from garry.herrington@gmail.com , 27 March 2018

Problem with a Claimed Disproof of the Riemann Hypothesis
> Hwang 1990

There are three papers by Hwang on Matthew Watkins' webpage of claimed proofs/disproofs of the Riemann Hypothesis (a link to this webpage is in the references below).

The Hwang papers are undated but a footnote in one of them suggests it is 1990. The other two Hwang papers refer to the 1990 paper as "recent" and rely on its claimed disproof of the Riemann Hypothesis. For reference purposes I will refer to the three Hwang papers as Hwang 1990, Hwang 1990a and Hwang 1990b (titles in the references below).

After noting that the numerical evidence supports the Riemann Hypothesis, Hwang 1990 claims in Theorem 1 that the Riemann Hypothesis is false.

Quote from Hwang 1990, Theorem 1

However, I disprove this hypothesis as follows:

Theorem 1. Let $\zeta^*(s)$ be the function generated from $\zeta(s)$ by replacing each ρ_n by $\rho_n^* = ½ + i\gamma_n$, then we have $\zeta^*(s) > \zeta(s)$, for each $s > 1$.

Clearly, Riemann's Hypothesis means $\zeta^*(s) \equiv \zeta(s)$ which is false due to Theorem 1.

End of quote

One problem is that Hwang 1990 does not define the relation ">" on $\mathbb{C}$ used in the statement of Theorem 1. The absence of a definition renders Theorem 1 meaningless.

The notation ">" used in Theorem 1 of Hwang 1990 is standard notation for a "total ordering". Total orderings can be defined on the set $\mathbb{C}$ but there is no total ordering of $\mathbb{C}$ for which the field $(\mathbb{C}, +, x)$ is an "ordered field" because any total ordering of $\mathbb{C}$ is incompatible with the field properties of $(\mathbb{C}, +, x)$. The field properties of $(\mathbb{C}, +, x)$ are needed to define the Riemann zeta function and derive relations involving it. There are links in the references below to definitions of a "total ordering" and an "ordered field" and to a proof that any total ordering of $\mathbb{C}$ is incompatible with the field properties of $(\mathbb{C}, +, x)$.

As both Hwang 1990a and Hwang 1990b rely on Theorem 1 in Hwang 1990 then a problem for Hwang 1990 is also a problem for these other papers.

References

1. Hwang,J., 1990 (undated but probably 1990), "On the Falsity of Riemann's Hypothesis".

2. Hwang,J., 1990a (undated but assumed circa 1990), "The Quickest Disproof of Riemann's Hypothesis".

3. Hwang,J., 1990b (undated but assumed circa 1990), "On the First Exceptional Zero of Riemann's Zeta-function".

4. Hwang's papers are at http://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/RHproofs

5. Definition of a "total ordering" : https://en.wikipedia.org/wiki/Total_order

6. Definition of an "ordered field" : https://en.wikipedia.org/wiki/Ordered_field

7. Incompatibility of a total ordering of $\mathbb{C}$ with the field properties of ($\mathbb{C}$, +, x) : https://proofwiki.org/wiki/Complex_Numbers_cannot_be_Totally_Ordered

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