Fluctuation dynamo amplified by intermittent shear bursts

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Thanks to my collaborators:
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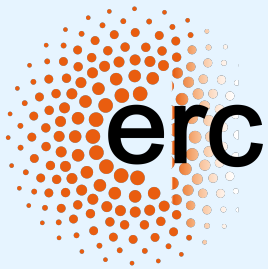


The Fluctuation Dynamo

Boussinesq MHD Convection Simulations

Shear Bursts: Amplified Energy Production

Magnetic Helicity

Introduction

The term dynamo in magnetohydrodynamics describes the systematic and sustained generation of magnetic fields (and therefore magnetic flux and magnetic energy) as a result of the dynamics of the velocity field. Dynamo action in turbulent plasmas is responsible for stellar and planetary magnetic fields.

Outline

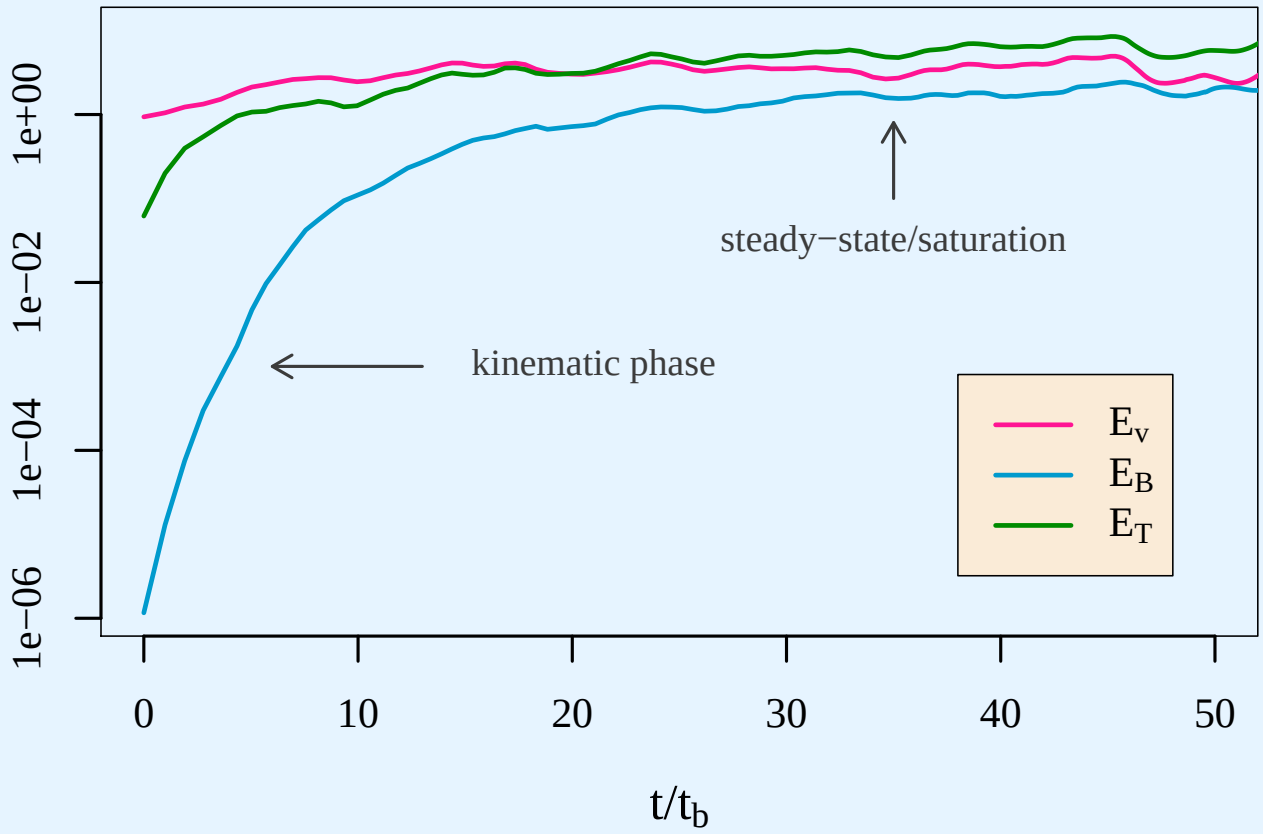
1. Terminology and basic concepts of dynamo theory.
2. Highlight interesting topics of dynamo research.
3. Shear bursts: periods of heightened magnetic energy amplification during steady-state MHD convection.

