THE OCEANIC THERMOCLINE

Authors' Foreword to the Two Following Papers on the Oceanic Thermocline

Each of the two following papers attempts to provide a theoretical framework for explaining the oceanic thermocline, and the associated thermohaline circulation of the ocean. They were developed independently, and then the authors exchanged copies of their papers. As they read, the two theories are not compatible, but the results of each are similar in certain respects, and apparently resemble the actual ocean situations as we actually know it. Because of the basic differences in the two theories we have decided not to try to combine them into a single paper, but to publish them together in the same number of this journal, so that they should both appear in the literature simultaneously, and so that the marked successes in the basic formulae should be immediately evident to the reader.

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The Oceanic Thermoline and the Associated Thermohaline Circulation

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Abstract

A study is made of the thermal structure of an ocean which is dynamically geostrophic, bounded by an eastern coast, and driven by an imposed surface irrotational distribution and wind-forces. The heat equation contains vertical diffusion in the form of a virtual eddy mixing parameter and non-linear terms of vertical and internal wave advection. Comparison of the results with the North Atlantic shows an agreement with the observed temperature distributions in the thermocline region, including the deepening of isotherms in mid-latitudes. The value of the vertical component of velocity at the bottom of the thermocline is presumed, and a numerical value for the eddy-conductivity obtained.

1. Introduction

Ninety years ago the question of whether the wind-stress or thermally produced differences of density are the predominant cause of oceanic circulation was a subject of public debate.2 Since that time, this central issue of physical oceanography has been treated with more caution, and is only indirectly alluded to in present-day textbooks. That the

1 Contribution No. 1969 from the Woods Hole Oceanographic Institution.
2 Present address, Dep. of Physics, Harvard University. Part of the material presented here has been included in a doctoral thesis presented to the Dep. of Physics, Harvard University.
3 Telsia XI (1939).
vertical integral of mass transport of the ocean can probably be attributed to the wind-stress acting upon the surface was established by Stromness in 1952. This important concept, and its further elaboration in the hands of Moss (1953), Nayar (1950), and others, are described in a survey article by Stromness (1957). Computation of a vertical integral, however, gives no idea of the distribution of property in the vertical, whereas the geostrophic calculation of currents from observed hydrographic station data gives only the vertical distribution of the velocity vector, but not its integral. Moreover, the vertical integration completely cancels the contribution of the thermohaline circulation. Consequently, there was a tendency, starting in 1950, to minimize the possible role of the thermohaline circulation. The first attempts to formulate a model concerning active thermal-convective processes were (1) an important Russian paper by P. S. Likhin (1953) and (2) an unpublished doctoral dissertation by Fondane (1954). (1) chapter xi in a book on the Gulf Stream, written in 1954-5 by Stromness (published in 1957). Studies (1) and (2) had no meridional boundary — the key feature of the Sverdrup wind-driven model—and so were not directly applicable to actual oceanic basins. Study (3) treated only the horizontal patterns of flow that would exist in a meridionally bounded basin on the assumption of a fixed vertical mass flux at mid-depth, and did not treat the important vertical heat flux portion of the problem. Later, Voreades and Stromness (1957) and Likhin (1957) were able to modify study (1) so that the important influence of the variation of the Coriolis parameter with latitude could be incorporated; and obtained expressions for the depth of the thermocline which seemed more related to reality. On the basis of these models, Stromness (1958) indicated how they might be applied to give a rather detailed picture of the patterns of flow in the thermohaline circulation of the world ocean. Nevertheless, as explained in the survey article (Stromness, 1957) neither of these models imitates the actual oceanic density distribution very closely; hence it was desirable to construct a theoretical model of the oceanic thermohaline circulation with an eastern wall to inhibit zonal flow, driven by a meridional temperature gradient fixed at the surface, and without using the totally unrealistic large vertical temperature gradient imposed by both Likhin and Voreades and Stromness on the model, mentioned above for the sake of linearity of the convection equation.

The present paper is an attempt to construct such a model, and we believe it is a substantial improvement upon previous studies. However, it contains—as they also do—a parameter treatment of the mixing processes embodied in an "eddy thermohaline conductivity" parameter. This parameter is assumed to be uniform and constant over the entire ocean basin—in contradiction to other studies which attribute the oceanic thermocline essentially to variations in \( x \) (for example, Davenport, 1936; Moss and Anturbain, 1944). Thus we envisage the ocean as being slowly and evenly stirred by some physical process which we cannot specify, and the necessity for so doing is a feature of how far these are of actually being able at present of giving a complete physical account of the thermal convective circulation of the ocean.

