 3(A_3 - A_1)\mu^2 + 5(A_5 - A_3)\mu^4 + \ldots$$
$$+ (2n+1)(A_{2n+1} - A_{2n-1})\mu^{2n} + \ldots \quad . \quad (12)$$

Equating coefficients,

$$A_1 - \beta B = 0 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$3(A_3 - A_1) = \beta k \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$5(A_5 - A_3) - \tfrac{1}{4}\beta A_1 = 0 \quad . \quad . \quad . \quad (15)$$

and in general

$$(2n+1)(A_{2n+1} - A_{2n-1}) - \frac{\beta}{2n}A_{2n-3} = 0 \quad . \quad . \quad (16)$$

Put

$$N_{n+1} = \frac{A_{2n+1}}{A_{2n-1}} \quad . \quad . \quad . \quad . \quad (17)$$

Then by (16)

$$N_{n+1} - 1 = \frac{\beta}{2n(2n+1)} \frac{1}{N_n} \quad . \quad . \quad . \quad (18)$$

[10] Lamb, *Hydrodynamics*, 1924, §216.

It is proved in works on tidal theory that the solution giving finite velocities at the poles satisfies

$$\lim_{n\to\infty} N_n = 0 \quad . \quad . \quad . \quad . \quad (19)$$

Hence

$$N_n = -\cfrac{\beta}{\cfrac{2n(2n+1)}{1-N_{n+1}}} = -\cfrac{\beta}{2n(2n+1) + \cfrac{\beta}{(2n+2)(2n+3)} \, {}_{1+} \cfrac{\beta}{(2n+4)(2n+5)} \, {}_{1+}} \quad (20)$$

which holds for values of $n \geqslant 2$. The resulting arithmetic gives

$$N_2 = -\frac{1}{2.08}\,;\; N_3 = -\frac{1}{4.04}\,;\; N_4 = -\frac{1}{6.51}\,;\; N_5 = -\frac{1}{9.85}\,;$$

$$N_6 = -\frac{1}{13.7}\,;\; N_7 = -\frac{1}{18.3}\,;\; N_8 = -\frac{1}{23} \quad . \quad (21)$$

Then (13), (14) and (15) give

$$A_1 = -\tfrac{1}{3}\beta k\,(1-N_2) = -2.71\,k \quad . \quad . \quad (22)$$

$$B = A_1/\beta = -0.225\,k \quad . \quad . \quad . \quad (23)$$

Hence

$$\frac{1}{\mu^2}\frac{dP'}{d\mu} = -k\,(2.71\,\mu - 1.304\,\mu^3 + 0.323\,\mu^5 - 0.0496\,\mu^7 + 0.0050\,\mu^9 - 0.0004\,\mu^{11} \quad . \quad . \quad . \quad (24)$$

$$P' = -k\,(0.225 + 0.677\,\mu^4 - 0.217\,\mu^6 + 0.0404\,\mu^8 - 0.0050\,\mu^{10} + 0.0004\,\mu^{12} - \ldots) \quad . \quad . \quad . \quad . \quad (25)$$

But

$$hp'_s = P' - Q$$
$$= -k\,(0.225 - 1.0000\,\mu^2 + 0.677\,\mu^4 - \ldots) \quad . \quad . \quad (26)$$

The last result is easily checked by means of the formula

$$\int_0^1 p'_s d\mu = 0 \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The numerical results are as follows:—

μ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$-10hp_s1/k$	2.25	2.15	1.86	1.40	0.81	0.14	−0.57	−1.26	−1.90	−2.43	−2.80

We notice that p'_s has a maximum at the poles, and diminishes steadily to a minimum at the equator. Accordingly there should be easterly prevailing winds at every part of the earth's surface.

To estimate Q, we may compare the results for England and the equator given by Dines in Table X. of the memoir quoted above. We have

$$Q = \int_0^{p_s}\int_0^z \frac{T'}{T_0}\,dp\,dz \quad . \quad . \quad . \quad (28)$$

and on comparing the values of this quantity for England and the equator we find after a numerical integration that the difference is 393 mb. km. The corresponding values of μ are 0.8 and 0. Thus k is about 600 mb. km., and the corresponding difference of pressure between the poles and the equator is, by the table,

$$\frac{5.05}{10}\,\frac{k}{h} = 41 \text{ mb.}$$

The velocities of the corresponding winds are of order

$$\frac{1}{2\omega a \cos\theta}\,\frac{\partial p'_s}{\rho\,\partial\theta} = 400\,\frac{\text{cm.}}{\text{sec.}}$$