Some terminology...

- The study of turbulent dynamos has been subdivided into two types: the large-scale/mean-field dynamo and the small-scale/fluctuation dynamo depending on whether magnetic fields are amplified on scales larger than or smaller than the size of the turbulent eddies (the energy injection scale).
- These describe two separately-developed historical approaches. The implied separation of scales is artificial, but a necessity for mean-field models, which solve the induction equation for the averaged field.
- A dynamo can be classified as slow, i.e. the growth-rate of the magnetic field is dependent on resistivity, or fast, i.e. the growth-rate of the magnetic field is independent of resistivity.
- kinematic dynamo (vs. nonlinear/self-consistent/dynamic dynamo): the back-reaction of the magnetic field on the velocity field is ignored, implying that the dynamical effect of the generated magnetic field is negligible. The kinematic dynamo has been used to study how the magnetic field produced varies with flow structure and speed.

Growth and saturation of the dynamo

- Kinematic growth phase of the dynamo: the magnetic field is weak, has no dynamical effect on the turbulent flow, and magnetic energy grows exponentially.



- Stationary phase/saturation of the dynamo: in currently realizable numerical simulations of fluctuation dynamos this is independent of Reynolds numbers and Prandtl numbers as long as the Reynolds numbers are large.

Fluctuation dynamo theory

- Mechanism: random Lagrangian stretching of the magnetic field by the turbulent fluid motion.
- The fluctuation dynamo is always excited when the magnetic Reynolds number is sufficiently large, i.e. in most astrophysical situations.
- The fluctuation dynamo is not problem-specific (like the mean-field dynamo), and even operates in homogeneous isotropic situations.
- Mean-field theories assume that all small-scale magnetic fluctuations result from the shredding of the mean field by the turbulence. But these induced small-scale fields vanish if the mean field vanishes/are different from the fields generated by the fluctuation dynamo.

The challenge of dynamo simulation

- Resolving the different numerical scales for astrophysical (stellar, ISM, galactic) dynamos remains a challenge for magnetoconvection simulation.
- The Reynolds number, Re, is limited by grid size.
- The magnetic Prandtl number $\mathrm{Pr_m} = \nu/\eta = \mathrm{Re_m}/\mathrm{Re}$ should be very small for a star, or very large in the ISM.

	solar convection zone	simulation range
Re	10^{13}	$\mathcal{O}(10^3) - \mathcal{O}(10^4)$
$\mathrm{Re_m} = \mathrm{RePr_m}$	10^{11}	$\mathcal{O}(10^2) - \mathcal{O}(10^4)$
$\mathrm{Pr} = \nu/\kappa$	10^{-5}–10^{-7}	10^{-1}–10^{1}
$\mathrm{Pr_m} = \nu/\eta$	10^{-4}–10^{-7}	10^{-3}–10^{3}
Ra	10^{23}	$\mathcal{O}(10^5)$ - O(10^6)

- Simulations should reveal general features and processes that will hopefully let us understand how the dynamo could work in realistic parameter regimes.

Low magnetic Prandtl number dynamos

- Stretching is performed by the flow motions on the viscous scale ℓ_ν.
- The scale of the magnetic field that needs to be stretched is the resistive length scale ℓ_η, i.e. the smallest possible scale at which the motions can still overcome resistive damping.
- When $\mathrm{Pr_m} >> 1$, then $\ell_\eta << \ell_\nu \Longrightarrow$ well-documented stretch-fold dynamo mechanism.
- When $\mathrm{Pr_m} << 1$, then $\ell_\eta >> \ell_\nu$, ℓ_η is in the middle of the inertial range, far from ℓ_ν and the length scale of energy injection.
- Open question of how the dynamo works for $\mathrm{Pr_m} << 1$. Does turbulent diffusion dominate over small-scale stretching?