2. Formulation

We consider an ocean thermally driven by an imposed north-south temperature distribution on the surface. The convective overturn is represented by a Coriolis parameter varying linearly with latitude. The ocean is taken to be infinitely deep and bounded only by an east coast. We integrate into the non-linearly coupled temperature and velocity fields in the study stage. Let \( u, v, w \) be the eastward, northward, and vertical respectively, with \( \rho, \theta, \phi \) the corresponding salinity components. The origin is fixed in the east coast at mid-latitude. The region of interest is \( x = 0, y = \pm y_0, z = 0 \). The fluid system to be considered is defined explicitly by the following assumptions. The equation of state of the fluid varies linearly with temperature only. Furthermore, the coefficient of thermal expansion, \( x \), is considered zero except when coupled with \( y \), the acceleration of gravity. In the equations of motion we neglect \( x \) and inertial...
the ocean thermocline driven by the South Thermohaline circulation. The constraints on rotation and Coriolis parameter varying with depth. The ocean is taken to be stratified and bounded by an isopycnal into the non-linear steady flow and velocity fields in 2002, the eastern wall to inhibit zonal flows and the meridional temperature at the surface, and without any unrealistic large vertical gradient imposed by both factors. The model is then solved for the sake of linearization equation. Then an attempt is made to construct the model in a substantial on previous studies. However, they also do a parametric mixing processes embodied in anisotropic conductivity parameter is assumed to be present over the entire ocean basin with the exception of some regions where the anisotropic parameter is zero (for example, DiAN, 1986, Mars, 1996). Thus we envisage anisotropy slowly and steadily stirring anisotropy process which we cannot neglect for doing so is a far short we are of actually assessing of giving a complete theory of the thermal convective ocean.

The fields of primary interest are \( T \) and \( w \); two equations in these alone can be obtained. By cross-differentiation of (1) and (2) and use of (4), we obtain

\[
\rho w - f \frac{\partial w}{\partial x} = 0
\]  

(6)

Cross-differentiation of (1) and (2) yields

\[
\rho \frac{\partial w}{\partial x} - g \frac{\partial w}{\partial z} = 0
\]  

(7)

Inserting \( v \) from (6) into (5) and (7) yields the desired equations:

\[
\rho \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} = 0
\]  

(8)

and

\[
\rho \frac{\partial w}{\partial x} - u \frac{\partial w}{\partial y} = 0
\]  

(9)

The boundary conditions imposed at \( z = 0 \) are \( T = T_{\infty}, T = T_{0} \) and \( w = 0 \). At the ocean surface, \( T = 0 \). This situation is not flow through the plant \( z = 0 \), but is otherwise physically unrealistic. However, in no real fluid or ocean would the above equations describe the region near such a boundary; therefore our detailed results will not be valid. Anticipating carrying the \( y \)-dependence parametrically, we reserve discussion of this boundary condition.

3. The Similarity Transformation

\( T \) and \( w \) are specified at \( z = 0 \) and approach arbitrarily close to their asympotic values along some curve \( z = f(x) \). In related problems it is often found that at a given \( x \), the approach of the fields to their asymptotic values depends only on the ratio of the actual depth to the value \( f(x) \), of the curve at this point (Getzvors, 1973). The formal analogy to these boundary layer problems occurs because only vertical mixing is allowed, so that only higher order derivatives with respect to \( x \) are present. Formally, we seek separated solutions of the form \( T = G(x)H(z), w = H(z)\psi(x) \), where \( \psi = \frac{x}{z} \). Thus, all func-
tions are tacitly functions of \(y\). This transformation is also convenient for satisfying the boundary conditions, e.g., at \(x = 0\), \(\xi = 0\) and \(T\) becomes a function of \(x\). In the following a prime denotes differentiation with respect to \(\xi\), and a subscript \(y\) differentiation with respect to \(y\).

Taking \(x\) and \(x\) derivatives in terms of \(x\) and \(\xi\), and inserting these in (8) and (9), we obtain

\[
-xF' - H\left(\frac{1}{2}\right)w'\theta' = 0 \quad (10)
\]

\[
HF'' + \frac{G\alpha w'}{f} \left(\frac{1}{2}C'\theta + \frac{C'G'}{F} \xi \theta'\right) = 0 \quad (11)
\]

The condition for separability of (10) in \(x\) and \(\xi\) is merely that \(H\) be a constant times \(F\); this constant can be absorbed into \(w\) so we require \(F = H\). For equation (11) to separate all three terms must have the same \(x\)-dependence. The second and third terms require that \(C'G'/G\) be a constant times \(F'/F\), or that \(G\) be an arbitrary power of \(F\). The first and second terms then require that \(F\) be some power of \(x + c\) where \(c\) is a constant.

In summary, under the general transformation

\[
\xi = x + c, \quad T = (x + c)^{\alpha + 1} \phi(x), \quad w = (x + c)^{\alpha'}(\theta)
\]

(9) and (10) take the form

\[
x'' - \alpha x'^2 - \frac{1}{2} w'\theta' = 0 \quad (12)
\]

\[
x'' - \frac{2G\alpha w'}{f} (x\theta + 1) \theta + \frac{C'G'}{F} \xi \theta' = 0 \quad (11)
\]

The constants \(\alpha\) and \(\epsilon\) are determined by the particular boundary conditions. To make the surface temperature independent of \(x\), we choose \(\alpha = -1\). The choice of \(\epsilon = 0\) gives \(\xi = x^2\); \(\xi = x\) for both \(x \to -\infty\) and \(x \to 0\), and the east coast temperature will automatically correspond to the boundary temperature.

