[23] R. Mazzeo. The Hodge cohomology of a conformally compact metric. *J. Diff. Geom.* **28** (1988), 309–339.
[24] R. Mazzeo. Elliptic theory of edge operators. *Comm. P. D. E.* **16** (1991), 1615-1664.
[25] R. Mazzeo. Unique continuation at infinity and embedded eigenvalues for asymptotically hyperbolic manifolds. *American J. Math.* **113** (1991), 25–56.
[26] R. Mazzeo, R. Melrose. Meromorphic extension of the resolvent on complete spaces with asymptotically constant negative curvature. *J. Funct. Anal.* **75** (1987), 260–310
[27] R. B. Melrose. *Geometric Scattering Theory.* New York, Melbourne: Cambridge University Press, 1995.
[28] R. B. Melrose, M. Zworski. Scattering metrics and geodesic flow at infinity. *Inventiones Math.* **124** (1996), 399–436.
[29] L. B. Parnovski. The Selberg zeta function for cocompact discrete subgroups of $SO^+(1,n)$. *Funktsional. Anal. i Prilozhen.* **26** (1992), 55-64 (Russian); *Functional Anal. Appl.* **26** (1992), 196–203.
[30] S. J. Patterson. The Laplacian operator on a Riemann surface I, II, III. *Compositio Math.* **31** (1975), 83–107, **32** (1976) 71–112, and **33** (1976), 227–259.
[31] S. J. Patterson. The limit set of a Fuchsian group. *Acta. Math.* **136** (1976), 241–273.
[32] S. J. Patterson. The Selberg zeta-function of a Kleinian group. In *Number Theory, Trace Formulas, and Discrete Groups: Symposium in honor of Atle Selberg, Oslo, Norway, July 14–21, 1987*, Ne w York, Academic Press, 1989, pp. 409–442.
[33] S. J. Patterson. On Ruelle's zeta-function. In *Festschrift in honor of I.I. Piatetski-Shapiro on the occasion of his sixtieth birthday*, ed. S. Gelbart, R. Howe, P. Sarnak. Jerusalem : Weizmann Science Press, 1990.
[34] S. J. Patterson, P. A. Perry. Divisor of the Selberg Zeta function for Kleinian groups in Even Dimension. To appear in *Duke Math. J.*
[35] P. A. Perry. The Laplace operator on a hyperbolic manifold, II. Eisenstein series and the scattering matrix. *J. reine angew. Math.* **398** (1989), 67–91.
[36] P. A. Perry. The Selberg zeta function and a local trace formula for Kleinian groups. *J. reine angew. Math.* **410** (1 990), 116–152.
[37] P. A. Perry. The Selberg zeta function and scattering poles for Kleinian groups. *Bull. Amer. Math. Soc.* **24** (1991), 327–333.
[38] P. A. Perry. The Selberg zeta function and scattering poles for Kleinian groups. In Mathematical Quantum Theory, I. Schrödinger Operators (Vancouver, B.C. 1993), 243-251.
[39] P. A. Perry. A trace-class rigidity theorem for Kleinian groups. *Ann. Acad. Sci. Fenn.* **20** (1995), 251–257.
[40] D. Ruelle. Zeta functions for expanding maps and Anosov flows, *Inventiones Math.* **34**, (1976), 231–242.
[41] A. Selberg. Harmonic analysis and discontinuous groups in weakly symmetric Riemannian spaces with applications to Dirichlet series. *J. Indian Math. Soc.* **20** (1956), 47–87.
[42] M. Zworski. Counting scattering poles. In Proceedings of the Taniguchi International Workshop, *Spectral and Scattering Theory*, M. Ikawa, ed., Marcel Dekker, New York, Basel, Hong Kong, 1994.
[43] M. Zworski. Dimension of the limit set and density of resonances for convex co-compact hyperbolic quotients. *Inventiones Math.* **136** (1999), 353–409.

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3. Study the asymptotic distribution of resonances for convex co-compact manifolds in three dimensions. It would be very interesting to see a relationship, similar to that found in two dimensions by Zworski [43], between the Hausdorff dimension of the limit set and the counting function for resonances in strips.
4. Study how the scattering resonances move under quasi-conformal deformations.

