Prime numbers as lawful creatures of the mind

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Abstract

Numbers (positive integers) are the most fundamental creatures of the human mind. The algorithm that programs the mind and makes the mind creative must be sufficient for the mind to create numbers. I found that prime numbers are directly linked to the creation algorithm of the mind. The essence of primes is the duality of uniqueness and uniformity together with the creation algorithm of the mind. This new understanding of the foundation of primes can deduce some of the best-known properties of primes, including the duality of regularity and randomness and the Riemann Hypothesis.

Key words: prime numbers, uniqueness, uniformity, the Riemann Hypothesis, the Prime Law

Introduction:

A prime number is commonly defined as a positive integer that has only two divisors, 1 and itself. Both the number 1 and 2 can be either included or excluded as primes by manipulating the definition of primes. Accordingly, the primality of the number 1 and 2 are decided by human agreements rather than objective logic or reason. The number 1 is not considered a prime today but was in the past (Gardner, 1984; Hardy, 1999; Hardy and Wright, 1979; Lehmer, 1914; Sloane and Plouffe, 1995). While 2 is considered a prime today, at one time it was not (Tietze, 1965). The odd primes have many properties not shared by 2, the only even prime. It is also easy to have a definition based on calculation that would include all primes except 2. Thus, a prime can be defined as a positive integer that cannot be expressed by the even number of sums of any single number except 1 and itself. For example, 1 is 0 (an even number) sum of 1 and itself; 3 is 2 sums of 1 and 0 sum of itself; but 2 is not a prime since it is 1 (an odd number) sum of 1.

To define numbers by calculation that is itself defined by numbers is a tautology, which merely describes ways of identifying some primes but reveals little about what a prime really is or the essence of primes. This leads to the dilemma that 2 is a prime in one definition based on division but not a prime in another equally plausible definition based on addition. It is arbitrary human convenience to favor one tautological definition over another. We can only resolve such dilemma with objective reasoning when we achieve a deeper understanding of primes that is based on knowledge more fundamental than calculation and numbers. Primes are the foundations of mathematics and should have a form of existence or definition that is independent of mathematics. If primes are atoms that build other numbers, then the primes must be built by its own building blocks, which would be equivalent to quantum particles.

Numbers are creatures of the mind. The essence of a creature is its building blocks together with a rule of manipulating the building blocks. The essence is what is ultimately responsible for the properties of a creature. The essence of matter, the quanta building block together with a law of manipulating the quanta, is what is ultimately responsible for the properties of the physical universe. A creature must be defined by its essence. The present definition of primes contains no concept of building blocks and does not reveal the essence of primes.

To avoid circularity, a creature must be defined by things that are more basic than it rather than more advanced. We must use quantum particles rather than molecules to define atoms, even though we discovered molecules before we knew about quanta. Just because calculation was discovered before prime numbers in human history does not mean that primes must be defined by calculation. A concept can only be defined by concepts lower or more basic in logical hierarchy. The hierarchy must be, from bottom/basic to top/advanced, the following: numbers (primes and non-primes), addition/subtraction, multiplication/division, etc. What is even more basic than numbers must be used to define primes and non-primes. The creation algorithm that programs the mind and makes the mind creative is obviously the foundation of all human creations. Since numbers (positive integers) may represent the most fundamental creatures of the mind, the creation algorithm of the mind should be able to create numbers.

Results and discussion:

The algorithm that makes the mind creative

The most fundamental capacities of a human mind may be to know and to imagine, which are essential to creativity. To know is to recognize the unique from a background of contrasting uniformity and vice versa. To imagine is to think of novel things that do not exist previously. By observing how the human mind creates, I have found a simple algorithm that programs the mind and makes the mind creative. If a computer can be programmed with this algorithm, it would become creative. This algorithm consists of a pair of opposite but complimentary yin and yang principles and a mind that coordinates the interplay of the two principles. A creation or creature is defined as the unique that does not exist previously, is distinguished from all other imagined things, and can exist subsequent to its creation by being able to initiate a population of followers that share a uniform pattern resembling the unique. A creation has the duality of uniqueness and uniformity. A follower of a creation is defined as the new thing that does not exist previously but shares some uniform property with a prior creation. A creation is a big jump in paradigm while a follower of a creation represents a small step advance within a paradigm.