4. Order of Magnitude Considerations

Equations (12) and (11) are still too complicated to be solved exactly. However, from the results in the previous sections, we can determine certain gross properties of the temperature and velocity fields: the general structure, the asymptotic values of \(w\) and \(T\) and the scale of \(\xi\) within which all asymptotic values are essentially obtained. Moreover, it would not be much more meaningful to have the details of the exact solutions because the mathematical model which we are using is itself not detailed, and also because the results are to be compared with the averaged observations of the real ocean.

In seeking unknown orders of magnitude, it is convenient to introduce known magnitudes in terms of slightly different parameters. Let \(\eta = 10^p, \gamma(y) = 10^{y_1}, \xi(y) = 10^{y_2}\) and \(y\) and \(y_2\) are of order unity while \(\eta\) is of order \(10^k\). We expect \(\eta\) also to be of order unity. To determine how \(\epsilon\) affects the amplitude of \(w\) and scale of \(\xi\) we write \(\xi = \xi_0 + \epsilon \xi_1 + \cdots\) and \(w = w_0 + \epsilon w_1 + \cdots\). We assume that the scale of \(\xi\) and amplitude of \(w\) are of order unity, and further that \(\theta\) and \(W\) are smooth functions of \(\xi\), i.e., the functions and all their derivatives are of order unity.

Under these assumptions, (12) and (11) become, with a prime now denoting differentiation with respect to \(\xi\)

\[
x'' + \alpha x'^2 + \frac{C'G'}{F} \xi \theta' = 0 \quad (14)
\]

\[
ox'' + \alpha x'^2 + \frac{C'G'}{F} \xi \theta' = 0 \quad (15)
\]

Requiring that the dynamical equation (15) always retain both terms gives \(b = 0\). In the heat equation (14), we notice that both advections are necessarily the same order of magnitude, insuring that vertical eddy-diffusion can be comparable to the advections yields \(a = 0\). The parameter \(\epsilon\) now disappears from (14) and (15), which now contain only terms of roughly unit magnitude. From the similarity transformation we have obtained the dependences of the asymptotic value of \(w\) and the scale depth on \(x\), from the above considerations the order of magnitude and major part of the \(y\) dependence of these quantities.

5. Parametric Linearization and Solution

The mathematical difficulties remaining are inherent in the two nonlinear terms in \(\xi\). To obtain approximate results, we shall replace

\[\xi = 0, a \text{ with an}\]

We then find a parametric solution to which \(\gamma\) and \(\xi\) apply.

\[\eta = 0, \gamma(y) = 0, \xi(y) = 0\]

(14) and (15) provide the sole approximations of parameters for \(\alpha = 0, \beta = 0\).
the asymptotic values of $\theta$, $\gamma$, and $W$ are unknown. However, we do know that $\theta_0$ ranges from $T_1$ to the surface to essentially zero at the depth $L$. We therefore crudely take $T_1$ as the average $\theta_0$, although the rate average will vary in $Y$ with the shape of $\theta$. However, we have absolutely no information about $\theta$. Therefore, we shall allow the unknown average value of $W$ to enter the equations and parameterize the solutions. Then, consistently taking the average of the parameterized solution actually obtained will yield an algebraic equation to the average $W$. Finally, the solution of the algebraic equations, in terms of the external parameters, is inserted back into the solutions for $\theta$ and $W$. Replacing (14) and rewriting (15), the final linear set of equations is

$$\begin{align*}
\omega^2 - \phi^2 - \frac{1}{2} \beta \gamma W &= 0 \\
\gamma^2 W' + \zeta &= 0 \\
\phi &= \frac{1}{2} W'(C) + C
\end{align*}$$

where $C$ is a constant. The above is not the most general form, but it is the most general form that can be obtained from the solutions of (16) and (17).

The above considerations suggest the following conclusions: by taking higher moments of $\omega^2$ and $W^2$, we shall obtain

$$\begin{align*}
\theta &= \theta_0 e^{-\frac{Y}{2}} \\
W &= W_0 (1 - e^{-\frac{Y}{2}})
\end{align*}$$