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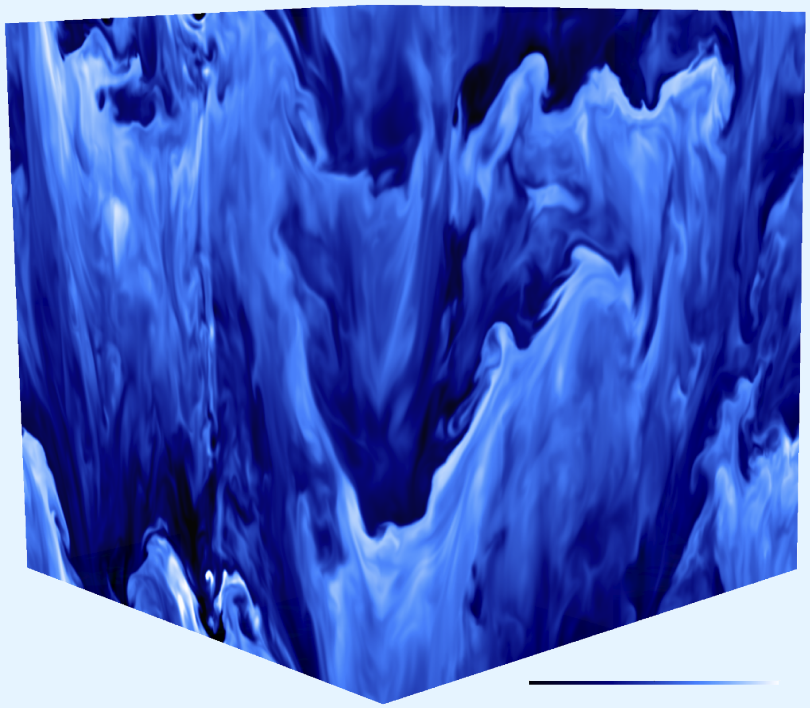
- Schekochihin et al. Simulations of the small-scale turbulent dynamo. The Astrophysical Journal 612.1 (2004): 276.
- Ponty et al. Numerical study of dynamo action at low magnetic Prandtl numbers. Phys. Rev. Lett. 94.16 (2005): 164502.
- Brandenburg. Dissipation in dynamos at low and high magnetic Prandtl numbers. Astronomische Nachrichten 332.1 (2011): 51-56.

References & Suggested Reading

- Glatzmaier. Introduction to Modeling Convection in Planets and Stars: Magnetic Field, Density Stratification, Rotation. Princeton University Press, 2013.
- Jones, Thompson, and Tobias. The solar dynamo. Space science reviews 152.1-4 (2010): 591-616.
- Brandenburg, Sokoloff, and Subramanian. Current status of turbulent dynamo theory: From large-scale to small-scale dynamos. Space science reviews 169.1-4 (2012): 123-157.
- Beresnyak. Universal nonlinear small-scale dynamo. Phys. Rev. Lett. 108.3 (2012): 035002.

Investigations of vigorously convecting plasma



Thermal fluctuations during steady state magnetoconvection.

- Pseudo-spectral DNS, at high resolution over extremely long times.
- Detailed treatment of turbulence, fundamental mechanisms of the fluctuation dynamo.
- Flows not dominated by boundary conditions, quasi-periodic: periodic with $k_z = 0$ modes explicitly suppressed.

J. Pratt, A. Busse, W.-C. Müller. Fluctuation dynamo amplified by intermittent shear bursts in convectively driven magnetohydrodynamic turbulence. A&A 557 A76 (2013).

R. Moll, J.P. Graham, J. Pratt, et al. Universality of the small-scale dynamo mechanism. The Astrophysical Journal 736.1 (2011): 36.