The yang principle is uniformity selection that allows the mind to recognize the unique or the creation. Uniformity abolishes individuality and selects for the unique. Uniformity selection drives the creation of the unique. The yin principle is uniqueness selection that allows the unique to initiate a population of followers sharing a uniformity pattern resembling the unique. The mind uses this principle to allow the unique to exist or survive subsequent to its creation. Uniqueness selection results in the formation of an ordered uniformity consisting of individuals who are fittest or most adapted to the unique. The process from the unique to a specific uniformity of a population of followers is essential for the unique to exist subsequent to its creation, which further serves to drive the creation of the next unique. The creative process of the mind is the iterative use of the same creation algorithm and an endless cycling process from uniformity to unique to new-uniformity. When the mind sees the unique, the mind strives to fit and follow. When the mind sees uniformity, the mind strives to be unique. All human minds are a unity of different degrees of the yin and yang principles.

To create, the mind needs to know what is known previously, which is termed the existing-uniformity. Selection by existing-uniformity allows the mind to know whether something is new with a meaningfully ordered pattern. In addition, all creations begin from the imagination of the mind. Within the imagined world, there exists a unique entity that is distinguished from the imagined-uniformity shared by other imagined entities. To create by uniformity selection is to bring into existence an imagined entity that is distinct from both the existing-uniformity and the imagined-uniformity.

Creating primes by the creation algorithm of the mind

Like creations of the mind, the odd primes including the number 1 also have the dual property of uniqueness and uniformity. A thing is unique if it is not an inherent part of something else and is different from uniformity. A number is an inherent part of a smaller number either because it is needed for the smaller number to have meaning or because it can be expressed as a pattern of a single smaller number >1. The number 2 lacks uniqueness because it is an inherent part of creating the number 1, as evidenced by the existence of civilizations that had invented only 1 and 2 and by the absence of civilizations that invented only 1 but not 2. We need 2 to invent 1 or for 1 to have any meaning. We need both 1 and 2 in order to invent the concept of number. However, we do not need 3 to invent 1 and 2. The author John Barrow observes:

"There are many primitive human groups who cannot count beyond two and have developed number-sense at all. Some Australian aboriginal tribes only possess words for the quantities 'one' and 'two'. All greater quantities are expressed by a word with the sense of 'many'. Some South African tribes exhibit a similar language structure for number words. In fact, there is even a vestigial remnant of it in modern Indo-European languages where we find that the original root for 'three' has a meaning of something like 'over' 'beyond', or 'afar', indicating that the early sense was merely that of something more than the particular words for one and two. In Latin, this affinity exists between *trans* meaning 'beyond' and *tres* meaning 'three'; the same relationship is manifest in French where we find *tres* for 'very' and *trios* for the number 'three'. Presumably, only later did words emerge to distinguish the various different varieties of 'many'. Some languages, for example Arabic, also retain a threefold treatment of quantity with differentiation between singular, dual (for two only), and plural (for more than two)." (Barrow, 1992).

All numbers are inherent in the number 1 as patterns of 1s but the property of uniqueness of the odd primes is not inherent in the pattern of 1s. Uniqueness is in contrast to uniformity and cannot exist independent of uniformity. While a prime can be expressed as a pattern of 1, its uniqueness cannot. Every number (positive integer) can be uniquely defined by a pattern of 1s but this makes every number equally unique. Thus none is unique. The uniqueness of a number is based on the existence of numbers greater than 1 and the existence of non-unique numbers. Primes and non-primes are like odd and even or yang and yin and cannot exist independent of each other. The number 1 is unique since oneness is synonymous with uniqueness. If 1 is unique, then 2 must be non-unique because it is an inherent part of creating the number 1.