References

[1] S. Agmon. On the spectral theory of the Laplacian on non-compact hyperbolic manifolds. Journées "Équations aux derivées partielles" (Saint Jean de Monts, 1987), Exposée No. XVII, École Polytechnique, Palaiseau, 1987.

[2] D. Borthwick. Scattering theory and deformation of asymptotically hyperbolic manifolds. Preprint, 1997.

[3] D. Borthwick, A. McRae, E. Taylor. Quasirigidity of hyperbolic 3-manifolds and scattering theory. *Duke Math. J.* **89**, 1997.

[4] R. Brooks, R. Gornet, P. Perry. Isoscattering Schottky Manifolds. Submitted to *Geometric and Functional Analysis.*

[5] J. Elstrodt, F. Grunewald, J. Mennicke. Analysis and number theory in hyperbolic 3-space. In *Algebraic structures and number theory (Hong Kong, 1988)*, Teaneck, New Jersey, World Scientific Publishing, 1990.

[6] C. Epstein. Envelopes of horospheres and Weingarten surfaces in hyperbolic 3-space. Preprint, Princeton University, 1984.

[7] D. Fried. The zeta functions of Ruelle and Selberg, I, *Ann. scient. Éc. Norm. Sup.* **19** (1986), 491–517.

[8] R. Froese, P. Hislop, P. Perry. The Laplace operator on hyperbolic three-manifolds with cusps of non-maximal rank. *Inventiones Math.* **106** (1991), 295–333.

[9] R. Gangolli. Zeta functions of Selberg's type for compact space forms of symmetric spaces of rank one. *Ill. J. Math.* **21** (1977), 1–41.

[10] I. C. Gohberg, E. I. Sigal. An operator generalization of the logarithmic residue theorem and the theorem of Rouché. *Math. U. S. S. R. Sbornik* **13** (1971), 603–625.

[11] L. Guillopé. Théorie spectrale de quelques variétés à bouts. *Ann. Sci. École Norm. Sup.* **22** (1989), 137–160.

[12] L. Guillopé. Fonctions zeta de Selberg et surfaces de geometrie finie. In *Zeta functions in geometry*, *Adv. Stu d. Pure Math.* **21** (1990), 33–70.

[13] L. Guillopé, M. Zworski. Upper bounds on the number of resonances for non-compact Riemann surfaces. *J. Funct. Anal.* **129** (1995), 364–389.

[14] L. Guillopé, M. Zworski. Polynomial bounds on the number of resonances for some complete spaces of constant curvature. *Asymptotic Anal.* **11** (1995), 1–22.

[15] L. Guillopé, M. Zworski. Scattering asymptotics for Riemann surfaces. *Ann. Math.* **145** (1997), 597–660.

[16] L. Guillopé, M. Zworski. The wave trace for Riemann surfaces. Submitted to *Geometric and Functional Analysis.*

[17] D. Hejhal. *The Selberg trace formula for* $PSL(2,\mathbb{R})$. *Lect. Notes in Math.* **548** (1976) and **1001** (1983).

[18] P. D. Hislop. The geometry and spectra of hyperbolic manifolds. Spectral and inverse spectral theory (Bangalore, 1993). *Proc. Indian Acad. Sci. Math. Sci.* **104** (1994), no. 4, 715–776.

[19] M. Joshi, A. Sá Barretô. Inverse scattering on asymptotically hyperbolic manifolds. Submitted to *Acta Mathematica.*

[20] P. Lax, R. S. Phillips. The asymptotic distribution of lattice points in Euclidean and non-Euclidean spaces. *J. Funct. Anal.* **46**, 280-350 (1982).

[21] P. Lax, R. S. Phillips. Translation representation for automorphic solutions of the non-Euclidean wave equation I, II, III. *Comm. Pure. Appl. Math.* **37** (1984), 303–328, **37** (1984), 779–813, and **38** (1985), 179–208.

[22] N. Mandouvalos. Scattering operator, Eisenstein series, inner product formula, and "Maass-Selberg" relations for Kleinian groups. *Mem. Amer. Math. Soc.* **400** (1989).

where $G_p(s) = \prod_{s_0 \in Z_p}(1 - s/s_0)$, $G_t(s)$ is an entire function with zeros at $k = 0, -1, -2, \cdots$ of multiplicity $(2k+1)\chi_N$, and

$$G_{scatt}(s) = \exp(P_3(s)) \prod_{s_0 \in Z_{scatt}} E(s/s_0, 3)$$

where $P_2(s)$ is a polynomial of degree 2.

