Optimal control of wave energy converters using deterministic sea wave prediction

G. Weiss\(^1\), G. Li\(^2\), M. Mueller\(^2\), S. Townley\(^2\) and M.R. Belmont\(^2\)

\(^1\)School of EE, Tel Aviv University, Ramat Aviv 69978, Israel
\(^2\)College of Engineering, Mathematics and Physical Sciences, University of Exeter, Harrison Building, North Park Road, Exeter EX4 4QF, UK

*Corresponding author: e-mail: gweiss@eng.tau.ac.il, Phone: +972 3 6405164

We demonstrate that deterministic sea wave prediction (DSWP) combined with constrained optimal control can dramatically improve the efficiency of sea wave energy converters (WECs), while maintaining their safe operation. Our analysis concerns a WEC consisting of a float moving vertically against a heave plate and employing a hydraulic/electric power take-off system. Maximising energy take-off while minimising the risk of damage is formulated as an optimal control problem with a disturbance input (the sea elevation) and with both state and input constraints. We demonstrate that a nearly optimal control is of bang-bang type. This paves the way to adopt a dynamic programming (DP) algorithm to resolve the on-line optimization problem efficiently. Simulation results show that this approach is very effective, yielding at least a two-fold increase in energy output as compared to control schemes which do not exploit DSWP.

Keywords wave energy; float; heave plate; synchronous machine; constrained optimal control; bang-bang control; dynamic programming; deterministic sea wave prediction

Wave energy converters (WECs) are still a relatively immature technology (compared to solar or wind energy) and is far from being commercially competitive with traditional fossil fuel or nuclear energy sources. Progress is hampered by inefficient energy extraction (often due to the fact that the parameters are not optimally tuned) and risk of device damage by large waves (which forces the devices to be shut down in bad weather). Extracting the maximum possible time average power from WECs, while reducing the risk of device damage involves a combination of good fundamental engineering design of the devices and effective control of their operation. The traditional approach to these issues exploits short term statistical properties of the sea [3] but it has been shown [4, 5] that doing so severely limits the average power that can be extracted. It will be shown that (as [6–18] demonstrated in the 1970s) methods for achieving the maximum power output are inevitably non-causal and require prediction of the shape of the incident waves. The recent development of deterministic sea wave prediction (DSWP) as a scientific discipline [9–25], particularly real time DSWP [9–16, 23, 25] now makes such an approach realistic, even though demanding.

![Figure 1: Schematic diagram of the WEC](image1)

![Figure 2: A mechanical circuit representation of the WEC](image2)

The dimensions of point absorbers are small compared with the wavelength of incoming waves and they are potentially very efficient if their frequency response function closely matches the spectrum of the incident waves (resonance). Passive control methods (such as impedance matching) have been explored to improve energy extraction by tuning the dynamical parameters of the devices [6, 26–28]. Most of these approaches are linear control schemes. A non-linear control method that has received some attention is latching, [29–34]. This
attempts to force the phase angle between the float and the wave at the WEC to be similar to conditions at resonance. The above control strategies do not use prediction of the forces acting on the WEC and thus inevitably lead to sub-optimal energy extraction. Since the early work [2, 6, 8] there have been a number of authors who have recognized the importance of DSWP in the control of a variety of floating body applications [4, 5, 14, 34, 35], but these have, as yet, not been incorporated into actual control schemes.