The opposite of uniqueness is uniformity or not being able to be singled out. A prime also exists in a pattern, e.g., 18 is a pattern of the prime number 3. In such a pattern, the number 3 could not stand out as a unique individual. The uniformity property of a prime makes it possible for other subsequent primes to be uncovered as the unique. The number 23 is a prime because it is not a pattern of any other numbers greater than 1. The number 2 is essential for the number 1 to be unique and for other odd primes to be unique. For uniqueness to exist, the uniformity background must co-exist. Two is the first number of non-uniqueness and therefore has some uniqueness property and the related uniformity property. It is the most unique (the first number of non-uniqueness) and the most uniform (present in more patterns of 1 than any other non-unique numbers) among non-unique numbers.

If the building block of non-primes is the prime, it is only fair and logical to go down the hierarchy to ask what is the building block of prime. That building block logically cannot be a number since prime number is the lowest level a number (positive integer) can be. If 1 is a prime, its building block must be 1 itself. The number 1 is also the building block of all other primes. A prime is a positive integer that can be built in only one way from its building block 1 by way of even number of sums of 1 but not of any other numbers greater than 1. How does a creative mind perceive the number 1? Of course, 1 represents uniqueness or oneness or a single smallest point of the whole. One is also uniformity or the single wholeness and is present everywhere or in every number or in every part of the whole. So, 1 embodies the ultimate duality of uniqueness and uniformity. To a creative mind, 1 and the duality are synonymous. The number equivalent of the duality concept is 1. Since 1 is the sole building block of primes, we can also say that the duality is the building block of primes. The duality as building block of primes expresses the meaning of 1 as building block in a more fundamental way that is directly linked to the creation algorithm of the mind. It can also be easily related to the quantum building block of matter, which also has the duality of uniqueness/particle and uniformity/wave. The following shows that the creation algorithm can use the duality as building block to create primes. The mathematical model of the creation algorithm is the orderly creation of primes.

Postulate 1. The imagined domain All things created by the mind comes from imagination and the imagined world of the mind is termed the imagined domain. The content of this domain consists of an infinite number of the basic building block of numbers, 1. There are infinite number of patterns of 1, each differ by its count of 1s. Each pattern, except that of a single 1, has the uniform property of having a count of 1s that is between two other patterns. The pattern of 2 is between the pattern of 1 and the pattern of 3. Since the contents of the imagined domain has no numbers smaller than 1, the pattern of a single 1 is not in between two other patterns and is therefore unique.

Postulate 2. The reality domain The reality domain is where the materialized creations of the mind exist. A prime is generated in the reality domain because of its uniqueness at the time of its creation. It subsequently exists in the reality domain because of its ability to initiate a pattern/uniformity. A prime is defined as a lawful creature of the mind that has the duality of uniqueness and uniformity. A non-prime is defined as a follower of a prime. The mind creates primes by following the two principles of the creation algorithm as postulated above: 1) to generate uniqueness by uniformity selection and 2) to maintain subsequent existence of the unique by uniqueness selection to form uniformity. Uniqueness selection is the process of species formation or forming follower numbers that share properties with the unique. For example, the follower numbers of 3 are 6, 9, 12, . . ., which share the uniform property of 3-ness and form the species of 3. A pattern of 1s or number moves from the imagined domain into the reality domain because it is either uniquely recognized by the mind or is necessary to maintain existence of the unique in the reality domain.

Creating primes. Prior to the creation of any numbers in the reality domain, the unique number in the imagined domain is 1. So the first goal is to generate 1 as the unique or prime in the reality domain. Since a prime must form a species or pattern in order to exist following its creation, the species of 1 is formed with 1 followed by the next closest number 2. In addition, to express uniqueness requires the simultaneous presence of uniformity. So, the species of 2 is formed to represent uniformity with 2 followed by the next closest number that shares the property of 2-ness, 4. Two is selected to represent uniformity because it is the only other pattern besides 1 that is available in the reality domain at this point when the species of 1 has not yet progressed beyond 2. Table 1A shows the contents of the reality domain at its time of creation. The prime/uniqueness/1/odd/yang and non-prime/uniformity/2/even/yin are generated simultaneously and cannot exist independent of each other.