Multiplying (16) and (17) by $\gamma$, integrating from zero to $Y$, infinity,

$$-\int_0^\theta (\omega^2 + \phi^2 - \frac{1}{2} \beta \gamma W_0 = 0$$

$$\int_0^\gamma W_0 = \frac{1}{2} \beta \gamma W_0 = 0$$

In (16) and (17) the derivatives and integral can be substituted from the form (16) and (17). From (19) we also have simply $\omega^2 = 2$. Solving the algebraic equations, we obtain

$$L = \frac{2}{\beta} (\phi Y - \frac{1}{2} \gamma W)$$

where $C$ is a constant. Equation (18) is independent of $\gamma$, and

$$W_0 - \phi_0 Y = 4$$

The above results could of course be improved by taking higher moments of $\omega^2$ and $W^2$ and introducing additional constraints into the forms (16) and (17). The approximate forms for the temperature and vertical velocity are

$$T = \theta_0 e^{-\frac{Y}{2}}$$

$$\frac{1}{2} \beta \gamma W^2 = 10$$

$$\frac{1}{2} \beta \gamma W^2 = 10$$

Taking the higher moments of $\omega^2$, $W^2$, and the solutions of (16) and (17) lead to more general solutions for $\theta$, $\gamma$, and $W$. The results of this paper are summarized as follows: by taking higher moments of $\omega^2$, $W^2$, and the solutions of (16) and (17) lead to more general solutions for $\theta$, $\gamma$, and $W$. The results of this paper are summarized as follows:
6. Finite depth

It is apparent that in mid-latitudes the thermocline is so shallow that it occupies only a small fraction of the actual ocean depth, and that the assumption of infinite depth is justified in making the calculations of $\Phi$ and $W$ in the thermocline region itself. James Cresson showed us in private correspondence concern- ing an earlier form of the thermocline model that this was likely to be the case. But the real ocean in fact has a bottom, and if this bottom were flat the vertical component of velocity $W$ must vanish there. Thus we regard the quantity $W_x$ as being a measure of the vertical velocity just beneath the thermocline (that is, at infinity so far as the thermal boundary layer is concerned). The form of the similarity transformation does not permit us to fix the bottom at $z = 0$; instead, it is necessary to put the bottom at $z = L$ where $L \gg h$, a very small compromise. Evidently we can still use the form of solution for temperature given by equation (19) and need modify equation (20) only as follows

$$W = W_x \left( 1 - \frac{L_z}{L \delta} e^{-\frac{L_z}{L \delta}} \right)$$

where the quantities $L_z$ are small at $z = L$ and the initial conditions are exactly satisfied.

7. Inclusion of wind-stress

The dynamical equations (1) and (2) are now viscous. For the wind stress to work upon the ocean it is necessary to allow viscosity to play a role somewhere, and thin has generally been done by a boundary-layer analysis of the form of Ekman. Experience suggests that the Ekman viscous boundary layer is confined to an upper one hundred meters in the ocean. At any point in the ocean the total vertically integrated mass transport in the Ekman layer is to the right (in Northern hemisphere) of the direction of the applied wind-stress, $T$, and of

$$T_a \frac{L_z}{L \delta}$$
\( e \) ocean. The only modification distribution of the vertical component \( W \) is that instead of an
asymptotic value \( W_{oo} \) at \( t \) approaches a maximum, the thermocline. Which we find \( W_{oo} \) nevertheless because finite depth so far as the the.
then, it is concerned, and then vary this maximum just under the the zero at the bottom. However, thermal components of velocity of
the thermocline are concerned, the finite depth has a major effect, from
some amplitude (in the case of the vanishing beneath the the-
you are calculated from equation of which no longer tend zero.
Of no meridional motion is \( u \)
which \( u = 0 \) at \( W = 0 \), hence at
Since throughout most of the
the latitudes \( \Delta \theta = L \), the depth of
motion [for reference level for cullens] \( L \approx \theta \), or quite
At the bottom of the thermocline, of wind-stress at the surface, \( W \)
both top and bottom of the
the vertically integrated value
action \( W \) must vanish. This is a
Sverdrup's earlier wind-driven thermohaline circulation is
theatrical and does not contribute to
all of horizontal velocity con-
stant in trying to
in a vertical circulation just the
wind-stress
acal equations (1) and (2) are
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of experience suggests that the
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hundred meters in the ocean.
the ocean the total vertically a
transport in the Ekmann layer
(Northern hemisphere) of
applied wind-stress, \( \tau \), and of
Folks Jr. (1989). 3
magnitude \( T \). Since both \( \tau \) and \( T \) vary as
functions of position over the ocean, the mass
transport of the Ekmann layer also varies from
place to place. If we form the horizontal
divergence of the Ekmann layer, we find that
this implies by equation (1) a vertical compo-
nent of velocity at the bottom of the Ekmann
layer of an amount

\[
\frac{\partial}{\partial \theta} \left( \frac{\partial u}{\partial \theta} \right) + \frac{\partial}{\partial \phi} \left( \frac{\partial u}{\partial \phi} \right)
\]

The definition of \( a \) is given by (28).
In these equations \( W_{oo}, \) \( \theta_{0}, \) \( \phi_{0} \) are
regarded as known, the quantities \( a, \) \( L, \) \( W_{oo} \)
are to be found. In this case the value of \( L \)
by the cube

\[
\frac{L^3}{8} \frac{\partial}{\partial \theta} (\gamma T_{\theta}) = L W_{oo} = a \phi
\]

instead of the simple cubic root (13). Once
having obtained this we find

\[
W_{oo} = \frac{8 \phi_{0}}{L}
\]

