Fluctuation dynamo amplified by intermittent shear bursts

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Introduction

The term dynamo in magnetohydrodynamics describes the systematic and sustained generation of magnetic fields (and therefore magnetic flux and magnetic energy) as a result of the dynamics of the velocity field. Dynamo action in turbulent plasmas is responsible for stellar and planetary magnetic fields.

Outline

1. Terminology and basic concepts of dynamo theory.
2. Highlight interesting topics of dynamo research.
3. Shear bursts: periods of heightened magnetic energy amplification during steady-state MHD convection.
Some terminology...

- The study of turbulent dynamos has been subdivided into two types: the large-scale/mean-field dynamo and the small-scale/fluctuation dynamo depending on whether magnetic fields are amplified on scales larger than or smaller than the size of the turbulent eddies (the energy injection scale).

- These describe two separately-developed historical approaches. The implied separation of scales is artificial, but a necessity for mean-field models, which solve the induction equation for the averaged field.

- A dynamo can be classified as **slow**, i.e. the growth-rate of the magnetic field is dependent on resistivity, or **fast**, i.e. the growth-rate of the magnetic field is independent of resistivity.

- **Kinematic dynamo** (vs. nonlinear/self-consistent/dynamic dynamo): the back-reaction of the magnetic field on the velocity field is ignored, implying that the dynamical effect of the generated magnetic field is negligible. The kinematic dynamo has been used to study how the magnetic field produced varies with flow structure and speed.
Growth and saturation of the dynamo

- **Kinematic growth phase** of the dynamo: the magnetic field is weak, has no dynamical effect on the turbulent flow, and magnetic energy grows exponentially.

- **Stationary phase/saturation** of the dynamo: in currently realizable numerical simulations of fluctuation dynamos this is independent of Reynolds numbers and Prandtl numbers as long as the Reynolds numbers are large.
Fluctuation dynamo theory

- Mechanism: random Lagrangian stretching of the magnetic field by the turbulent fluid motion.

- The fluctuation dynamo is always excited when the magnetic Reynolds number is sufficiently large, i.e. in most astrophysical situations.

- The fluctuation dynamo is not problem-specific (like the mean-field dynamo), and even operates in homogeneous isotropic situations.

- Mean-field theories assume that all small-scale magnetic fluctuations result from the shredding of the mean field by the turbulence. But these induced small-scale fields vanish if the mean field vanishes/are different from the fields generated by the fluctuation dynamo.
The challenge of dynamo simulation

- Resolving the different numerical scales for astrophysical (stellar, ISM, galactic) dynamos remains a challenge for magnetoconvection simulation.
- The Reynolds number, $Re$, is limited by grid size.
- The magnetic Prandtl number $Pr_m = \nu/\eta = Re_m/Re$ should be very small for a star, or very large in the ISM.

<table>
<thead>
<tr>
<th></th>
<th>solar convection zone</th>
<th>simulation range</th>
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<tbody>
<tr>
<td>$Re$</td>
<td>$10^{13}$</td>
<td>$O(10^3) - O(10^4)$</td>
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<tr>
<td>$Re_m = RePr_m$</td>
<td>$10^{11}$</td>
<td>$O(10^2) - O(10^4)$</td>
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<td>$Pr = \nu/\kappa$</td>
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<td>$Pr_m = \nu/\eta$</td>
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<tr>
<td>$Ra$</td>
<td>$10^{23}$</td>
<td>$O(10^5) - O(10^6)$</td>
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- Simulations should reveal general features and processes that will hopefully let us understand how the dynamo could work in realistic parameter regimes.
Low magnetic Prandtl number dynamos

- Stretching is performed by the flow motions on the viscous scale $\ell_\nu$.

- The scale of the magnetic field that needs to be stretched is the resistive length scale $\ell_\eta$, i.e. the smallest possible scale at which the motions can still overcome resistive damping.

- When $Pr_m \gg 1$, then $\ell_\eta \ll \ell_\nu \implies$ well-documented stretch-fold dynamo mechanism.

- When $Pr_m \ll 1$, then $\ell_\eta \gg \ell_\nu$, $\ell_\eta$ is in the middle of the inertial range, far from $\ell_\nu$ and the length scale of energy injection.

- Open question of how the dynamo works for $Pr_m \ll 1$. Does turbulent diffusion dominate over small-scale stretching?