All of these results can be generalized to convex co-compact hyperbolic manifolds of even dimension [34], and presumably to those of odd dimension.

4. Some Further Results

By methods of dynamics, it can be shown that the zeta function for a convex co-compact hyperbolic manifold in *any* dimension $n+1$ is a quotient of entire functions of order at most $n+1$ and finite type [34]. It follows that the counting function for scattering resonances,

$$N(r) = \#\{s \in Z_{scatt} : |s| \leq r\}$$

obeys the upper bound $N(r) \leq Cr^{n+1}$ if $\dim(X) = n+1$. A rather more precise version of the trace formula has been given in dimension two by Zworski and Guillopé [15], who have also shown that $N(r)$ is bounded below by a constant times $\mathrm{vol}(N)r^2$ in dimension two.

Two hyperbolic manifolds are *isoscattering* if their scattering poles correspond with multiplicities. Brooks, Gornet, and Perry [4] have constructed pairs of isoscattering Schottky manifolds in three dimensions, whose conformal structures 'at infinity' are the same but which are *not* isometric.

Borthwick-McRae-Taylor [3] and Joshi-Sá Barreto [19] have studied the inverse problem for the scattering operator. The first authors show that if two hyperbolic three-manifolds are diffeomorphic with quasi-conformally equivalent boundaries, the QC dilation can be estimated by the norm of the difference of scattering operators.

The second authors prove a rather sharp theorem about recovering the metric from the scattering operator. These authors work in the context of asymtotically hyperbolic manifolds where the metric is known to be of the form $\rho^{-2}h$ for a defining function ρ and a smooth nondegenerate metric h on X. The scattering operator $S_X(s)$ on such a manifold is a pseudodifferential operator of order $2\Re(s) - n$ if $\dim(X) = n+1$. They show that if two scattering operators differ by an operator of order $2\Re(s) - n - k$, then the Taylor series of the metrics as a function of the defining function agree to order k at the boundary.

5. Some Open Problems

It seems fitting to end the opening lecture in a workshop with a list of open problems.

1. Prove a trace formula for convex co-compact hyperbolic manifolds in any dimension. This has been done by Guillopé and Zworski in dimension two [16].
2. Compute the zeta function explicitly for Schottky manifolds in dimension three. The fundamental group is a free group on g generators so there is an easy and explicit coding for the geodesic flow. Compute the scattering resonances by finding the zeros of the zeta function.

so that the functional equation for the zeta function becomes

$$\frac{Z'_\Gamma(s)}{Z_\Gamma(s)} + \frac{Z'_\Gamma(1-s)}{Z_\Gamma(1-s)} = (2s-1)(0-\mathrm{Tr})(R_X(s) - R_X(1-s)) - \frac{\Gamma(s)\Gamma(1-s)}{\Gamma(s-1/2)\Gamma(1/2-s)}(0-\mathrm{vol})(X)$$

where the zero-trace of an operator on $C^\infty(X)$ with smooth kernel $K(w, w')$ is defined as

$$FP_{\varepsilon\downarrow 0}\int_{\rho\geq\varepsilon} K(w,w)$$

while the zero-volume is defined as

$$FP_{\varepsilon\downarrow 0}\int_{\rho\geq\varepsilon} \mathrm{dvol}\,.$$

The first term on the right is 'essentially'

$$\mathrm{Tr}(S_X(s)^{-1}S'_X(s))$$

This follows from several facts. First, the difference of resolvent kernels can be expressed in terms of the scattering eigenfunctions $E(\pi(w), b; s)$:

$$G(\pi(w),\pi(w');s) - G(\pi(w),\pi(w');1-s) = (2s-1)\int_{\partial X} E(\pi(w),b;s)E(\pi(w'),b;1-s)$$

Secondly, the integral of the right-hand side over a compact subset of X can be expressed in terms of the logarithmic derivative of the scattering operator. This calculation is actually quite difficult and is the technical core of [34]; its result is however closely analogous to results in scattering theory for the wave or Schrödinger equation on Euclidean space.