The point absorber model used is shown in Fig. 1 and roughly corresponds to the Power Buoy device PB150 developed by OPT Inc, see [36]. On the sea surface is a float, below which hydraulic cylinders are vertically installed. These cylinders are attached at the bottom to a large area antiheave plate whose vertical motion is designed to be negligible compared to that of the float. The heave motion of the float drives the pistons inside the hydraulic cylinders to produce a liquid flow. The liquid drives hydraulic motors attached to a synchronous generator (or synchronous machine, denoted by SM in Figure 3). From here, the power reaches the grid via back-to-back AC/DC/AC converters. The mechanical circuit corresponding to this simplified model is shown in Fig. 2. Here \( h_w \) is the water level, \( h_v \) is the height of the mid-point of the float and \( D \) is the hydrodynamic damping of the float including added damping due to the damping effect of the movement of the float [1]. \( K \) is the hydrostatic stiffness giving the buoyancy force, which can be calculated from the float geometry, while \( m \) is the mass of the float including “added mass” [1]. The friction force acting on the float is \( f_f = D_f \frac{d}{dt}(h_v) \). In order to simplify the model we neglect the frequency dependence of both \( D \) and \( m \) (see, e.g., [26]). We also neglect the static component of the friction force \( f_f \). For a more thorough investigation of the modeling issues of point absorbers, see [1, 37, 38]. The control input is the \( q \)-axis current in the generator-side power converter, to control the electric torque of the generator [39]. The generator torque is proportional to the force \( f \) acting on the pistons from the fluid in the cylinders. Since the motion of the float imposes a velocity \( v \) on the piston, the extracted power \( P(t) \) at time \( t \) is expressed as

\[
P(t) = f(t)v(t). \tag{1}
\]

This power is smoothed by the capacitors on the DC bus of the converters, see Fig. 3. In our model the modest power losses in the hydraulic transmission, the generator and the converters will be neglected.

Figure 3: Schematic diagram of the hydraulic and electric systems on a WEC

To avoid damage, and for overall performance reasons, two constraints have to be considered in any real WEC. One concerns the relative motion of the float to the sea surface (it should neither sink nor raise above the water and then slam), which is the first restriction shown below:

\[
| h_w - h_v | \leq \Phi_{\text{max}}, \quad | f | \leq \gamma. \tag{2}
\]

The other constraint is on the control signal set by limitations on the allowable converter current: this is the second condition in (2). The control objective is to maximize the extracted energy (the integral of \( P \) from (1) over a certain receding time horizon) subject to the constraints (2). We remark that there is a further constraint on the motion of the float because of the limited excursion of the piston with respect to the cylinder (see Fig. 1). This constraint has the form \( | h_v | \leq \lambda \). However, we shall not consider this constraint, since we assume that \( \lambda \) is large enough compared to the expected excursion.

In this research, whose details are reported in [48], the constrained optimal control problem is solved using fundamental principles from optimal control theory [45–47] and real time deterministic sea wave prediction [9–
We use an interior penalty term to replace the state constraint. The system to be controlled is second order LTI with the constrained input $f$. Using Pontryagin’s minimum principle, we demonstrate that a nearly optimal control is of bang-bang type, meaning that $f$ is always at one edge of the allowed range. As will be shown, for an arbitrary sea wave input known over an interval of time (not a sine wave), direct numerical computation of the optimal control scheme is not realistic. Consequently, we employ dynamic programming (DP) [43,44], which is well suited for constrained optimal control problems [48]. This makes it possible to compute the discretized control input step by step without having to solve the canonical equations. However, Pontryagin’s minimum principle is still important for telling us that a nearly optimal control for the original problem (8) is of bang-bang type, which dramatically simplifies the DP algorithm.

In the on-line implementation of the DP algorithm for WECs, at each sampling instant, the DP algorithm determines the optimal control input to achieve maximal energy extraction for the WEC over the prediction horizon, while satisfying input and state constraints. Denote the sampling period by $T_s$. Then the N-stage receding horizon is $t_0 + H_p$, where $H_p = N \times T_s$ is the prediction time and $t_0$ is the present time. The receding horizon slides forward by one sampling period after each execution of the algorithm. At $t_0$, the DP algorithm produces an optimal control input sequence for the interval $[t_0, t_0 + H_p]$; however, the control action which is applied to the system is only the first value (at time $t_0$) of this control input sequence. The next execution of the algorithm computes the optimal input for the system within the interval $[t_0 + T_s, t_0 + H_p + T_s]$, with updated sea surface/wave prediction, but only the control input value at $t_0 + T_s$ is applied to the system, and so forth. This on-line optimization control method is known as model predictive control (MPC), and also as moving horizon control [49,50]. For this on-line implementation of the DP algorithm in the WEC control, forward dynamic programming (FDP) is better suited than the more common backwards dynamic programming. The FDP algorithm is also based on the principle of optimality, but it calculates the cost by sweeping from the initial stage to the last stage. There are two reasons for us adopting FDP as an on-line optimization algorithm. First, FDP can significantly reduce the on-line computational burden. In FDP, at each sampling instant only the optimal control signal corresponding to one initial state is calculated, i.e., the state measured at the present time. Second, in FDP there is no constraint imposed on the final state.