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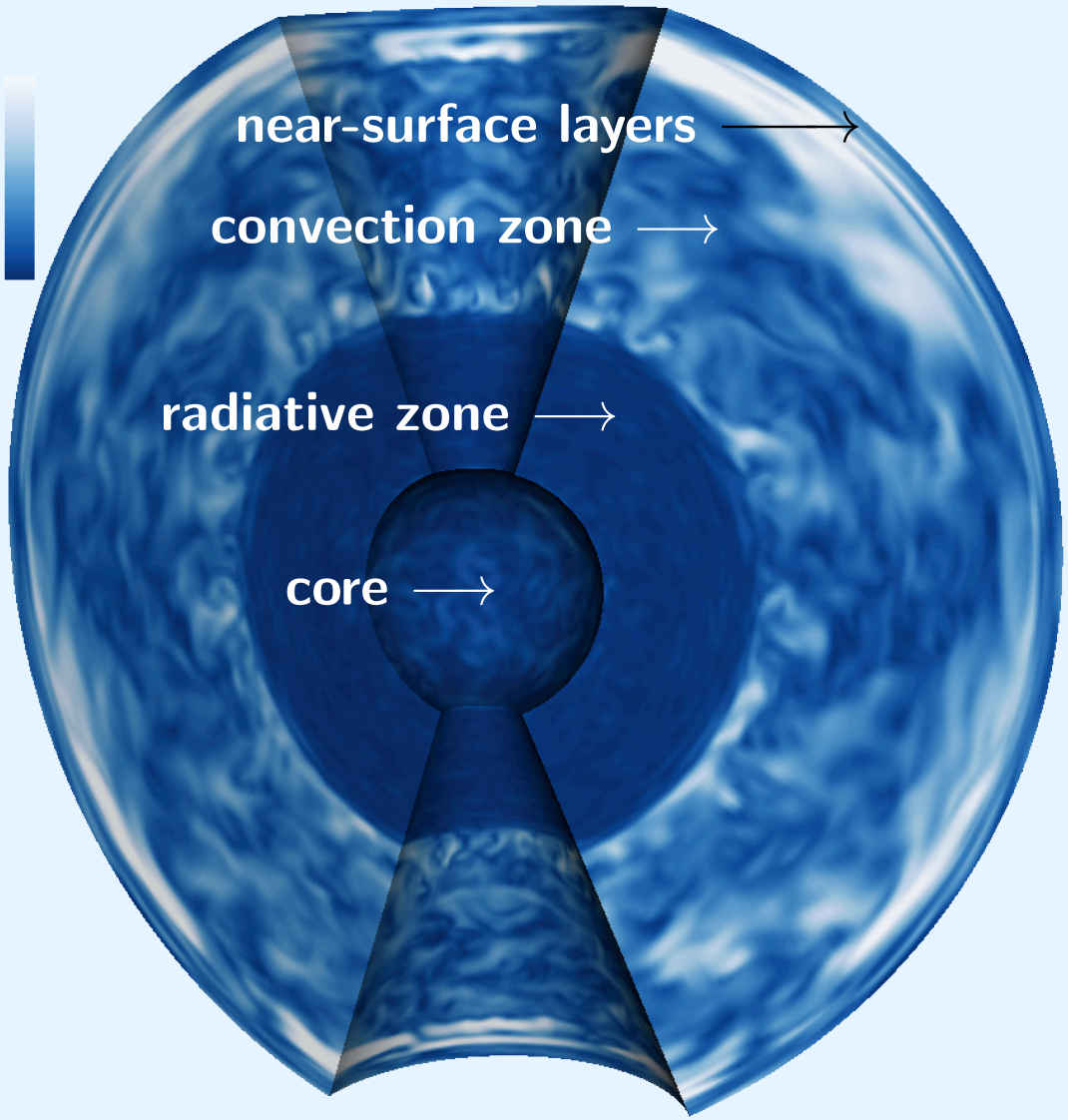
Boundary conditions

- With fully-periodic boundary conditions (homogeneous Rayleigh–Bénard), the macroscopic elevator instability can be readily identified (Calzavarini *et* al. 2006, Phys. Rev. E, 73, 035301).
- Homogeneous, incompressible flow can gain vast amounts of energy by coherent large-scale vertical motions.
- elevator instability: parallel, vertical *jets* ($k_z = 0$) of alternating direction throughout the volume. Not a wave that can couple non-linearly to other waves in the flow.
- This obscures the type of turbulent flows we are interested in studying.
- Mean flows parallel to gravity are excluded in our quasi-periodic set-up. We see no elevator modes.
- Our quasi-periodic boundary conditions enlarge the interval of scales on which vigorous turbulent fluctuations can be observed towards large-scales.

Stellar dynamos



Velocity magnitude during steady-state convection, in a realistic wedge of our sun at ~ 1 Myr, simulated with a the three-dimensional, time-implicit, compressible LES, stellar evolution code MUSIC (ERC Advanced grant, University of Exeter).

The Boussinesq approximation (incompressible fluid with buoyancy force, not compressible) is often used to study the fluctuation dynamo.

In Boussinesq simulations we learn about general characteristic properties of the fluctuation dynamo, that may be relevant to understanding dynamo action on small scales in the convection zone of a realistic star.