The second term is exactly

$$\frac{\Gamma(s)\Gamma(1-s)}{\Gamma(s-1/2)\Gamma(1/2-s)}\chi_N$$

the Euler characteristic of the manifold N.

From these results, one gets:

Theorem 1. *The zeros of $Z_\Gamma(s)$ are:*

- *Discrete eigenvalues of Δ_X*
- *Scattering poles of $S_X(s)$*
- *'Topological' zeros at $s = 0, -1, -2, \cdots$.*

From this Theorem there follows what may be regarded as one form of the trace formula: a Weierstrass product representation of the zeta function in terms of spectral data. Namely if

$$E(z,n) = (1-z)\exp(z + z^2/2 + \cdots + (-1)^n z^n/n)$$

we have

$$Z_\Gamma(s) = G_p(s)G_t(s)G_{scatt}(s)$$

for functions $u \in C^\infty(X)$. Although L^2 solutions to this equation are rather rare (and may not exist at all) there are many C^∞ solutions. A special form of importance to scattering theory are those associated to $\Re(s) = 1/2$ which take the form

$$u \sim \rho^s f_+ + \rho^{1-s} f_- + \mathcal{O}(\rho)$$

for $f_\pm \in C^\infty(X)$. Roughly speaking these solutions represent a scattering experiment in which an incoming wave with initial amplitude $f_-|_{\partial X}$ gives rise to an outgoing wave of amplitude $f_+|_{\partial X}$. For $\Re(s) = 1/2$ solutions of this type span the continuous spectrum in a sense that can be made precise.

It can be shown that there is a unique solution of the eigenvalue equation of this form if $f_-|_{\partial X}$ is prescribed, so that in particular the map

$$S_X(s) : f_-|_{\partial X} \to f_+|_{\partial X}$$

from $C^\infty(\partial X)$ to itself is well-defined. This map is called the *scattering operator* Δ_X. The uniqueness also implies that there is a unique map from $f_- \in C^\infty(\partial X)$ to the solution u: the integral kernel $E(\cdot, b; s)$ of this map (so that the identity

$$u(\pi(w)) = \int_{\partial X} E(\pi(w), b; s) f_-(b)$$

holds) is called the Eisenstein function (by analogy with similar functions occuring, for example, in the harmonic analysis of $PSL(2, \mathbb{Z})$

It extends to a meromorphic, pseudo-differential operator-valued function of s having order $2\Re(s) - 1$ and the following poles:

- Poles arising from L^2-eigenvalues of the Laplacian. If u is an L^2 solution of the eigenvalue equation, it is not hard to see that $u \sim \rho^s f_+$ for $f_+ \in C^\infty(X)$, i.e., $f_- = 0$.
- Poles arising from scattering resonances–we denote the set of such poles by Z_{scatt}.
- 'Trivial' poles of infinite rank at $s \in \mathbb{N}$.

It has been shown by Guillopé and Zworski [15] that the set of resolvent resonances coincides, with multiplicity, with the set of scattering poles; this result has been generalized to higher (even) dimensions recently by Borthwick and Perry. Thus the resolvent resonances, closely associated to the continuous spectrum, are the 'new' discrete data which play a role very similar to the role played by eigenvalues in the case of compact X.

What sort of trace formula should we expect? The first observation is that for $\Re(s) > 1$, there is *no essential change* in the derivation that leads to (2.4), so that the same relationship holds for non-compact X. The difference is what happens for $\Re(s) < 1$. The automorphic function defined by (2.3) belongs to $\rho^s C^\infty(X)$ and since the volume element behaves like $\rho^{-2} d\rho$ this function is no longer integrable. Instead, one replaces (2.4) with the formula

$$Z'_\Gamma(s)/Z_\Gamma(s) = (2s-1) FP_{\varepsilon \downarrow 0} \int_{\rho \geq \varepsilon} \varphi_s$$

- The Zeta function obeys a functional equation

$$\frac{Z_\Gamma'(s)}{Z_\Gamma(s)} + \frac{Z_\Gamma'(1-s)}{Z_\Gamma(1-s)} = (2s-1)\,\mathrm{Tr}(R_X(s) - R_X(1-s)) - \frac{\Gamma(s)\Gamma(1-s)}{\Gamma(s-1/2)\Gamma(1/2-s)}\chi_X$$

  where χ_X is the Euler characteristic of X.