Case 2: \( W_{oo} = 0 \), the Ekmann layer is convergent.
Here there is a reversal of the vertical component
of velocity \( W \), with depth, and therefore we must divide the ocean into two
layers: one from \( \zeta = 0 \) to \( \zeta = \infty \), and the other from \( \zeta = -\infty \) to \( \zeta = 0 \) where \( W = 0 \) which we treat exactly as be-
fore. For our purposes, if we define the unk-
temperature at \( \theta = \phi \) we can write
of the temperature and vertical velocity in the
lower layer as

\[
\frac{\partial}{\partial \theta} (\gamma T_{\theta}) = L W_{oo} = a \phi
\]

The quantity \( h \) is also unknown, of course,
however, we can write the quantities \( L, \) \( h \),
and in terms of \( \theta, \) by equations (23) and (24).
The only change here is that we can possibly
the advection term \( \frac{\partial}{\partial \theta} \) in the expansion
for \( \zeta \), because we will incorporate the merid-
ional advection exactly in the upper layer.
As we have already seen, this term does not
play a major quantitative role in the results.
If the upper layer is not very thick a very
rough approximation of the vertical velocity
is given by a linear relation (see numerical comparisons, Stommel (1956)).

The average of $W$ from $z = 0$ to $z = h$ is obviously $W_0/2$ and the vertical derivative $W_z/z$ is obviously $W_0/h$, so the upper layer is $T_U$ so that we now can insert these approximations in the non-linear temperature equation and obtain (we can obtain a more accurate form for $W$ later by solving the dynamical equation (22))

$$W = W_0 (1 - 2z/h)$$  \hspace{1cm} (29)

The solution of this equation is of the form

$$\theta = C_1 e^{-\frac{2z}{h}} + C_2 e^{-\frac{2z}{h}}$$  \hspace{1cm} (30)

where $C_1$ and $C_2$ are constants of integration. The temperature must be equal to $\theta_0$ at $z = 0$ and to $\theta_1$ at $z = h$; therefore

$$C_1 + C_2 = \theta_0$$  \hspace{1cm} (31)

$$C_1 e^{-\frac{2h}{h}} + C_2 e^{-\frac{2h}{h}} = \theta_1$$  \hspace{1cm} (32)

Also, the temperature gradients at $z = 0$ must equal each other in both layers

$$C_1 e^{-\frac{2z}{h}} e^{-\frac{2h}{h}} - \frac{\theta_1}{2}$$  \hspace{1cm} (33)

and so must the vertical derivative of the vertical velocity component:

$$W_z e^{-\frac{2h}{h}} = \frac{\theta_1}{2}$$  \hspace{1cm} (34)

Eliminating the constants of integration, $C_1$ and $C_2$, and also $W_0$, we obtain three equations in which there are three unknowns $h$, $\theta_1$, and $\theta_0$

$$\left(1 - e^{-\frac{2h}{h}}\right) \frac{2z}{h} W_0 e^{-\frac{2z}{h}} \frac{\theta_0}{2} + \theta_0 e^{-\frac{2z}{h}} + \theta_1 e^{-\frac{2h}{h}} = 0$$  \hspace{1cm} (35)

$$W_0 \frac{\theta_1}{2} = 2e^{-\frac{2h}{h}}$$  \hspace{1cm} (36)

$$L = 2z(\theta_0)^{\frac{1}{h}}$$  \hspace{1cm} (37)

One of these is, unfortunately, transcendental so that in general solution cannot be obtained.

The different methods will be appropriate for different values of the parameters, involved. Rather than delving into a lot of algebra, we would like to point out that when dealing with the application of these ideas to the ocean, we do not really know $h$ to begin with. Instead we form an estimate of $h$, $L$, and $\theta_1$, from the shape of the vertical temperature sounding. We obtain a rough estimate of $W_0$ from charts of wind-transport of the surface layers, and then we do not use equations (29), (30), (31) in the same way. Equation (35) is used to obtain an estimate of $\theta_0$ and equations (36) and (37) can be used as a consistency check on the applicability of the model to the ocean. We know all the quantities that go into them, and they ought to be satisfied if the theory is applicable. We can then use equation (35) to calculate the vertical velocity just beneath the thermocline, $W_0$. A further point, because of the nature of the similarity transformation, it is not easy to introduce vertical velocity at the surface, $v_0$, with an arbitrary $x$ dependence: in fact we are restricted to $v_0$ distributions of the form $x^{-\nu}$.