After the beginning stage of generating the reality domain, the mind is aware of both the imagined domain and the reality domain. By comparing the two domains, the mind is looking for the next prime or unique pattern among patterns in the imagined domain that have no match in the reality domain. This pattern is now 3 and it is unique because it is the smallest while all other patterns share the uniform property of having counts of 1s that are between two patterns. To express 3 as a prime, the species of 3 (3, 6, 9) is formed in the reality domain. To apply the new concept of 3-ness, all species are extended to the 3^{rd} position. The reality domain has now advanced from the beginning stage of 1 and 2 to the next stage of 3-ness. At this stage, a number larger than 3 such as 4 expresses only the concepts already established such as 2-ness (2 units of 2-ness). After the stage of 3-ness, the mind is again ready to look for the next unique pattern remaining in the imagined domain, which is now 5. From the concepts of 3-ness and 5-ness, the 4-ness of 4 is now recognized as the intermediate between 3 and 5. By applying the concept of 5-ness and 4-ness, all number species are extended to the 5th position. The species of 5 (5, 10, 15, 20, 25) is formed to express 5 as a prime. The mind is then ready to look for the next unique pattern that remains in the imagined domain (Table 1B). In this way of iteratively applying the same creation algorithm, an infinite number of primes can be generated. Because this creation algorithm of the mind can create primes, it is hereafter called the Prime Law. Since primes have the same property and meaning as creations of the mind, the word ‘prime’ and the word ‘creation’ are interchangeable or synonymous. Therefore, the ‘Prime’ Law also literally means the ‘Creation’ Law.

On indivisibility of primes

Uniqueness means that a number is not an inherent part of a smaller number greater than 1. A prime is not a pattern of any smaller number greater than 1, which means indivisible by any smaller number greater than 1. Indivisibility is therefore a secondary property of the essence of primes as the unique and should not in and of itself confer primality. The number 2 is indivisible but is not a prime because it lacks the uniqueness essence. It is an inherent part of creating the number 1 as the unique or prime.

On the duality of randomness and regularity of primes

It is well known that primes seem to exhibit the duality of randomness and regularity. Such seemingly impossible unity of yin and yang opposites is what makes primes so interesting and mysterious. The mathematician Don Zagier once said in a 1975 lecture: “There are two facts about the distribution of prime numbers of which I hope to convince you so overwhelmingly that they will be permanently engraved in your hearts. The first is that, despite their simple definition and role as the building blocks of the natural numbers, the prime numbers grow like weeds among the natural numbers, seeming to obey no other law than that of chance, and nobody can predict where the next one will sprout. The second fact is even more astonishing, for it states just the opposite: that the prime numbers exhibit stunning regularity, that there are laws governing their behavior, and that they obey these laws with almost military precision." (Zagier, 1977). However, the first fact of unpredictability or randomness remains unproven. The mathematician Robert C. Vaughan said that "It is evident that the primes are randomly distributed but, unfortunately, we don't know what 'random' means." It is the seeming randomness that makes the regularity of primes so striking and interesting. If primes were ever proven to be predictable, it would cease to have the duality property and would in turn cease to be interesting and mysterious.

A proof on the unpredictability or randomness of primes would also have practical benefits since it would save people from wasting time on trying to develop formulas to predict primes. There exist a variety of formulas for either producing the N th prime as a function of N or taking on only prime values. However, all such formulas require either extremely accurate knowledge of some unknown constant, or effectively require knowledge of the primes ahead of time in order to use the formula. They do not really count as prediction. A true predictive formula should not make use of the knowledge of existing primes in order to predict the next future prime. It must be the opposite of the non-predictive way of generating primes, which is to use knowledge of existing primes such as indivisible by known primes.

Since the apparent duality of randomness and regularity is what makes the primes interesting, it is essential to prove that the duality is real. The newly discovered essence of primes can easily deduce or prove the duality of randomness and regularity. Randomness means unpredictability and primes can be easily proven to be unpredictable. When something can be predicted, it must belong to a pattern. As such, it is not unique and hence, by definition, not a prime. The essence of uniqueness rules out prediction of primes as a viable possibility. There is also another easy way to prove this. To predict primes means to predict uniqueness and in turn uniformity since uniqueness needs uniformity to have meaning. Uniformity is made of existing primes. So to predict primes is to predict *existing* primes, which is a logical non-sense.