4. *The General Circulation (further discussion).*

The strength of the prevailing winds of the globe, inferred from the above theory, is in rough agreement with observational knowledge. But the occurrence of easterly winds everywhere is at variance with the facts. In the actual atmosphere there are belts of high pressure about latitude 30°, while the pressure diminishes systematically from these belts towards both poles, apart from irregularities attributable to the distribution of land. Between the belts of high pressure the prevailing winds are easterly; outside them they are westerly or south-westerly. The only neglected factor that could conceivably so alter the results is friction; and indeed it is easily seen that friction would prevent the persistence of such a motion as has just been investigated. For surface friction against a wind from the east is a force tending to push it back towards the east. Since we are considering velocities from west to east as positive, this amounts to increasing the angular momentum of the atmosphere. The process can stop only when the easterly winds at the surface have been annulled, or when westerly winds have been produced to such an extent that friction against them just balances the effect of friction against the easterly winds. The order of magnitude of the time needed for such a change is easily found. With surface winds of 4 m/s, the surface friction is about 0.5 dyne/cm.^{2}. The momentum of a column of air, 1 cm. in cross section, extending the whole height of the atmosphere, moving with velocity 4 m/s, is 4×10^5 gm. cm./sec. Thus the surface velocities would be annulled by friction in about 8×10^5 sec., or 10 days, if no other change occurred. Further, mere symmetrical heating would not regenerate them; it is inevitable that friction must on the whole push the atmosphere as much eastwards as westwards.

For the motion of the atmosphere to be steady, the angular momentum about the axis of the air between any two parallels of latitude must be steady. But the existence of any systematic general circulation implies by definition that the mean velocity is in much the same direction all round a parallel of latitude, and therefore that the surface friction on the air between two parallels is systematically either increasing or reducing its angular momentum. This effect must be balanced in some way to keep the motion steady. Ordinary viscosity is inadequate, and the only alternative is the interchange of air with other parts of the atmosphere; that is, convection (in the literal meaning of the term). Let us consider then the southward convection of angular momentum across a parallel of latitude. The distance from the axis is $\varpi + z \sin\theta$ for points in the same vertical line, and the angular momentum about the axis is therefore $\rho\,(\varpi + z \sin\theta)^2\,(\omega + \dot{\phi})$ per unit volume. The southward velocity being u, the southward flux of angular momentum across the whole parallel, at any height, is $2\pi \int_0^\infty \rho\,(\varpi + z\sin\theta)^3\,(\omega+\dot{\phi})\,u\,dz$. But the total mass north of the parallel considered cannot be varying systematically in one direction, and therefore

$$\int_0^\infty \rho\,(\varpi + z \sin\theta)\,u\,dz = 0 \quad . \quad . \quad . \quad (1)$$

Let us write

$$\int_0^z \rho\,(\varpi+z\sin\theta)u\,dz=q \quad . \quad . \quad . \quad (2)$$

so that q is proportional to the rate of flow southwards at heights less than z. Evidently q vanishes when z is 0 or ∞. The southward flow of angular momentum is then

$$M=2\pi\int(\varpi+z\sin\theta)^2(\omega+\dot{\phi})\,dq \quad . \quad . \quad . \quad (3)$$

taken through the atmosphere. Integrating by parts,

$$M=-2\pi\int_0^\infty q\frac{d}{dz}\{(\varpi+z\sin\theta)^2(\omega+\dot{\phi})\}\,dz$$

$$=-2\pi\int_0^\infty q\left[2\omega(\varpi+z\sin\theta)\sin\theta+v\sin\theta+(\varpi+z\sin\theta)\frac{dv}{dz}\right]dz \quad (4)$$

remembering that

$$(\varpi+z\sin\theta)\dot{\phi}=v \quad . \quad . \quad . \quad . \quad (5)$$

This equation is exact. Now we can put ϖ equal to $a\sin\theta$ with sufficient accuracy and neglect z in comparison with ϖ. Then

$$M=-2\pi a\int_0^\infty q\sin\theta\left(2\omega a\sin\theta+v+a\frac{dv}{dz}\right)dz \quad . \quad . \quad (6)$$

With actual values, the terms inside the bracket are respectively of order 4×10^4, 4×10^2, 4×10^5 cm./sec. The last is therefore very much the greatest. Thus approximately

$$M=-2\pi a\int_0^\infty q\sin\theta\frac{dv}{dz}dz \quad . \quad . \quad . \quad (7)$$

$$=2\pi a\sin\theta\int_0^\infty \rho va\sin\theta\,.\,u\,dz,$$

giving on integrating by parts again,

$$=2\pi a^2\sin^2\theta\int_0^\infty \rho uv\,dz. \quad . \quad . \quad . \quad (8)$$