![Figure 4: Energy extracted over time (simulation).](image)

We see that the WEC controlled with the FDP algorithm with perfect prediction over a 1sec prediction horizon performs best (it is the top curve). When we add random prediction errors of a realistic size, the results are slightly weaker (the second curve from top). All other algorithms (not prediction based) perform significantly weaker, extracting about half of the energy.

We have sacrificed some detail in the hydrodynamic modeling, leading to a model of manageable complexity for on-line FDP. The implementation of FDP on a WEC control system is based on the assumption that the sea surface shape can be predicted for a short time period. This requirement has been a bottleneck for the
development of suitable optimal control strategies for WECs. However, the developments in deterministic sea wave modeling techniques have made real time sea wave prediction for a short time period realizable, [9–16, 23,25]. A key finding from this work is that the prediction horizons required are considerably smaller than those resulting from the previous studies [40–42]. For our particular model (described below) we found that there is no benefit in predicting the sea wave for more than 1 sec. For the sake of comparison, simulations have also been performed for various non-prediction-based WEC control strategies. The results demonstrate the following benefits of our approach. 1) Significant increase in energy output. We get up to a two-fold increase in energy output when compared to rival control algorithms which do not exploit sea state prediction. This is shown in the simulation results in Figure 4. 2) Robustness to the prediction accuracy and prediction horizon of DSWP. Especially important is the possibility of reducing the prediction horizon, because the difficulties associated with real time DSWP increase significantly with the prediction horizon.

The numerical parameters used in our SIMULINK simulations approximately reflect those of a moderate sized point absorber such as the PB150, but are not intended to be a precise description of any actual system. The float diameter is \( \Delta_h = 9 \text{m} \). The height of the float is \( \Delta h = 2.4 \text{m} \), hence the range of the float heave motion is \([-1.2, 1.2]\text{m}\). The stiffness is \( K = 6.39 \times 10^3 \text{N/m} \). The mass of the float is \( m_f = 10^5 \text{ kg} \). We estimate the added mass to be \( m_a = 7 \times 10^3 \text{ kg} \). Then the total mass is \( m = m_f + m_a = 8 \times 10^4 \text{ kg} \). The damping coefficient is taken as \( D = 2 \times 10^4 \text{ Nm/s} \). The damping ratio corresponding to the friction is \( D_f = 2 \times 10^4 \text{ Nm/s} \). We took \( \Phi_{max} = 1.2 \text{ m} \). The maximum control input is chosen to be \( \gamma = 3 \times 10^5 \text{ N} \). Real sea wave data (50 sec) gathered off the coast of Cornwall, UK have been used. The sampling period (and also the time step in the discrete time model and in the FDP algorithm) is \( T_s = 0.04 \text{ sec} \), but the simulation and the computation of the energy were done at a much finer time step. We have used a prediction horizon of 1 sec (25 steps), with and without prediction error. The prediction error is Gaussian and filtered through an unstable filter, so that it increases in time. We have compared our algorithm with various reasonable algorithms that do not use sea wave prediction. For lack of space we do not describe these alternative algorithms, but we refer to our paper [48]. The energy extracted over time for the various algorithms is shown in Figure 4, which demonstrates the advantages of using a FDP based optimization using deterministic sea wave prediction.

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References