The uniformity essence of primes demands that the formation of uniformity from a newly created prime or uniqueness must be regular and predictable. So the uniformity forming property of primes gives rise to the regularity of primes. Primes exist as regularly as possible in the uniformity. The creation of primes is fully determined by the orderly formation of uniformity by existing primes. The lawful rather than random way of creating primes explains why primes should follow some regularity patterns, such as the Prime Number Theorem. The unpredictability of individual primes explains why such a pattern cannot be completely precise or free of error margins. Thus, the essence of primes fully explains the duality of regularity and unpredictability or randomness.

On the Riemann Hypothesis

The German mathematician Bernhard Riemann formulated the Riemann Hypothesis in 1859. The hypothesis is widely regarded as the most important unsolved problem in all of mathematics. It is believed by most mathematicians to be true. The Riemann Hypothesis essentially says that the primes are as regularly distributed as possible given their seemingly random occurrence on the number line. According to the Prime Number Theorem of Gauss, the number of primes less than N is approximately the logarithmic integral Li(N) or less precisely N/ln(N). If the Riemann Hypothesis is true, the error between Li(N) and the true number of primes is at most of the order of the square root of N. This is the error margin expected by the theory of probability for some *seemingly* random events such as a coin toss. If one can prove that primes are *seemingly* random in the sense of a coin toss, one proves the Riemann Hypothesis and vice versa. I use the phrase ‘seeming or pseudo-randomness’ to describe an event that is lawfully caused but remains unpredictable, like the creation of primes by the Prime

Law. A population of such seemingly random events should show a regularity pattern reflecting the lawfulness and regularity in the causes of these events. However, even the most precise pattern should still show some error margin reflecting the unpredictability or seeming randomness of the individual event. I define 'real randomness' as an event that is caused by chance accidents like selecting a unique or prime number from an infinity of numbers by playing a dice. Both kinds of randomness are unpredictable but the error margin or variation from a pattern could be greater with real randomness. A population of lawfully created and predictable events follows a precise pattern without any error margin. A population of lawfully created but unpredictable events follows a less precise pattern with some error margin like the square root of N. A population of lawlessly created events follows a rough pattern with huge error margins which could be so high as to render the pattern meaningless or equivalent to no pattern. If whatever number that is selected from an infinity of numbers by playing a dice is defined as primes, we would obviously detect no meaningful patterns of primes in most cases, which is equivalent to saying that we could only have patterns with huge error margins. The error margin for a pattern of events that are lawfully caused but unpredictable must necessarily be the smallest among patterns that cannot predict individual event. Any smaller error margin would mean some degree of predictability. If we know that certain position of the tossing hand could cause a higher chance of landing heads while another position favoring tails, we could improve on the error margin but then the coin toss would not qualify as truly unpredictable. True unpredictability is shared by all kinds of randomness. Among these, seeming randomness caused solely by true unpredictability has the least amount of randomness or the smallest error margin.

There is a pattern that a fair coin toss follows, which says that the number of heads is equal to half of the number of toss N with an error of the square root of N. A fair coin toss must not have irregular or random bias toward the head or tail. Each landing of head or tail is fully determined by laws, such as the gravitational law, the exact position of the tossing hand, the wind, etc. A coin toss is as regular as possible and is only seemingly random because of unpredictability. It is unpredictable because humans cannot measure all the physical parameters that determine the fall of a coin. Also, the laws are not biased to favor either head or tail and remain unchanged timelessly. If a divine were to suddenly intervene for no reason to cause more landing of the head, the coin toss would be lawlessly caused and would display much wider error margin. If we only detect a seeming randomness in our coin toss with an error of the square root of N, we would be confident that everything is well and regular and no laws have been broken by either random accidents or deliberate intentions. But if we see a much wider variation than the square root of N, we would know that something is wrong or that some laws have been broken either accidentally or deliberately. The coin toss would be considered as unfair. The creation of primes is like the fall of a coin. It is lawfully caused and yet truly unpredictable. The Prime Law is also not biased toward any numbers. The error margin in the prime distribution pattern cannot be significantly smaller than that of coin toss since a smaller margin would mean some degree of predictability. It also cannot be significantly larger since that would mean some degree of lawlessness in the creation of primes.