Note that one can derive from this zeta function a trace formula for any function of the Laplacian using the fact that

$$(2s-1)\,\mathrm{Tr}(R_X(s) - R_X(1-s))$$

is essentially the trace of the spectral density for the Laplacian Δ_X.

3. Selberg's Trace Formula for a Non-Compact Surface

We now come to the main example. We will see that, in many respects, the situation for a non-compact Riemann surface is very similar to the 'classical' situation, and in other ways it is very different. More precisely: the formulas look the same, but the spectral data are very different owing to the presence of continuous spectrum. The situation we have in mind is the case where $X = \Gamma\backslash\mathbb{H}^2$, $\mathrm{vol}(X)$ is infinite and there are no cusps.

Such a manifold X has the structure of a compact manifold N with geodesic boundary consisting of finitely many closed geodesics, to which are attached infinite 'funnels' isometric to the annular region $1 \leq |z| < \exp(\ell)$ where ℓ is the length of the geodesic. It is useful to compactify this picture and think of X as a compact manifold with boundary which is only 'at infinity' because the metric takes the form $\rho^{-2}h$ for a smooth metric h on X and a defining function ρ for ∂X, i.e., a smooth positive function which vanishes exactly to first order on ∂X.

The study of the Laplacian on such manifolds (and their generalization to higher dimensions) has been carried out by a number of mathematicians, among them Agmon [1], Froese-Hislop-Perry [8], Guillopé [11, 12], Lax-Phillips [20, 21], Mandouvalos [22], Mazzeo-Melrose [26], Patterson [30, 31, 32], and myself [35, 36, 37, 39]. The Laplacian Δ_X has at most finitely many discrete eigenvalues below $1/4$, absolutely continuous spectrum in $[1/4, \infty)$, and *no* eigenvalues in this interval. We denote by Z_p the (finite and possibly empty) set of s with $\Re(s) > 1/2$ and $s(1-s)$ an L^2-eigenvalue of Δ_X.Thus the L^2-resolvent

$$R_X(s) = (\Delta_X - s(1-s))^{-1}$$

is meromorphic in the half-plane $\Re(s) > 1/2$ with a branch cut on the line $\Re(s) = 1/2$. It can be shown that the resolvent admits a meromorphic continuation to the complex s-plane [26] if one views it not as a map from $L^2(X)$ to itself but rather as a map from $\rho^N L^2(X)$ to $\rho^{-N} L^2(X)$ for N depending on s. The meromorphically continued resolvent has poles whose interpretation is elucidated by introducing the scattering operator.

The scattering operator is defined with reference to solutions of the eigenvalue equation

$$(\Delta_X - s(1-s))u = 0$$

yields another expression for the trace of $f(\Delta_X)$:

$$\operatorname{Tr}(f(\Delta_X)) = \sum_{\gamma\in\Gamma} \int_{\mathcal{F}} K_f^0(w,\gamma(w))\, dvol \tag{2.2}$$

where $\mathcal{F}$ is a fundamental domain for the action of Γ in $\mathbb{H}^2$. This sum can be regrouped by observing that Γ may be partitioned into hyperbolic conjugacy classes. Using Selberg's theory of point-pair invariants, one can reduce the right-hand side to a sum over hyperbolic conjugacy classes, obtaining an expression of the form

$$\operatorname{Tr}(f(\Delta_X)) = \sum_{\gamma \text{ primitive}} \sum_{n=1}^{\infty} \int_C K_f^0(w,\gamma^n(w))\, dvol$$

where C is a fundamental domain for the abelian group generated by γ. Since the integral kernel of K_f^0 can be computed explicitly, this yields the trace formula.

One version of the trace formula that will be particularly helpful for generalizations occurs if we take $f(\lambda) = (\lambda - z)^{-1}$, even though the resulting operator $f(\Delta_X)$ isn't trace class! It will be convenient to consider the resolvent operators

$$\begin{aligned} R_X(s) &= (\Delta_X - s(1-s))^{-1} \\ R_0(s) &= (\Delta_{\mathbb{H}^2} - s(1-s))^{-1} \end{aligned}$$

and their integral kernels $G_X(\pi(w),\pi(w')'s)$ and $G_0(w,w';s)$. It turns out that the function

$$G_X(\pi(w),\pi(w');s) - G_0(w,w';s) \tag{2.3}$$

on $\mathbb{H}^2 \times \mathbb{H}^2$ has a smooth restriction to the diagonal and defines a Γ-automorphic function $\varphi_s(w)$. This function can be integrated over X and yields a 'renormalized trace of $R_X(s)$'.