Let us now consider the moment of the frictional forces about the axis. The eastward frictional force per unit area is $-\kappa\rho_s v_s(v_s^2+u_s^2)^{\frac{1}{2}}$, where κ is the coefficient of skin friction, about 0.002. Thus the eastward friction will produce forces on the air north of co-latitude θ, whose moment about the axis will be

$$-\iint\kappa\rho_s v_s(v_s^2+u_s^2)^{\frac{1}{2}}\,dS\,.\,a\sin\theta$$

$$=-\int_0^{2\pi}\int_0^\theta\kappa\rho_s a^3 v_s(v_s^2+u_s^2)^{\frac{1}{2}}\sin^2\theta\,d\theta\,d\phi$$

$$=-2\pi a^3\int_0^\theta\kappa\rho_s v_s(v_s^2+u_s^2)^{\frac{1}{2}}\sin^2\theta\,d\theta \quad . \quad . \quad (9)$$

Here dS denotes an element of surface area. The air north of co-latitude θ is by hypothesis not gaining or losing angular momentum, so that the gain by friction must just equal the loss by convection to the south. The expressions (8) and (9) must therefore be equal. Hence

$$-\kappa a\int_0^\theta \rho_s v_s (v_s^2+u_s^2)^{\frac{1}{2}} \sin^2\theta d\theta = \sin^2\theta\int_0^\infty \rho uv dz. \quad . \quad (10)$$

It is easy to see that this equation cannot be satisfied if the pressure distribution is symmetrical about the axis and the winds geostrophic, even with the usual modification near the surface needed to allow for friction. For u is in these conditions negligible except in the lowest kilometre, and there it is only about $\frac{1}{4}v$. Thus the ratio of the two sides is practically κa to a quarter of a kilometre, or 50 to 1. Allowance for the variation of $\sin\theta$ within the range of integration may reduce the ratio to 20 to 1, but in any case there is obviously a serious discrepancy. It is evident, therefore, that such wind velocities as actually occur at the earth's surface are incompatible with a steady symmetrical distribution of pressure. The maintenance of the polar circulation against friction requires a greater supply of angular momentum from without than can be provided merely by the drift of air across the isobars produced by surface friction.

Even if we abandon the restriction to symmetry, these considerations impose a severe restriction on the types of motion that are possible. The only change required is that u and v will involve ϕ, and the above argument leads to the equation

$$\kappa a\int_0^\theta\int_0^{2\pi} \rho_s v_s (v_s^2+u_s^2)^{\frac{1}{2}} \sin^2\theta d\theta d\phi = -\sin^2\theta\int_0^\infty\int_0^{2\pi} \rho uv dz d\phi \quad (11)$$

instead of (10). If air moves to a large extent in directions deviating widely from the parallels of latitude, the products uv may become great enough to satisfy this equation.[11] But we notice that, in the northern hemisphere for instance, air moving northwards cannot proceed far without acquiring a velocity from the west, and air moving southwards acquires a velocity from the east. Considering any interchange of air across a parallel of latitude, then uv must be negative both for the air moving north and for that moving south. This corresponds to the observed preponderance of south-westerly and north-easterly winds over those in the other two quadrants. Hence the expression on the right of (11) is necessarily positive. But every factor in the expression on the left of (11) is positive except v_s. It follows that v_s is preponderatingly positive within the region considered. This result is independent of the temperature distribution. We have therefore the further result, which is easily seen to be applicable to both hemispheres, that if there is any circulation of air in polar regions at all, this circulation can be maintained against friction only by interchange of air with regions in lower latitudes; and that such interchange must always supply angular momentum in the positive sense to the polar regions, and therefore the circulation in polar regions must always be from west to east, whatever the temperature distribution may be. This is in agreement with what is observed in the southern hemisphere. The conditions in the northern hemisphere are more complicated on

[11] At this stage it becomes clear that ρuv has an intimate relation to the Reynolds shearing stress $\rho u'v'$.

account of the large land masses, but the region of low pressure over Greenland,[12] in both summer and winter, may be considered confirmatory, especially as it is a cold region and would therefore be a region of high pressure on the simple thermal theory without allowance for friction.