Boussinesq MHD Convection Equations

vorticity — kinematic viscosity — buoyancy force

$$\frac{\partial \boldsymbol{\omega}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega} + \boldsymbol{j} \times \boldsymbol{B}) = \nu \nabla^2 \boldsymbol{\omega} - \nabla\theta \times \boldsymbol{g}_0$$

magnetic field — magnetic diffusivity

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = \eta \nabla^2 \boldsymbol{B}$$

thermal fluctuations — thermal diffusivity — mean temperature gradient

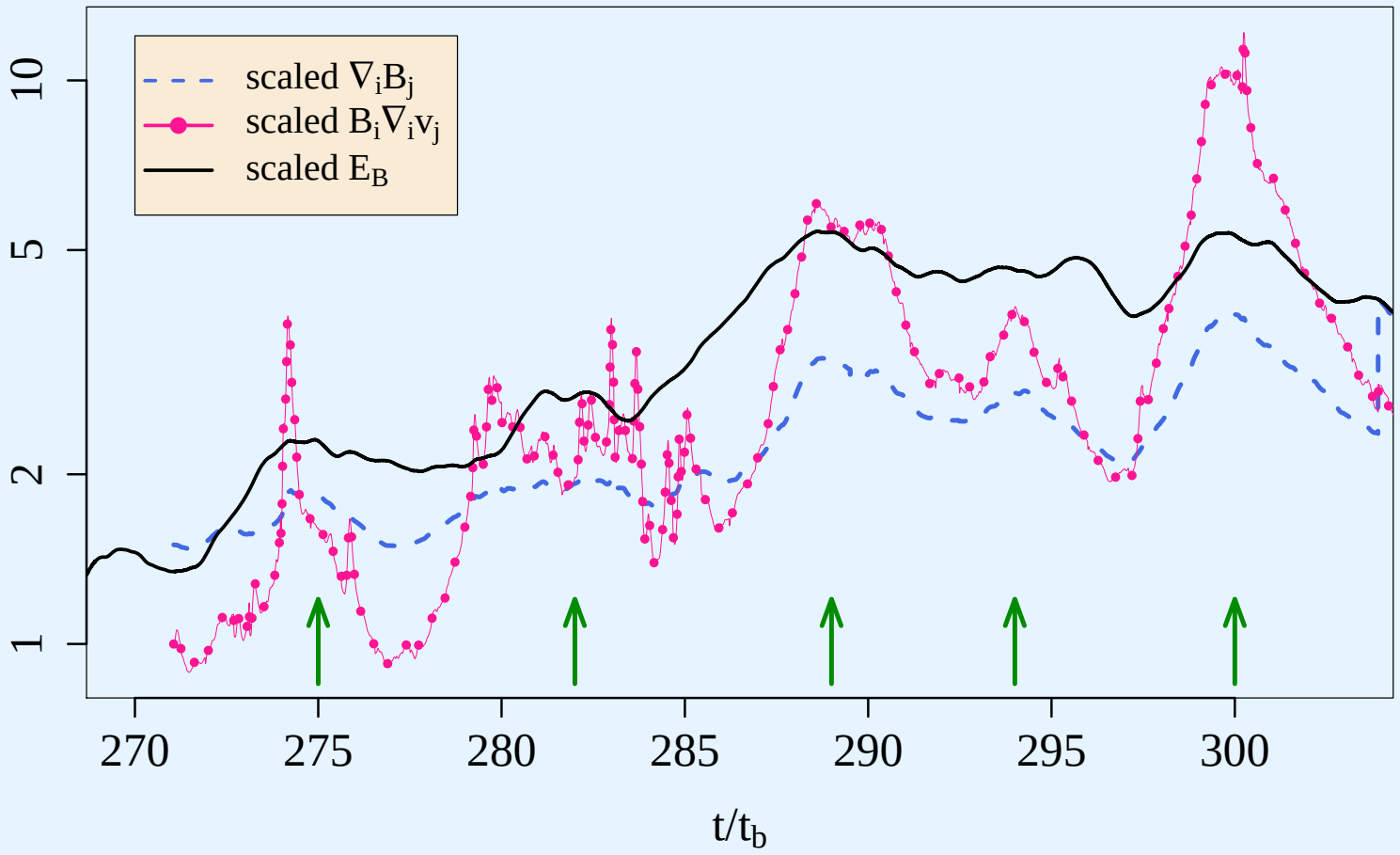
$$\frac{\partial \theta}{\partial t} + (\boldsymbol{v} \cdot \nabla)\theta = \kappa \nabla^2 \theta - \boldsymbol{v} \cdot \nabla T_0$$

