Summary of my Work – Riemann Hypothesis

In my approach, I show that if GRH is true then, series of g-primes $\sum g_n^{-s}$ should have an analytic continuation for $\sigma > \frac{1}{2}$.

Those g-primes are the numbers generating all the primes $p_n = 2qg_n + 1$ where q is the module of the Dirichlet L-fonction .

The modular PNT theorem implies that (g_n) has almost the same statistics than primes, it means $\pi(g_n, x) \sim \frac{x}{Logx}$ whatever the module q.

So I conjecture that the arithmetical properties are not important for the existence of the analytic continuation but only the statistics $\pi(g_n, x) \sim \frac{x}{Logx}$.

I have found the technique to prove the analytic continuation of $\sum g_n^{-s}$, using a "periodification" and a renormalisation of the sum. It is based on my formula for series having periodic coefficients (this is a generalization of the Euler-MacLaurin formula).

The proof is constructive. I mean that it is possible to compute the analytic continuation.

It is very important that the g-primes are distinct. Otherwise the series may have a pole on the real axis between $1 > \sigma > \frac{1}{2}$ and the corresponding L-function a zero.

So the result follows.

For instance the Riemann zeta function writes : $\zeta(s) = \exp\left(\sum_p p^{-s}\right) \prod_p \left(1 - p^{-s}\right)^{-1} e^{-p^{-s}}$

The modified Euler product converges for $\sigma > \frac{1}{2}$ (it's a classical result of Weierstrass), it has neither pole nor zero.

So, as by my theorem $\sum_p p^{-s}$ has an analytic continuation (except s=1) on $\sigma > \frac{1}{2}$, $\zeta(s) \neq 0$.

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