Considering the matter quantitatively, we see that $\int_0^\infty \rho uv dz$ must be about 20 times as great as for a symmetrical system with similar resultant velocities. If u and v are comparable, instead of u being about $\frac{1}{4} v$ as was previously supposed, and if this remains true up to a height of about 5 km. instead of 1 km., the density being supposed uniform, the facts can be reconciled. Actually the density diminishes to some extent with height, and the height required will therefore need some increase to, say, 6 or 7 km. A further increase will be needed to allow for the fact that the correlation between u and v is incomplete, so that uv is on an average less than the product of the standard deviations of u and v. The polar circulation must therefore be maintained by streams of air whose depths are comparable with the height of the homogeneous atmosphere. The depth, indeed, appears to be so great that the north-easterly and south-westerly streams cannot exist one above the other; they must occur over different parts of the earth's surface. The surface wind, then, must blow from nearly opposite directions over different parts of the same parallel of latitude; but this implies a corresponding variation of pressure along a parallel of latitude.

Since the isobars must in any case be closed curves, this indicates a system of cyclones, whose heights must be comparable with those of the homogeneous atmosphere and the troposphere. This agrees with Dines's result that the pressure anomalies in temperate region cyclones extend up to the tropopause, but the constitution of the cyclone as fundamentally a combination of south-west and north-east winds appears to agree with the model of Bjerknes rather than with the symmetrical model of earlier writers; I think, however, that the apparent difference is one of emphasis and method of approach rather than of fact. On the other hand, the suggestions that cyclones represent either an instability of the general circulation, or oscillations about a steady general circulation, appear to be incorrect. These suggestions agree in assuming that a steady general circulation is possible; whereas it has been shown here that friction renders a steady general circulation impossible *ab initio*. Cyclones are then only the irregularities inevitable in any circulation when skin friction over the earth's surface is taken into account.[13]

The foregoing inferences rest on the postulate that there actually are prevailing winds on the earth's surface comparable with those inferred from the theory of a circulation without friction. This is a fact of observation, but it must be pointed out that the theoretical reason for it is far from obvious. Imagine first an atmosphere with all the level surfaces isothermal and isobaric, so

[12] *The Air and its Ways*, Pls. XX. and XXI.

[13] Exner, in the second edition of his *Dynamische Meteorologie*, pp. 216-7, suggests that cyclones are an essential component of the general circulation. His argument is quite different from that given here.

that there are no winds at any height. Suppose now the temperatures adjusted very slowly until they reach their actual values, and then kept constant. The vertical expansion in the heated regions makes the pressure at a given height (other than zero) greater than in a cold region, on account of the extra mass of air above that height, and the pressure difference tends to push the air outwards. This tends to reduce the surface pressure in the heated regions and to increase it in the cooled regions, thus producing a system of easterly winds over the surface. Apart from friction, this process would ultimately produce a steady state, namely, that investigated in §3.2.

But when friction is taken into account it is evident, by what has been already shown, that the changes are not at an end at this stage. Surface friction can arise only when winds have been developed on the surface, so that it would produce little effect until a circulation had been established by the above process. But when a circulation exists the effect of friction is to make the air drift across the isobars towards the low-pressure side, thus tending to destroy any pressure differences that may exist. There is no obvious reason why the process should not continue until the whole surface of the earth is at uniform pressure, with no surface winds at all anywhere. The vertical distribution of temperature, however, would still depend on the latitude, and therefore there would still be winds in the upper air. The fact that we actually have winds on the ground indicates that such a state as this could not be permanent, and the reason for impermanence may probably be found in the variation of the wind over horizontal surfaces. This may well lead to horizontal eddies, or cyclones, giving rise to the horizontal Austausch of Defant,[14] but the mechanism is not clear. Friction will presumably tend to make the total energy a minimum, but this again, if the friction were small and proportional to the velocity, would generate the motion of 3.2. It would perhaps be more natural to try to make the total energy a minimum, subject to the additional condition that the surface velocities must be such that the total angular momentum is undergoing no secular change; but it is not easy to see how to apply such a method, since the energy of the cyclones themselves should appear in the total energy, and we lack a means of evaluating it.

In any case, however, there must either be surface winds or no surface winds when the air is subject to differential heating. The latter alternative is not in accordance with observation, though not for any evident dynamical reason. Taking into account, however, the observational fact that surface winds do exist, it has been proved by pure dynamical theory that the only possible motion consists of westerly circulations around the poles, with a corresponding easterly circulation in middle latitudes or near the equator (the difference probably depending on the temperature distribution) and with a system of cyclones closely similar to those we know. No steady general circulation of the atmosphere, without cyclones, is dynamically possible when friction is taken into account.