On the other hand, one can evaluate the trace in the spirit of (2.2)–note that the conjugacy class of the identity element is absent from the sum. One obtains the surprising formula [5]

$$Z_\Gamma'(s)/Z_\Gamma(s) = (2s-1)\operatorname{Tr}(R_X(s) - R_0(s)) \tag{2.4}$$

where the right-hand side means the 'renormalized trace,' and $Z_\Gamma(s)$ is the Selberg zeta function for Γ, defined as follows. For each primitive hyperbolic element, set $\ell(\gamma) = \operatorname{dist}_{\mathbb{H}^2}(0,\gamma(0))$ and set

$$\begin{aligned} Z_\Gamma(s) &= \prod_{\substack{\gamma \text{ inconjugate} \\ \text{primitive}}} \prod_{k=0}^{\infty} (1 - \exp(-(s+k)\ell(\gamma)) \\ &= \exp\left(\sum_{\gamma} \sum_{n=1}^{\infty} (1 - e^{-n\ell(\gamma)})^{-1} e^{-s\ell(\gamma^n)} \right) \end{aligned}$$

This infinite product is well-defined for $\Re(s) > 1$ and can be shown using methods of dynamics to have a meromorphic continuation to all of $\mathbb{C}$. Indeed, these methods give a representation of $Z_\Gamma(s)$ as a quotient of entire functions of order 2.

Thus we get some simple connections:

- The zeros of $Z_\Gamma(s)$ are determined by poles of $R_X(s)$ (eigenvalues of Δ_X) and poles of $R_0(s)$ (in even dimension of X, the topological poles at $s = -\mathbb{N}$

RESONANCES, ZETA FUNCTIONS, AND TRACE FORMULAS FOR KLEINIAN GROUPS

PETER PERRY

ABSTRACT. We discuss scattering theory and zeta functions for convex co-compact Kleinian groups. This lecture was given at the International Workshop on Spectral Geometry, 13–16 June, 1999, at the Technion-Israel Institute of Technology.

1. Introduction

Selberg's trace formula [41] for a compact Riemann surface X relates the length spectrum of closed geodesics on X and the eigenvalues of the Laplace operator on X. It is the ancestor of the Duistermaat-Guillemin trace formula and its variants which show how spectral invariants encode geometric information about X for any smooth compact manifold without boundary.

Today I want to survey progress made on analogues of Selberg's trace formula for noncompact X, particularly manifolds of *infinite* volume. The essential difference here is that the spectrum of the Laplacian may include finitely many or even no eigenvalues, so that 'most' of the spectral data is encoded in the continuous spectrum of the Laplacian. In these cases the role of eigenvalues is in some sense played by so-called *scattering resonances* which are poles of the meromorphically continued resolvent or the meromorphically continued scattering operator. I will begin by recalling Selberg's trace formula for a compact surface and its relationship to the Zeta function, and then develop analogues in the infinite-volume case.

2. Selberg's Trace Formula for a Compact Surface

If X is a compact surface of constant negative curvature, the uniformization theorem states that $X = \Gamma\backslash\mathbb{H}^2$ where $\mathbb{H}^2$ is real hyperbolic two-dimensional space and Γ is a discrete group of isometries of $\mathbb{H}^2$. The trace formula results from computing the trace of a function f of the Laplacian Δ_X two ways: one using the spectral resolution for the Laplacian, and the other using an explicit parametrix. The first method yields

$$\mathrm{Tr}(f(\Delta_X)) = \sum_{n=0}^{\infty} f(\lambda_n) \tag{2.1}$$

Supposing that $K_f^0(w,w')$ is the integral kernel for $f(\Delta_{\mathbb{H}^2})$, that $\pi : \mathbb{H}^2 \to X$ is the natural projection, and that $K_f(\pi(w),\pi(w'))$ is the integral kernel of $f(\Delta_X)$, the formula

$$K_f(w,w') = \sum_{\gamma\in\Gamma} K_f^0(w,\gamma(w'))$$

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