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Problem with a Claimed Proof of the Riemann Hypothesis
> Logan 2018

Logan 2018 claims a proof that all the zeros of the Riemann Xi function are real which is equivalent to claiming a proof of the Riemann Hypothesis (e.g. Conrey 2003, p.345).

There is a problem with the derivation in Logan 2018, section 3.2,
"(Real + Imaginary) and (Real – Imaginary) Part Zeros".

For brevity let,

1. $f_1 \equiv \sum_{n=0}^{\infty} a_{2n}\, r^{2n}\cos(2n\theta)$ and $f_2 \equiv \sum_{n=1}^{\infty} a_{2n}\, r^{2n}\sin(2n\theta)$.

2. $f\,'$ ≡ the series f after term-by-term differentiation by r

3. θ' ≡ θ after differentiation by r.

Logan 2018, section 3.2 starts with $f_1 = 0$ and $f_2 = 0$ and derives from $f_1 + f_2 = 0$ and $f_1 - f_2 = 0$ two different expressions for θ' where (r,θ) is a common solution of $f_1 = 0$ and $f_2 = 0$.

A problem with the derivation is that it works not only for $af_1 + bf_2 = 0$ with the choices a,b = 1,1 and a,b = 1,-1 but also when a,b are arbitrary but independent of r and θ. The derivation then gives the paradoxical result that θ' is dependent on two arbitrary parameters a,b.

The derivation in Logan 2018, section 3.2 makes implicit assumptions which include:

A1. If (r,θ) is a common solution of $f_1 = 0$ and $f_2 = 0$ then $\theta(r)$ is a differentiable function of r.

A2. The series $f_1'$and f_2' are uniformly convergent so the series f_1 and f_2 can be differentiated term-by-term by r.

A3. The series $f_1'$and f_2' are absolutely convergent so their terms can be rearranged.

The problem with the derivation is likely to come from one or more of these implicit assumptions being false.

References

1. Conrey, J., "The Riemann Hypothesis", Notices of the AMS, March 2003

2. Logan, A., "Proof of the Riemann Hypothesis", viXra : 1802.0124

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