[14] *Geografiska Annaler*, 1921, pp. 209-265; *Met. Zs.*, **39**, 1922, 8-14.

DISCUSSION.

Mr. L. F. RICHARDSON said that Dr. Jeffreys had discovered a very pretty analogy between a region in the atmosphere where the temperature was higher than normal, and a region on the sea where the equilibrium height of the surface was lower than normal on account of the attraction by the sun and moon. The analogy would be even prettier if equation (16) were multiplied through by some factor such as to make its terms of the same physical dimensions as the terms of the analogous equation (19).

Colonel E. GOLD said they were all very much indebted to Dr. Jeffreys for turning his powerful analysis to these meteorological problems. A paper like this would need time to study before one could discuss it adequately. One of the things which appealed most to him in it was on p. 100. It had always been one's idea that it was the anticyclones in the sub-tropical region which interfered with the regular trade winds, and that these anticyclones were due to the distribution of land and water. It was interesting to find from Dr. Jeffreys's analysis that it does not matter whether we have land and water or a uniform distribution of water over the whole globe, there will be no belt of SE or NE trade winds extending continuously around the earth. He would like to ask Dr. Jeffreys to explain why, in equation (6), p. 91, it is fair to cancel out a factor which is going to be zero. Another point was that Dr. Jeffreys says he is using the word " convection " in the literal sense of the word. We all of us think we do that, but he was afraid we did not always mean what he thought Dr. Jeffreys meant in his paper, that is, horizontal mass transference on a large scale, such as that involved in the trade wind systems. He thought it would economise speech to restrict the meaning of convection to transfer of heat by exchange of mass, the exchange proceeding in both directions simultaneously, and the currents involved being of small cross-section : they might be numerous and so extend over a wide area : they can certainly be horizontal as well as vertical. Perhaps Dr. Jeffreys meant that he used the word in the " lateral " sense. Dr. Jeffreys neglects the variation of g with height. That appears permissible in comparing pressure at a given level, but the variation of g with height affects the absolute pressure at 10 km. by half a millibar. Dr. Jeffreys did not make clear in his paper that he makes a due allowance for the variation of gravity over the surface of the globe. No doubt he does so implicitly, but it might be worth while making it explicit.

Mr. L. C. W. BONACINA said that in reading the paper one or two points had worried him greatly. Dr. Jeffreys throughout seemed to imply that the westerly circulation extended practically to the Poles with a definite centre of low pressure over the Poles, but observations showed that on the polar side of the 60th parallel in each hemisphere you again got prevailing east-wind. It is shown in the Antarctic and over the North Polar basin, the average pressure over the North Polar basin being not very much less than that in latitude 30°N. The low pressure belt is along the 60th parallel. In the accounts of various explorers in Greenland, such as Nansen, Peary, and De Quervain, it had been demonstrated that over the interior of Greenland there existed a glacial anticyclone, as a result of the overflowing winds and a very low temperature. The relatively high pressures over the North Polar basin, Greenland and the Antarctic, may be very shallow, but they do point to a surface temperature effect. It therefore seemed to him (Mr. Bonacina) that we must conclude that Dr. Jeffreys's results would apply to a uniform globe, that is to say, a globe divested of continentality, oceanity and glaciation. In that case Dr. Jeffreys's frictional cyclones would not bear much resemblance to our actual Bjerknesian systems, which depend upon contrasts of temperature which are themselves the effects of continentality, oceanity and glaciation : for if there existed an an entirely uniform globe, without continentality or monsoonal effects, the simple planetary circulation would be undisturbed, and winds changing latitude would quickly adjust their temperatures. Another point which struck him was that Dr. Jeffreys spoke repeatedly of the permanent belt of low pressure in polar regions. He would like to point out that the most intense anticyclones on the Earth's surface occur not in the region of high pressure in latitude 30°N, but in the low pressure belt, about 60°N, where there are frequently such intense cyclones. Even within the limits of Great Britain, this principle could be seen; for whereas in Scotland where very deep winter cyclones below 28.0 inches occur, there have been several instances of winter anticyclones over 31.0 inches; in England a sea-level pressure of 31 inches had never quite been recorded.

He felt that those points ought to be brought out, but the necessity for doing so did not lessen one's admiration of Dr. Jeffreys's masterly analysis.