The uniqueness essence of primes proves that primes cannot be predicted. A number is either a prime (head) or a non-prime (tail) but cannot be predicted to be such. This in turn proves that primes must display at least *seeming* randomness in the sense of a coin toss. The Prime Law shows that primes are lawfully created and thus cannot have *real* randomness. Primes can only be *seemingly* random. The essence of primes entails *seeming* randomness in the sense of a coin toss, which manifests in the form of the Riemann Hypothesis. The mathematician Harold M. Edwards wrote: “One of the things which makes the Riemann Hypothesis so difficult is the fact that there is no plausibility argument, no hint of a reason, however, un-rigorous, why it should be true.” (Edwards, 1974). The essence of primes is the plausibility reason that the Riemann Hypothesis must be true. If primes are lawfully caused, a population of primes must show some kind of regularity pattern. If the individual creation of a prime is unpredictable, then the most precise regularity pattern, such as the Prime Number Theorem, simply cannot be free of an error margin that is inherently associated with any seemingly random event such as a fair coin toss.

Some mathematicians have suggested that Gauss had modeled his Prime Number Theorem by tossing a coin (du Sautoy, 2003). To Gauss, primes look like they are generated by nature flipping a coin, heads it's prime, tails it's not. The coin could be weighted so that instead of landing heads half the time, it lands heads with probability of 1/ln(N). If the prime coin is tossed in an unbiased and unpredictable fashion, it should show an error margin of the square root of N, the same as a real coin toss. The Riemann Hypothesis says that the prime coin is fair and random with an error margin similar to that of a real coin toss. Obviously, if primes were generated by a prime coin toss, they would not be reproducible. If nature were to toss it again, 5 may not show up as heads or primes while it did the first time, even though the number of primes under a number N would stay similar or reproducible. However, primes generated by the Prime Law are rigid and precise and reproducible every time. Also, if primes were generated by a prime coin toss, one would expect to find at least 2 heads in a row at some point on the number line. But odd primes are always separated by at least one non-prime number. Also, the prime coin toss would not be biased toward only odd numbers. So, individual prime is clearly not generated in a random fashion by nature flipping a prime coin, even though the regularity pattern of a population of primes can be modeled by a coin toss. The probability for a prime coin toss to generate the actual prime sequence is extremely small although not zero. Lawfulness is usually associated with predictability but a major exception is the lawful creation of uniqueness or primes by the Prime Law. The Prime Law can generate something not by way of a coin toss while also makes it look like an unpredictable outcome of a coin toss.

In mathematics, a proof is a demonstration that, assuming certain axioms, some statement is necessarily true. A proof is derived by principles of deduction from certain axioms or common truth. To prove the Riemann hypothesis, one needs to be able to logically deduce it from some basic fundamental truth or axioms or essence of primes. In mathematics, axioms are neither derived by principles of deduction, nor are they demonstrable by formal proofs. Instead, an axiom is taken for granted as valid, and serves as a necessary starting point for deducing logically consistent propositions. But today's axioms can easily become tomorrow's deductions. From the essence of primes as uniqueness, we can deduce or prove that primes are indivisible. That prime is uniqueness is not an axiom but is a proven statement because the Prime Law can create primes by simply creating uniqueness.