8. Application to a real ocean basin

The fact that the field of density calculated from equation (25) superficially resembles that naturally occurring in the ocean is demonstrated in figures 1 and 2. Figure 1 is a dimensional perspective diagram of the solution (45) with an imposed surface temperature distribution of the form

$$\theta_0(x) = 20^\circ C - 25^\circ C e^{\frac{x}{\lambda}}$$  \hspace{1cm} (38)

where we have included the non-dimensional lengths $x$, $x'$ by the relations $x = -Lx'$, $x' = \lambda = Dx'$, $x = Dx'$, $L$ and $L_0$ are dimensional constants defining the scale of the phenomena, and chosen so as to make the numbers $x'$, $y'$ in a range near unity. Thus the actual formula used in constructing the diagram was

$$T = (20 - 3.5x_0) \exp (-\frac{x}{\lambda})$$  \hspace{1cm} (39)

$$L_0 = 5 \times 10^6 \text{ cm}$$

$$D = 10^6 \text{ cm}$$

where

$$\theta_0(x) = 20^\circ C - 25^\circ C (22 + 12.5x_0)^{\frac{1}{4}}$$

$$\lambda_0 = 10^6 \text{ cm}$$

Allan 31 (1935).
...and conclude by 

...and conclude by noting that when dealing with these ideas the
not really know \( x \) to begin with. An estimate of \( L \), the
length of the vertical, could be used in a more detailed
formulation of the problem. We know all the quantities \( m \), \( n \), \( \phi \), and \( \nu \), and they ought to be impor-
tant. We can then use eq (5) to calculate the vertical
length of the thermocline, \( D \), because of the nature of the
formation, it is not easy to translate velocity at the surface, \( \nu \),
to depth dependence. In fact we would find that the depth
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dependence is not as important as the more

detailed...
surfaces are deepest in mid-latitudes—a fact which hitherto has been attributed to the surface convergence of wind-driven surface layers. Here of course, in figure 1, there is no wind acting; whereas, in the real ocean diagram, figure 2, we must be on the lookout for the effect of the wind which actually does act on the North Atlantic.

Before attempting to account for the effect of the wind, first we calculate various quantities such as the parameter \( k \) and the vertical component of velocity by fitting the two figures 1 and 2 to the wind. Thus the real Atlantic Ocean is roughly 5,000 km wide and 10,000 km long, and taking \( L_w = 5,000 \) km, \( L_y = 10,000 \) km makes the horizontal scales of the two figures agree. Setting \( \beta = 2 \times 10^{-3} \text{ cm}^{-1} \text{ sec}^{-2}, \alpha = 2 \times 10^{-8} \text{ cm}^{-1} \text{ sec}^{-2}, \gamma = 10^3 \text{ cm}^{-1} \text{ sec}^{-2} \). From figure 2 we can see that the value of \( k \) is equal to 1 corresponding to a depth of 1,000 meters at \( y = 0 \) and \( x = -1 \), and we can get the value of the parameter \( k \):

\[ k = 10 \text{ cm}^2 \text{ sec}^{-1} \]

The vertical velocity at the bottom of the thermocline-thickness \( x \), defined as the depth at which the temperature is \( T \) of the surface temperature locally. This useful relation is obtained by elimination of \( u' \) between equations (25) and (50):

\[ u' = \left( \frac{T_v + T_1}{\beta} \right) \frac{a h}{a y} \]

If the thermocline thickness were 1,000 meters (as it appears in the first glance to be at \( y = 0 \) and \( x = -1 \)), the sub-thermocline vertical velocity is

\[ w(x) = 5 \times 10^{-3} \text{ cm sec}^{-1} \]

The expression for the vertical velocity in terms of thermocline thickness is interesting because it does not involve the parameter \( k \) explicitly. It is very similar to the expression used by STOMMEL (1958) in an analysis of the abyssal circulation, but having the much more primitive model of W仪式和 SYMONZ (1957). We believe that both these values, of \( a h \) (Table I) are of magnitude too high. The reason is that we are computing a model in figure 1 with no winds, to a schematic representation of the real North Atlantic Ocean which does suffer being acted upon by winds. In mid-latitudes Montgomery (1957) has computed that wind ought to be an important Ekman layer at the surface, directly beneath, which there is a vertical component of velocity downward, \( u = -5 \times 10^{-3} \text{ cm sec}^{-1} \).

We expect therefore that part of the warm water layer to the upper portions of figure 2 really must correspond to the wind-driven layer of thickness \( k \), and that below this lay\er the proper thermocline regime begins. The thickness of the thermocline which we used before was too big. It included both \( k \) and \( x \). A result, better idea of the rate of exponential decay of temperature in the main thermocline \( x = -1 \), \( y = 0 \) can be obtained from curve NA in figure 3.

Let us proceed by first writing the quantities \( L, k, \) and \( x \) in terms of the original \( x \) coordinates. This is advisable because the dimensions of these quantities are rather odd

\[ \frac{L}{2} = (\alpha h)^{2/3} \]

\[ x = (\alpha h)^{2/3} \]

\[ \frac{d}{d} = (\alpha h)^{2/3} \]

where \( k \) is the "thickness" of the thermocline, estimated by the slope of the temperature curve. We have below the infection point at \( \theta = 10^3 \text{ cm} \) in the temperature curve, and \( x \) is the "depth" of the wind-driven layer which we identify in figure 3, for example, as the depth of the infection point. The temperature at the infection point is defined as \( \theta \). The information we obtain from the North Atlantic curve in figure 3 is as follows

\[ a_k = -3.5 \times 10^6 \text{ cm} \]

\[ a_k = -9.0 \times 10^6 \text{ cm} \]

\[ \theta = 10^3 \text{ C} \]