Mr. C. S. Durst referred to Dr. Jeffreys's statement that from the analysis there was no obvious reason for the existence of surface winds, and asked if the release of energy due to condensation of water vapour in the equatorial belt might not be the explanation of their existence.

Dr. Jeffreys, in reply to Mr. Bonacina, said that the circulation near the North Pole was very irregular. It was probably much affected by the large glaciated land masses; if these were sufficiently important they could make air sweep across the pole to such an extent that the systematic circulation would be unnoticed. The outflowing winds over Greenland were not in themselves conclusive evidence of an anticyclone circulation; they might be antitriptic winds of small vertical and horizontal extent. Pilot balloon observations or accurate levelling across Greenland would be needed to decide the point. The argument from friction in the paper is independent of whether the globe is uniform or not; but in any case, if it is shown that a steady circulation is impossible on a uniform globe, it hardly seems likely that the imposition of irregularities on the surface would make one possible. There is no antagonism between this theory and the Bjerknes view of the structure of the cyclone; the theory, indeed, goes far towards explaining why the Bjerknes structure is necessary. The intense anticyclones in temperate latitudes spoken of by Mr. Bonacina are part of the irregularities shown to be essential, not of the mean distribution of pressure that gives the general circulation.

In reply to Colonel Gold's question about the cancelling of the frequency, Dr. Jeffreys said that in problems of tidal type without friction the analysis was always complicated by the possibility of free steady motions, which could not be said to be due to the disturbing forces. If the disturbing forces were first supposed to have a finite period, it could be definitely said that the periodic solution was due to the disturbing forces. If then the period is made indefinitely great this periodic solution tends to a definite limit, which can be said to be the motion due to the disturbing forces. If a direct attack was made on the problem of the effect of a steady distribution of temperature it would be impossible to separate the true effect of the temperature from the motion due to accidental initial conditions, but the device in the paper makes it possible.

With regard to the meaning of "convection," Dr. Jeffreys thought it should cover all changes of the value of an element at a place due to the arrival of matter from somewhere else where that element is different. In meteorological literature it was often restricted to the effects of vertical motion, but those of horizontal motion were often as important though less spectacular. Often, again, it was used to denote the whole motion of the air, and not merely this particular type of effect produced by the motion. He himself confessed to having used it in a case where this effect was justifiably entirely neglected; but he considered that the sense first defined was far the most useful and satisfactory.

The allowance for the variation of g with height would be inappreciable. The motion arises from variations of temperature over the surfaces where $U + \frac{1}{2}\,\omega^2\varpi^2$ is constant, and the variation of g only enters multiplied by these small quantities.

With regard to Mr. Durst's inquiry, there was no difficulty about the amount of energy available; the energy in the frictionless motion considered was greater than in the actual motion. The problem was to explain how the energy attained its actual distribution in the atmosphere. The effects of humidity could be treated by a slight adjustment of the temperature.

PROCEEDINGS AT THE MEETINGS OF THE SOCIETY.

November 18, 1925.

At the *Ordinary Meeting* the following candidates were balloted for and elected Fellows of the Society:—

- Ernest George Beilby, Inverleith, Matale, Ceylon;
- Jakob Bjerknes, D.Ph., Värvarslingen på Vestlandet, Bergen, Norway;
- Lance Harold Browning, Flight-Lieut., R.A.F., 3 (F.) Squadron, Upavon, Wilts;
- Edgar Harold Butler, 24, Westbury Road, New Southgate, N.11;
- William Patterson McFerran, B.A., King Edward VI. Grammar School, Retford, Notts;
- Alan Cameron Macintyre, 174, Papanui Road, Christchurch, N.Z.;
- Mrs. Iris MacLulich, Haslemere Hotel, Montpelier Road, Brighton;
- Harold Victor Michell, Flying Officer, R.A.F., Air Headquarters, Baghdad, Iraq;
- Major Myles Herbert Roffey, University, Hong Kong;
- Henry Trevor Roller, 42, Clanricarde Gardens, W.2; and
- John Wardale, 35, St. George's Square, S.W.1.

Messrs. Ball, Baker, Ash & Co., Chartered Accountants, were appointed auditors of the Society's accounts for 1925.

December 16, 1925.

At an *Ordinary Meeting* the following candidate was balloted for and elected a Fellow of the Society:—

- Victor Dury, King Edward VII. School, Taiping, Federated Malay States.

The President announced that a loyal Address had been forwarded in the name of the Fellows to His Majesty the King on the occasion of the death of Queen Alexandra, and a reply from the Secretary of State was read to the meeting.