incompressible fluid — divergence free magnetic field

$$\nabla \cdot \boldsymbol{v} = \nabla \cdot \boldsymbol{B} = 0$$

- nondimensionalized equations. Fluctuations of fields, i.e. $x = X - \bar{X}$ with zero means (except mean temperature gradient).
- characteristic time scale is the buoyancy time: $t_\mathrm{b} = 1/\sqrt{\alpha g |\nabla T_0|}$ where α is the volume thermal expansion coefficient.

Shear Bursts: Energy production from Magnetic Stretching



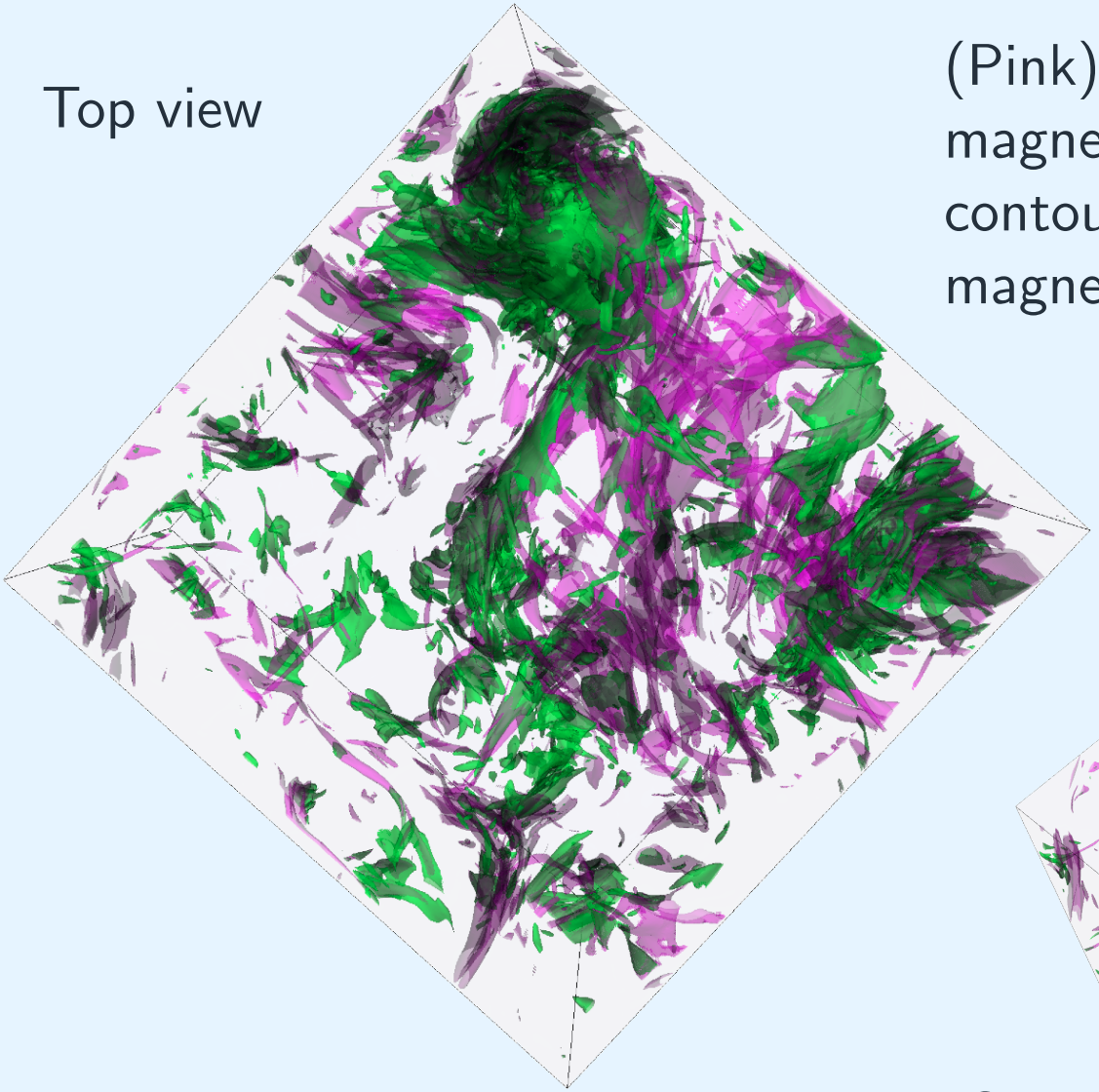
Magnetic stretching (pink) and magnetic shear (blue) due to intermittent high-shear flows boosts energy over several buoyancy times. When successive shear-bursts happen frequently, magnetic energy (black) is amplified over long times. Consecutive shear bursts are indicated by green arrows.