These estimates are for a geographical position corresponding to

| \( x = -5 \times 10^6 \text{ cm} \) |
| \( y = 0 \) |
| \( z = 10^{-5} \text{ cm sec}^{-1} \) |

Table XI (1959). 3
and from Montgomery's wind-transport charts \( \omega_{\text{wind}} = 5 \times 10^{-4} \text{ cm sec}^{-1} \)

The quantities for other ocean basins are somewhat different. Using these quantities we compute

\[
Z = 0.44 \text{ cm}^3(\text{sec} \cdot \text{C})^{-1/2} \\
W_z = 4.0 \text{ cm}^3 \text{sec}^{-1/2} \text{C}^{-1/2} \\
h = 0.72 \text{ cm}^3 \text{sec}^{-1} \text{C}^{-1/2} \\
\beta = 10^6 \text{ C}^{-1} \\
\lambda = 0.81 \text{ cm}^3 \text{sec}^{-1/2} \text{C}^{-1/2} \\
W_{\text{up}} = -3.5 \text{ cm}^3 \text{sec}^{-1} \text{C}^{-1/2} \\
\text{Thus the actual upwelling velocity beneath the thermocline obtained from by transformation to the original coordinate} \\
\omega_{\text{wind}} = 3.1 \times 10^{-4} \text{ cm sec}^{-1}
\]

We believe that the order of magnitude of the missing parameter \( \lambda \) and the deep upwelling velocity \( \omega_{\text{up}} \) computed from the North Atlantic temperature distribution in this manner is much more in accord with the estimates of the magnitude of mixing processes and of the deep-ocean transports, than the numbers obtained neglecting the wind. Thus in applying these ideas to the actual ocean it is essential to
include the wind stress in the model. The two causes of oceanic circulation (wind-stress and thermohaline process) are connected in a basically non-linear fashion and cannot be superposed as additive solutions. Fofonoff has already pointed this out for a wind ocean. Of course, in this model, vertical integrals of transports depend upon processes above as has now been known for twelve years. The reader can now substitute the various constants into the expressions for the temperature in the upper layer and ten that it affords a reasonable approximation to the facts of that actually observed.

The state of affairs in the North Atlantic Ocean is not markedly different from that in other oceans. The vertical distribution of temperature at $90^\circ$ latitude near the western sides (the west of the eastern boundary currents) of other meridionally bounded oceanic basins is shown in figure 2. \(N:\) North Atlantic; \(S:\) South Atlantic; \(S:\) South Indian; \(N:\) North Pacific; \(S:\) South Pacific. Actually the thermocline in the North Atlantic is somewhat anomalously deep compared to those in other oceans, but the shape of the main thermohalines themselves are very similar, i.e., the value of \(z\) is about the same. The fact that the Pacific is twice as wide as the Atlantic is of no consequence because width of the ocean appears only as a cube root factor. Some of the complications in the temperature curves shown in figure 3 are due to salinity effects on the density field—but these are derated which at the present stage of the theory do not merit immediate consideration.

The model shown in figure 1 exhibits a number of general features which ought to be mentioned. First, the warm water in mid ocean does not tend to move toward the poles—but flows with an equatorward meridional component. At low latitudes it turns west (since it cannot cross the equator, according to equation (6)) where peninsually it forms a western boundary current when it encounters the western boundary of the ocean. The western boundary current is a higher-order dynamical regime which can occur under fairly general conditions of stratification, so there is no reason to doubt that the thermal circulation can be "closed" on the western side by such a boundary current—although we have not attempted a formal demonstration for this particular model. Everywhere in deep water there is a local upward component of velocity. The way in which the solution for a single basin can be connected together with solutions for other basins to form a scheme descriptive of the general circulation of the world ocean has already been pointed out by Stommel (1952).

9. Comments on the parametric treatment of melting, the quantity $a$. The inclusion of an eddy-conductivity, $a$, is a parametric way of including small-scale vertical mixing processes into the theoretical model. Although this is a very common thing to encounter in the earlier oceanographical literature, it is a technique which present-day oceanographers are very reluctant to employ, except as a last resort. It means that we simply do not know anything about the physics of the mixing process. Despite our ignorance of the cause of mixing at the mechanisms, we do have some idea, as we shall show below, of the order of magnitude of the mixing. We take $a$ by these other considerations, be of order of magnitude unity (c.g.s.), or five hundred times the molecular diffusion coefficient. The assumption has important consequences on how the model is to be set up, as was shown in equation (14). If $a$ were much greater than unity, equation (14) would become $\theta' = \omega$, yielding a linear form for $\theta$ which would have no thermocline-like character, if $a$ were much less than unity equation (14) would become $W' = -\alpha \mu y^2$, which describes a purely advective process. This equation, together with (15), has been solved by the method outlined in the beginning of section 4. The solution exhibits something like a thermocline, but its dependence on latitude does not agree with observations. Therefore, the choice for $a$ of order unity seems necessary. Is the results we note that only the cube root of $a$ appears in the depth $L$, so that $a$ may vary by an order of magnitude without changing the results significantly.

