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Problem with an Implicitly Claimed Proof of the Riemann Hypothesis
> Carella 2008

§ 1 The Claim

Carella 2008 implicitly claims a proof of the Riemann Hypothesis in Theorem 7,

"Let $N_k = 2 \cdot 3 \cdot 5 \cdots p_k$. Then $N_k / \varphi(n_k) > e^{\gamma} \log\log n_k$ for all sufficiently large integers N_k "

This implicit claim was noted by G.Caveney (reference below).

Nicolas 1983, Theorem 2(b), proved that if the "Riemann Hypothesis is false" (≡ not RH) then "the inequality in Theorem 7 is false for infinitely many k" (≡ P), that is, "not RH $\longrightarrow$ P".

The contra-positive of "not RH $\longrightarrow$ P" is "not P $\longrightarrow$ RH" and Carella 2008 in Theorem 7 claims to have proved "not P" thereby implicitly claiming a proof of the Riemann Hypothesis.

§ 2 A Problem

A problem with the proof of Theorem 7 in Carella 2008 is that it is incomplete.

Immediately following line (10) the proof states,
"Next, since the Chebyshev's function satisfies $p_k > \vartheta(p_k) = \log N_k$, and …"

Grosswald 1967 says it was proved in Schmidt 1903 that there are infinite, divergent sequences x_k in $\mathbb{R}(0<)$ on which $\vartheta(x_k) - x_k$ changes sign infinitely often, where $\vartheta(x) \underset{1}{\equiv} \sum_{p \le x} \log p$ and p denotes a prime. This was confirmed in Landau 1905 by a different method and in Littlewood 1914 it was proved that $\vartheta(x_k) - x_k = \Omega_{\pm}(x_k^{½} \log\log\log x_k)$.

It follows from Schmidt 1903 that the sequence x_k has an infinite, divergent sub-sequence x_k' on which $\vartheta(x_k') \underset{2}{>} x_k'$. Let q_k be the largest prime $\le x_k'$ then $x_k' \underset{3}{\geq} q_k$ and the definition in relation(1) implies $\vartheta(x_k') \underset{4}{=} \vartheta(q_k)$. Relations(2,3,4) then imply $\vartheta(q_k) > q_k$ for infinitely many k.

The case $\vartheta(q_k) > q_k$ is not covered in the proof of Theorem 7 in Carella 2008 and hence the proof is incomplete.

References

1. Carella,N., 'Divisor and Totient Functions Estimates", 2008, at
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2. G.Caveney comments are at link above for Carella,N.
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4. Landau,E., Math. Ann. 59 (1905), pp 527-550.
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6. Nicolas,J-L., "Petites valeurs de la fonction d'Euler",
   J. Number Theory 17 (1983), no. 3, pp 375–388.
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