Kurt Godel has proven that axioms drained of meaning and truth could never be used to prove that they will never lead to deductions that could contradict the axioms or other deductions. He further proved that any consistent axiom system free of human intuition is necessarily incomplete in that there will be true statements that can't be deduced from the axioms. Godel believed that mathematics had failed to prove the Riemann Hypothesis because its axioms are not sufficient to explain the Hypothesis. Any mathematical edifice or formal system built on axioms drained of meaning and truth cannot have the certainty of truth and reality. The reality of mathematics is directly related to the meaning and truth of the axioms and in turn the intuition of the mind. But since not all intuitions are trustworthy, formalists like David Hilbert have tried to get rid of intuitions out of mathematics, which has been proven to be futile by Godel. Here I suggest a novel approach to get rid of any uncertainty about the intuitions or intuition-based axioms. To have certainty and truth in the mathematical edifice, intuitions or axioms need to be replaced by *proven* truth. Mathematics needs to be based on proven truth rather than axioms or intuitions. The ultimate law of the universe needs to have a firm foundation in proven truth rather than axioms. For the starting points to be proven truth rather than unproven axioms, they have to prove themselves or be a deduction of themselves. They must be self-caused. The essence of prime is the starting point of primes, which has three parts, the duality of uniqueness and uniformity and the mind programmed with the Prime Law.

The duality starting point is a proven truth because it has caused by way of the Prime Law an infinite number of itself, the primes, which are nothing but the duality. The Prime Law is also self-caused because it is caused or created by the human mind that is already programmed with the Prime Law. The creation law that programs the human mind must be the cause of every law or theory that is formulated by the mind, including the creation law or the Prime Law itself. In this sense, the Prime Law has caused itself. Therefore, the starting point of primes, the duality building block and the Prime Law, has caused or proved itself. Thus, mathematics built upon the essence of primes must be complete and consistent. Since the mind is part of the Prime Law, mathematics simply cannot be complete and consistent without the mind or the *proven* intuition of the mind. This independently confirms what Godel has found.

One can try to prove the exact statement of the Riemann Hypothesis, all non-trivial zeros of the zeta function have real part one-half, using advanced mathematics or number theory. If it is proven in this way, it would mean that primes have seeming randomness behavior like that of a coin toss. But it would not reveal whether the seeming randomness is *caused* by real randomness or nature flipping a coin. It would not answer the question whether the regularity of primes is *caused* randomly rather than lawfully. Some properties of prime must be a direct deduction of its essence and simply cannot be deduced from other secondary properties. The unpredictability property is a direct deduction from the uniqueness essence of primes. To prove the exact statement of the Riemann Hypothesis by starting from the secondary properties of primes is to prove the seeming randomness or unpredictability property without relying on the uniqueness essence. This is like trying to prove the hardness property of diamond by using other secondary properties such as rareness and beauty. We may never know if it can be done and there is a real possibility that it cannot be done. Godel is most likely correct in his view, as the mathematician Marcus du Sautoy wrote: “Godel himself had voiced such concerns in relation to the Riemann Hypothesis: perhaps the axioms that formed the foundations of the mathematical edifice were not broad enough to carry the required proof, in which case you might continue building upwards and never find a connection to the Hypothesis. However, he did offer some consolation. Godel believed that any conjecture of genuine interest cannot be forever out of reach. It was just a matter of finding a new foundation stone to extend the base of the edifice. Only by going back to the subject's foundations and seeking to broaden them would you be able to build up to the missing proof." (du Sautoy, 2003). The trinity of uniqueness, uniformity and the Prime Law is the new and broadened foundation for the primes. The Riemann Hypothesis suggests that primes are lawfully created and yet unpredictable. This has now been proven by the Prime Law. Primes are lawful but unpredictable creatures of the creation algorithm of the mind.

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Table 1. Creating primes.

A. The contents of the reality domain at the time of creation. **B.** Subsequent progression of the reality domain. From left to right represents the number species with each number increasing in value from the previous number by the unit value of the beginning number; the species terminates at the Nth position where N is the numeric value of the last known prime (N>2). The species of 2 is listed not because 2 is a prime but because it is an inseparable part of creating the first prime 1. Successive prime numbers from small to large are listed on the left side column in the order from top to bottom. The Table can be expanded in a prime by prime manner over time to infinity, in both the vertical direction from top to bottom and the lateral direction from left to right.

A.

1	2
2	4

B.

1	2	3	4	5	...	N
2	4	6	8	10	...	2N
3	6	9	12	15	...	3N
5	10	15	20	25	...	5N
…	…	…	…	…	…	…
N	2N	3N	4N	5N	…	NN