In the calculations we have taken $a$ to be a constant. A more valid $\alpha$ would probably be general a function of depth. Were this the case, the first term of the heat equation (16) would be replaced by $a[\phi/(\phi)]$. In the resulting calculation, the first term of (1) would be replaced by $a [\phi]$, and (10) would replace $a$ in all the remaining equations.
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acid by \( s \sqrt{\gamma/(\beta^0)} \), and\( \beta \)
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form of \( s \) is not critical. If the
values of equations (10) were taken, in-
properties of \( s(\zeta) \) would enter.
The mean distribution of many oceanic
ables like salinity, oxygen, carbon-14, etc,
more to a large extent on the same
velocity field and parameter values as
which determine the mean thermal field.
For the most part these properties have
important dynamical effect—except, of course,
salinity which can play a dominant role, but in
isogalactic areas usually plays a definitely
ary role in determining the density
ration. Therefore these properties are used
five tracers whose distribution depends upon
thermal circulation and the parametric
mixing, but which does not in turn effect
them. They provide a means for testing
model of the oceanic circulation for inter-
al consistency. In order to make such a test
does not seem likely that very small scale
experiments, or transient phenomena will be
very convincing because we are so completely
agreement about the nature of the mixing process
that we do not even know to scale or spectrum,
or how uniformly it acts in time. Under
the circumstances we prefer to employ tracer pheno-
mena whose horizontal and vertical and
discussion are all comparable to those
of the thermocline itself; and we want these
to be located quite close to or within
the thermocline. We offer two examples from
the Atlantic. In both cases salinity is the tracer.

Case 1: The first case is the layer of subtropical
intermediate water in the South Atlantic
Ocean. At Meter Profile VIII in the eastern
half of the ocean, between Stations 177 and
181, the salinity minimum lies at a depth
of 700 meters, where the temperature is about
\( 3^\circ C \), thus lying immediately beneath
the main thermocline. The thickness of the layer
is between 500 meters and 1,000 meters—depend-
ing upon how it is defined. The salinity in
the minimum here is about 0.2%, lower than
that 200 meters above and below the minimum
(\( s(\beta^0) \approx 10^{-12} \text{ cm}^2 \text{ sec}^{-1} \)).
The salinity gradient in the east direction is small
(\( \beta(200) \approx 0 \)) as compared to that in the northward direction
(\( \beta(700) \approx 7.10^{-16} \text{ cm}^2 \text{ sec}^{-1} \)). After (1955) has
made dynamical calculations of the velocity
between stations 177-181 at 700 meters and
that a mean northward velocity of as little as
\( 0.2 \text{ cm sec}^{-1} \). The mixing parameter com-
puted from this situation is \( \text{cm}^2 \text{ sec}^{-1} \)
by \( \text{cm} \). According to our
theory there ought to be an upward velocity
of about \( 2 \times 10^{-5} \text{ cm sec}^{-1} \) at this level, so that
we might expect the level of the minimum to
slope upward toward the north with a slope
of about 1 to which would indicate

Case 2: The second case to be discussed is the
layer of maximum salinity which covers much of
the southern North Atlantic in the area of the
Nor E Göteborg Current, and pans westward
through the entire Caribbean, storing which
passage the intensity of the maximum decreases
by about 10%. (\( s(\beta^0) \approx 0.2 \times 10^{-12} \text{ cm}^2 \text{ sec}^{-1} \)).

Upon entering the Caribbean layer is at a
level of 253 meters and spent leaving through
the Yucatan Channel it is 200 meters deep.
The mean westward velocity of this layer of
water is about \( 3 \text{ cm sec}^{-1} \) (\( s \approx 3 \text{ cm sec}^{-1} \))
and its salinity maximum is about 0.6%
greater than the salinity 100 meters above it
(\( s(\beta^0) = 2.1 \times 10^{-12} \text{ cm}^2 \text{ sec}^{-1} \)).

Thus we see that the Sawyer layer
at a temperature of \( 20^\circ C \), hence it lies near to the
of the thermocline—and the vertical veloc-
should be downward here; this latter is
in accord with the downward slope of the layer
as it crosses the Caribbean (slope of \( 1 \times \text{sec}^{-1} \)).

10. The Abyssal Circulation

At the reader can judge, this model of the
thermocline circulation implies many definite
conclusions about the circulation of the
deep waters of the ocean. For example, we
should be able to derive the total amplitude
of the deep circulation of the ocean by inter-
egrating the deep upwelling velocity over
the whole water-covered globe (in spherical co-
dinates instead of the beta-plane). In this way
we should be able to predict the average age
of the deep waters, a topic of current interest
geochemists, and a subject of much present
day experimental enquiry, and so-going activity.
This further extension of the ideas here pre-
tected is treated in a separate paper by A. B.
Stommel and Henry Stommel which is being
prepared for publication after this one.
REFERENCES


