from garry.herrington@gmail.com , 27 March 2018

Problem with a Claimed Proof of the Riemann Hypothesis
> Bergstrom 2008

§ 1 The Claim

Bergstrom 2008 claims a proof of the Riemann Hypothesis in section 9, "Proof of Conjecture 1.1" , page 8, and repeats this claim in FAQ #8 on page 39.

Quote from Bergstrom 2008, section 9, page 8

We then get

$$\lim_{N\to\infty} \tfrac{1}{2} \mid N^{(2s-1)} \pi^s (-4^s + 8^s) \,/\, \cos(\tfrac{1}{2} s \pi) (-2 + 2^s) (s + 2) (s + 1) s \Gamma(s - 3) \mid = 1 \qquad (20)$$

This equation can be true only if the modulus of $N^{(2s-1)}$ is equal to $N^0 = 1$, which thus requires that,

$\sigma = \tfrac{1}{2}$

This thus proves Conjecture 1.1 that Re(s) must be equal to ½ for all zeros of the Riemann zeta-function ζ(s) in the range 0 < Re(s) < 1.

End of quote

The equation in line (20) is false because two of the possible values of the limit do not equal 1 and the third possible value does not necessarily equal 1.

§ 2 Reformulation of the limit in (20)

If $a \in \mathbb{R}(0<)$ and $z, z_1, z_2 \in \mathbb{C}$ then $|a^z| = a^{\mathrm{Re}(z)}$, $a|z| = |az|$ and $|z_1 z_2| = |z_1||z_2|$.

These relations can be used to reformulate (20) as $\lim_{N\to\infty} N^{(2\sigma-1)} f(s) \underset{1}{=} 1$ where,

$$f(s) \equiv | \pi^s (-4^s + 8^s) \,/\, 2 \cos(\tfrac{1}{2} s \pi) (-2 + 2^s) (s + 2) (s + 1) s \Gamma(s - 3)|$$

In (20), N is an arbitrary member of $\mathbb{Z}(\geq 1)$ and s an arbitrary member of $\mathbb{C}'$ which is $\mathbb{C}$ after excluding the singularities of $f(s)$.

As N and s are independent, relation(1) can be reformulated to $f(s) \lim_{N\to\infty} N^{(2\sigma-1)} \underset{2}{=} 1$.

§ 3 The Problem

The equation in (20) is equivalent to relation(2). The possible values of the limit in relation(2) are,

- $2\sigma - 1 > 0 \longrightarrow f(s) \lim_{N\to\infty} N^{(2\sigma-1)} = \infty$
- $2\sigma - 1 = 0 \longrightarrow f(s) \lim_{N\to\infty} N^{(2\sigma-1)} = f(s_1)$ where $s_1 = ½ + it \in \mathbb{C}'$
- $2\sigma - 1 < 0 \longrightarrow f(s) \lim_{N\to\infty} N^{(2\sigma-1)} = 0$

Hence, as stated in Bergstrom 2008 (see quote in § 1 above), if (20) is true then $\sigma = ½$. However if (20) is true then also $f(s_1) = 1$ and this is not necessarily true, for example $f(½ + i) = 0.801$, from references (3,4).

Bergstrom 2008, FAQ #8, page 39 claims to prove $f(s_1) = 1$ but there is an error at the beginning of the proof.

Quote from Bergstrom 2008, FAQ #8, page 39

FAQ #8 (pages 8-9)

"Can you please show me the details of how the rest of the expression on the left-hand side of (20) becomes equal to the middle expression (which is unity) in Remark 9.2 on pages 8-9 in your preprint."

<u>ANSWER</u>: Step1.

First I need to show that

$$|(s+2)(s+1)\, s\, \Gamma(s-3)| = |\Gamma(s)| \qquad (1)$$

for s = ½ + it as I say on top of page 9 in the preprint.

The following recurrence relation is valid for the gamma function (choose n = 4 in Abramowitz and Stegun, eq 6.1.16)

$$(z+3)(z+2)(z+1)\,\Gamma(z+1) = \Gamma(4+z) \qquad (2)$$

End of quote

The relation in line (2) is valid for $z \in \mathbb{C}$ after excluding $\mathbb{Z}(\leq 0)$ which is the set of singularities of $\Gamma(z)$.

When s = 4+z then line (2) becomes, $(s-1)(s-2)(s-3)\,\Gamma(s-3) \underset{3}{=} \Gamma(s)$

Relation(3) is valid for s $\in \mathbb{C}''$ which is $\mathbb{C}$ after excluding $\mathbb{Z}(\leq 0) \cup \{1,2,3\}$, the set on which Γ(s-3) and/or Γ(s) is undefined.

The left side of line(1) is defined when s $\in \mathbb{C}''$.

If s $\in \mathbb{C}''$ then equating the left sides of relation(3) and line (1), and after rearrangement,

$$(|s-1|/|s+1|)\ (|s-2|/|s+2|)\ (|s-3|/|s|) \underset{4}{=} 1$$

Relation(4) is false because each factor in brackets is $\in \mathbb{R}(0,1)$.

As relation(3) is true then the falsity of relation(4) implies the falsity of line (1) for s $\in \mathbb{C}''$ which includes all values of s with Re(s) = ½.

References

1. Abramowitz,M. and Stegun,I., "Handbook of Mathematical Functions", Dover, 7e
2. Bergstrom,A., "Proof of Riemann's Zeta-Hypothesis", arXiv:0809.5120v18 [math.GM])
3. Calculation of functions of a complex variable, Microsoft Excel 2007 > Formulas > More Functions > Engineering
4. Online calculator of Γ(z) for complex inputs, ttp://members.chello.nl/k.ijntema/gamma.html

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