Description of Shear Bursts

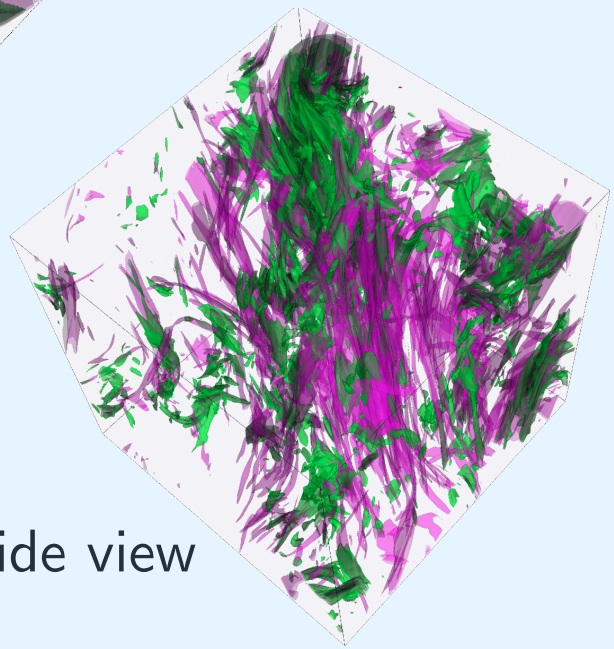
- Shear flow shows no preference for any fixed spatial direction, but follows a complicated, curved path in a localized section of the simulation volume.
- The flow path of the intense shear changes during the evolution of the shear burst, and is different for each shear burst.
- Shear bursts are embedded into the turbulence and have limited coherence and lifetime with regard to the full flow-field.
- Shear bursts exhibit uneven growth of energy, with a growth rate independent of Prandtl number.

