from garry.herrington@gmail.com , 23 April 2018

Problems with a Claimed Proof of the Riemann Hypothesis
> Anthony 2008

Anthony 2008 in Theorem 2 claims a proof of the Riemann Hypothesis.

Quote from Anthony 2008, Theorem 2

1. Theorem 2: All the roots of $\zeta(z) = 0$, lie on ½ - line, $z = ½ + iy$, where y is a
2. positive, negative, or complex number.

3. Consider equation (10) above
4. $$\zeta(z) = \frac{2^{z-1}}{2^z-1}\,\frac{z}{z-1} + \frac{2^z}{2(2^z-1)}\int_0^\infty \frac{i[[1+iX]^{-z} - [1-iX]^{-z}]}{e^{\pi X}-1}\,dX \qquad (10)$$
5. Now, the terms $(1 \pm iX)^{-z}$ in (11) can be expanded as a convergent series, for $X > 1$,
6. so that one can now represent the function for values of z as:
7. $$[1 \pm iX]^{-z} = \sum_{n=0}^{\infty}\left[\frac{(\pm iX)^n\,\Gamma(1-z)}{\Gamma(-z-n+1)\,n!}\right] \qquad (16)$$

and so, …

End of quote

A problem with the statement of Theorem 2 is line 2. If y could be a complex number, $a + bi$, where $a,b \in \mathbb{R}$ and $b \neq 0$ then z would have the form $z = ½ - b + ai$ and so z will not lie on the "½-line".

A problem with the proof of Theorem 2 is that $[1 \pm iX]^{-z}$ in (10) above is replaced with the series from (16) above. The series in (16) is divergent for $|X| > 1$ and hence the statement in line 5 of the quote is false.

The series in (16) can be rearranged to the form in Bromwich 1908, art 89, p.225,

$$1 + \sum_{n=1}^{\infty}\left[\frac{(\mp iX)^n\, z(z+1)\ldots(z+(n-1))}{n!}\right] \qquad \text{("the Binomial Series")}$$

by using the recursive relation $\Gamma(w+n+1) = (w+n)(w+n-1)\ \ldots\ (w+1)\,\Gamma(w+1)$

in Abraham and Stegun, p.256 with $w \equiv -z-n$.

According to Bromwich 1908, art 89, the Binomial Series is absolutely convergent for $|X| < 1$ and divergent for $|X| > 1$. It may converge, diverge or oscillate for $|X| = 1$ depending on the value of X and the value of Re(z)

Hence using (16) to substitute into (10) creates an integrand that is the difference of two divergent series on most of the range of integration.

References

1. Abramowitz,M. and Stegun,I., "Handbook of Mathematical Functions", Dover, 7e
2. Anthony,M., "A Simple Proof of the Riemann Hypothesis", 2008, http://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/RHproofs
3. Bromwich,T., "An Introduction to the Theory of Infinite Series", MacMillan, London, 1908

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