§ 1 The Claim

Carella 2008 implicitly claims a proof of the Riemann Hypothesis in Theorem 7,

"Let \( N_k = 2 \cdot 3 \cdot 5 \cdots p_k \). Then \( N_k / \varphi(n_k) > e^\gamma \log \log n_k \) for all sufficiently large integers \( N_k \)"

This implicit claim was noted by G. Caveney (reference below).

Nicolas 1983, Theorem 2(b), proved that if the "Riemann Hypothesis is false" (≡ not RH) then "the inequality in Theorem 7 is false for infinitely many \( k \)" (≡ P), that is, "not RH \( \implies \) P".

The contra-positive of "not RH \( \implies \) P" is "not P \( \implies \) RH" and Carella 2008 in Theorem 7 claims to have proved "not P" thereby implicitly claiming a proof of the Riemann Hypothesis.

§ 2 A Problem

A problem with the proof of Theorem 7 in Carella 2008 is that it is incomplete.

Immediately following line (10) the proof states,

"Next, since the Chebyshev's function satisfies \( p_k > \vartheta(p_k) = \log N_k \), and ..."

Grosswald 1967 says it was proved in Schmidt 1903 that there are infinite, divergent sequences \( x_k \) in \( \mathbb{R} \) on which \( \vartheta(x_k) - x_k \) changes sign infinitely often, where \( \vartheta(x) \equiv \sum_{p \leq x} \log p \) and \( p \) denotes a prime. This was confirmed in Landau 1905 by a different method and in Littlewood 1914 it was proved that \( \vartheta(x_k) - x_k = \Omega \) \( (x_k^{1/3} \log \log \log x_k) \).

It follows from Schmidt 1903 that the sequence \( x_k \) has an infinite, divergent sub-sequence \( x_k' \) on which \( \vartheta(x_k') > x_k' \). Let \( q_k \) be the largest prime \( \leq x_k' \) then \( x_k' \geq q_k \) and the definition in relation(1) implies \( \vartheta(x_k') = \vartheta(q_k) \). Relations(2, 3, 4) then imply \( \vartheta(q_k) > q_k \) for infinitely many \( k \).

The case \( \vartheta(q_k) > q_k \) is not covered in the proof of Theorem 7 in Carella 2008 and hence the proof is incomplete.
References


2. G.Caveney comments are at link above for Carella,N.


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