High magnetic helicity structures grow

Top view

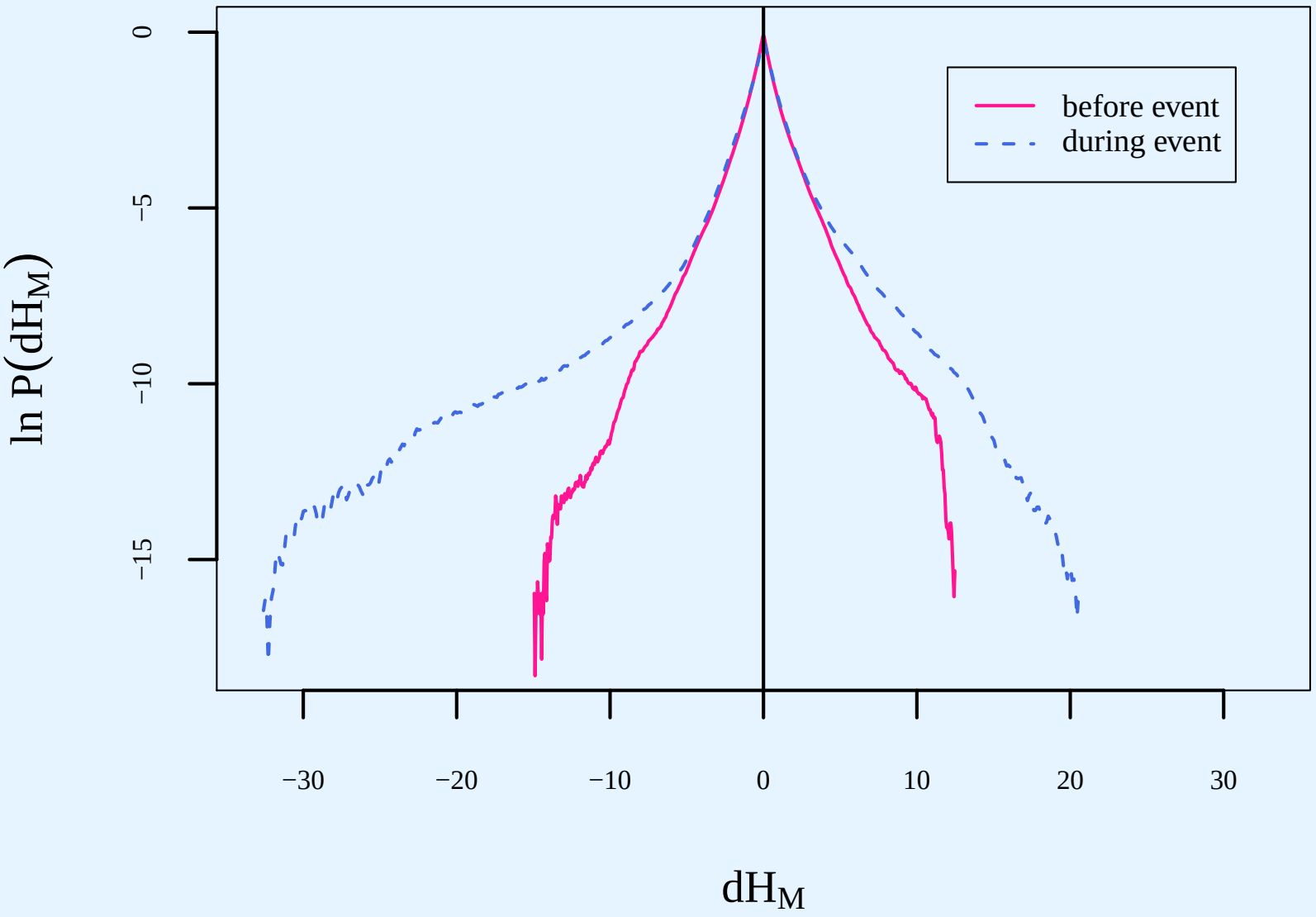


(Pink) contours of high magnetic energy. (Green) contours of high positive magnetic helicity.



Side view

Highly polarized magnetic helicity



Summary

- Bursts of shear-flow intermittently and spontaneously arise in our fluctuation dynamo simulation.
- Shear bursts create and interact with large-scale energy-containing magnetic structures.
- Consecutive shear bursts can produce elevated energies over long periods of time.
- These are observations. We are still looking for conceptual explanation for the shear burst.
- Conceptually a fluctuation dynamo can work at any scale. This is clear in our simulations because of our highly non-restrictive boundary conditions.

References & Suggested Reading

- Glatzmaier. Introduction to Modeling Convection in Planets and Stars: Magnetic Field, Density Stratification, Rotation. Princeton University Press, 2013.
- Jones, Thompson, and Tobias. The solar dynamo. Space science reviews 152.1-4 (2010): 591-616.
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Example Simulation Parameters

	MC1	MC2	MC3	MC4	MC5	MC6	MC7	MC8
Re (10^3)	3.1	5.2	5.4	5.6	3.5	5.1	4.0	2.4
$\mathrm{Re_m}$ (10^3)	6.2	5.2	5.4	2.8	7.0	7.65	8.0	4.2
Pr	1	1	0.5	1	1	2	1.5	1.76
$\mathrm{Pr_m}$	2	1	1	0.5	2	1.5	2	1.76
η (10^{-3})	7.88	7.39	6.29	6.7	7.8	8.9	9.2	12.5
$\tau_\eta(10^{-2})$	3.11	2.73	2.64	2.98	3.02	2.6	2.8	3.6
time (τ_η)	1219.2	1944.8	1433.7	2683.6	431.8	2203.3	2312.9	2531.0
LCT (τ_η)	1219.2	529.9	741.7	685.3	431.8	807.7	691.4	581.0
r_A	1.28	2.20	1.43	2.79	1.12	1.78	1.72	2.00
ℓ_bo	0.08	0.09	0.08	0.06	0.09	0.12	0.12	0.12