References & Suggested Reading


Investigations of vigorously convecting plasma

- Pseudo-spectral DNS, at high resolution over extremely long times.

- Detailed treatment of turbulence, fundamental mechanisms of the fluctuation dynamo.

- Flows not dominated by boundary conditions, quasi-periodic: periodic with $k_z = 0$ modes explicitly suppressed.

Thermal fluctuations during steady state magnetoconvection.


Boundary conditions

- With fully-periodic boundary conditions (homogeneous Rayleigh–Bénard), the macroscopic elevator instability can be readily identified (Calzavarini et al. 2006, Phys. Rev. E, 73, 035301).

- Homogeneous, incompressible flow can gain vast amounts of energy by coherent large-scale vertical motions.

- Elevator instability: parallel, vertical jets ($k_z = 0$) of alternating direction throughout the volume. Not a wave that can couple non-linearly to other waves in the flow.

- This obscures the type of turbulent flows we are interested in studying.

- Mean flows parallel to gravity are excluded in our quasi-periodic set-up. We see no elevator modes.

- Our quasi-periodic boundary conditions enlarge the interval of scales on which vigorous turbulent fluctuations can be observed towards large-scales.
The Boussinesq approximation (incompressible fluid with buoyancy force, not compressible) is often used to study the fluctuation dynamo. In Boussinesq simulations we learn about general characteristic properties of the fluctuation dynamo, that may be relevant to understanding dynamo action on small scales in the convection zone of a realistic star.

Velocity magnitude during steady-state convection, in a realistic wedge of our sun at ~ 1 Myr, simulated with a the three-dimensional, time-implicit, compressible LES, stellar evolution code MUSIC (ERC Advanced grant, University of Exeter).
Boussinesq MHD Convection Equations

\[
\frac{\partial \mathbf{v}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{\omega} + \mathbf{j} \times \mathbf{B}) = \nu \nabla^2 \mathbf{v} - \nabla \theta \times \mathbf{g}_0
\]

\[
\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = \eta \nabla^2 \mathbf{B}
\]

\[
\frac{\partial \theta}{\partial t} + (\mathbf{v} \cdot \nabla) \theta = \kappa \nabla^2 \theta - \mathbf{v} \cdot \nabla T_0
\]

\[
\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0
\]

- nondimensionalized equations. Fluctuations of fields, i.e. \( x = X - \bar{X} \) with zero means (except mean temperature gradient).

- characteristic time scale is the buoyancy time:
\[
t_b = \frac{1}{\sqrt{\alpha g |\nabla T_0|}} \text{ where } \alpha \text{ is the volume thermal expansion coefficient.}
\]
Magnetic stretching (pink) and magnetic shear (blue) due to intermittent high-shear flows boosts energy over several buoyancy times. When successive shear-bursts happen frequently, magnetic energy (black) is amplified over long times. Consecutive shear bursts are indicated by green arrows.
Description of Shear Bursts

- Shear flow shows no preference for any fixed spatial direction, but follows a complicated, curved path in a localized section of the simulation volume.

- The flow path of the intense shear changes during the evolution of the shear burst, and is different for each shear burst.

- Shear bursts are embedded into the turbulence and have limited coherence and lifetime with regard to the full flow-field.

- Shear bursts exhibit uneven growth of energy, with a growth rate independent of Prandtl number.
High magnetic helicity structures grow

(Pink) contours of high magnetic energy. (Green) contours of high positive magnetic helicity.
Highly polarized magnetic helicity

\[ \ln P(dH_M) \]

before event
during event

\[ -30 -20 -10 0 10 20 30 \]

\[ -15 -10 -5 0 \]

\[ dH_M \]
Summary

- Bursts of shear-flow intermittently and spontaneously arise in our fluctuation dynamo simulation.

- Shear bursts create and interact with large-scale energy-containing magnetic structures.

- Consecutive shear bursts can produce elevated energies over long periods of time.

- These are observations. We are still looking for conceptual explanation for the shear burst.

- Conceptually a fluctuation dynamo can work at any scale. This is clear in our simulations because of our highly non-restrictive boundary conditions.
The Fluctuation Dynamo

Boussinesq MHD Convection Simulations

Shear Bursts: Amplified Energy Production

Magnetic Helicity

References & Suggested Reading


Thanks!
# Example Simulation Parameters

<table>
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<tr>
<th>Parameter</th>
<th>MC1</th>
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