Contents

1 Phase preserving canonical identification and Riemann hypothesis

Riemann hypothesis states that the nontrivial zeros of Riemann Zeta function lie on the axis $x = 1/2$ [Riemann$_1$]. Since Riemann zeta function allows interpretation as a thermodynamical partition function for quantum field theoretical system consisting of bosons labelled by primes [Riemann$_2$], it is interesting to look Riemann hypothesis from the perspective of physics. In TGD p-adicization is essential part of quantum theory and this provides interesting new views to Riemann hypothesis. In particular, Riemann hypothesis can be sharpened and a connection with arithmetic quantum field theories results.

1.1 Rieman Zeta function as thermodynamical partition function

Rieman zeta function [Riemann$_1$, Riemann$_2$] can be defined as

$$\zeta(z) = \sum_n \frac{1}{n^z} \quad . \tag{1}$$

This sum converges for $s > 1$ only. One can extend the definition of Riemann zeta function to entire complex plane by analytical continuation. An especially interesting expression for Riemann Zeta function from the point of view physicist is as

$$\begin{aligned}
\zeta(z) &\equiv Z_B(z) = \prod_p Z_B(p,z) \quad , \\
Z_B(p,z) &= \frac{1}{1-x_p(z)} \quad , \\
x_p &\equiv p^{-z} \quad .
\end{aligned} \tag{2}$$

The product is over primes and the factor $Z_B(p,z)$ is partition function for a harmonic oscillator with frequency/temperature ratio

$$\frac{\omega}{T} = log(p)z \quad .$$

This representation can be derived from the the basic representation by using the decomposition of integer n to a product of powers of prime. Identifying $1/z$ formally as temperature, one can say that there is infinite number of oscillators with frequencies $\omega_p = log(p)$ in thermal equilibrium. Of course, in physical situation only real $z > 0$ corresponds to physical temperature but one can continue the partition function to the complex plane analytically.

The formula making possible analytical continuation of Riemann Zeta to negative values of $x = Re(z)$ is

$$\begin{aligned}
\Lambda(z) \equiv \pi^{-z/2}\Gamma(z/2)\zeta(z) &= \pi^{-(1-z)/2}\Gamma((1-z)/2)\zeta(1-z) = \Lambda(1-z) \quad , \\
\Gamma(z) &= \int_0^\infty t^{z-1}exp(-t)dt \quad .
\end{aligned} \tag{3}$$

$\Gamma(z)$ denotes Gamma function.

The transformation $z \rightarrow 1-z$ acts as a symmetry with respect to the line $x = 1/2$ so that zeros of Riemann Zeta must be located symmetrically around the line $x = 1/2$. Furthermore, the identity

$$\Lambda(z) = \overline{\Lambda}(\overline{z})$$

implies that $\Lambda(z)$ must be real at line $x = 1/2$. Hence Riemann hypothesis stating that all zeros of Riemann zeta function are on this line, is very natural.

Poles of gamma function imply trivial zeros of Riemann zeta function at $z = -2n$, $n \geq 1$. $1/z = 1$ can be interpreted as the analog of Hagedorn temperature since for $1/z > 1$ the sum representation of Riemann function does not converge. Riemann zeta function has pole at $z = 1$.

1.2 Fermionic version of Riemann hypothesis

Riemann zeta can be regarded as bosonic partition function. Fermionic partition function can be defined as

$$\begin{aligned} Z_F(z) &= \sum_{n \in U} \frac{1}{n^z} \ , \\ U &= \{n | n = \prod_i p_i^{k_i}, k_i \leq 1\} \ . \end{aligned} \tag{4}$$

Whereas all integers are allowed in the definition of the bosonic partition function giving rise to Riemann zeta, only integers containing prime powers smaller than two are allowed in the definition of the fermionic partitition function. This is in accordance with fermionic statistics allowing occupation numbers 0 and 1 in the states labelled by primes. It is easy to see that fermionic partition function can be written in the form:

$$\begin{aligned} Z_F(z) &= \prod_p Z_F(p,z) \ , \\ Z_F(p,z) &= = 1 + x_p(z) \ , \\ x_p(z) &= 1 + p^{-z} \ . \end{aligned} \tag{5}$$

Bosonic partition function $Z_B \equiv \zeta$ and fermionic partition function are related by simple identity

$$Z_F(z) = \frac{Z_B(z)}{Z_B(2z)} \ . \tag{6}$$

This identify follows from the algebraic identity $(1-x)(1+x) = 1 - x^2$. By using the identity one finds that the zeros of Rieman zeta Z_B are also zeros of the fermionic partition function Z_F and that $Z_F(z/2)$ vanishes when z is zero of Riemann zeta function. This can be interpreted as supersymmetry. In fact Z_F has no other zeros than those of Z_B plus the zero at $z = 1/2$ coming from from the pole of $Z_B(2z)$ at $2z = 1$.

1.3 Phase preserving canonical identification maps real quantum TGD to its p-adic counterpart

In TGD p-adics are essential part of basic quantum physics and one can say that real physics is mapped to its p-adic counterpart by phase preserving canonical identification with pinary cutoff determined by the requirement that p-adic structures obey same defining equations as corresponding real structures. This is possible since p-adic differential equations allow as integration constants p-adic pseudoconstants, which are constant functions only in length scales below resolution defined by pinary cutoff. The first thing coming into mind is to look under what conditions the partition function $Z_p(x_p)$ could be regarded as a canonical image of corresponding p-adic partition function such that canonical identification commutes with the property of being partition function.

Phase preserving canonical identification is defined as follows.

i) Let

$$z = \rho exp(i\phi)$$

be a complex number and restrict phase angle ϕ to the set of angles for which real and imaginary parts of $exp(i\phi)$ are rational numbers. This means that allowed phase angles ϕ corresponds to Pythagorean triangles, which are orthogonal triangles with integer valued sides. The explicit expression for Pythagorean phase factors

$$\begin{aligned} cos(\phi + \epsilon\pi/2) &= \frac{r^2 - s^2}{r^2 + s^2} \ , \\ sin(\phi + \epsilon\pi/2) &= \frac{2rs}{r^2 + s^2} \ , \\ \epsilon \in \{0, 1\} \ . \end{aligned} \tag{7}$$

where r and s are integers which are relative primes. From this it is clear that Pythagorean phases are in natural one-one correspondence with rationals. A convenient manner to represent Pythagorean phases is as squares of phase factors U:

$$\begin{aligned} exp(\phi + \epsilon\pi/2) &= U^2 \ , \\ U &= \frac{r + is}{\sqrt{r^2 - s^2}} \ . \end{aligned} \tag{8}$$

In this representation the product of Pythagorean phases reduces to multiplication of complex integers:

$$(r_1, s_1) \circ (r_2, s_2) = (r, s) = (r_1 r_2 - s_1 s_2, r_1 s_2 + r_2 s_1) \ . \tag{9}$$

ii) Define canonical identification mapping as a mapping, which maps rational phase factors numerically to themselves but re-interpreted as p-adic numbers. Rationals are indeed "common" to both p-adics and reals and p-adics and reals are different completions of rationals.

iii) Map the modulus ρ having pinary expansion

$$\rho = \sum_n x_n p^{-n}$$

to p-adic number using canonical identification

$$\rho = \sum_n x_n p^{-n} \rightarrow \sum_n x_n p^n \quad .$$

This map is continuous and single valued when one selects the pinary expansion of ρ to have finite pinary digits when this is possible ($1 = .99999...$ tells that there are two possible manners to select the pinary expansion for finite number of digits).

iv) Canonical identification thus maps only a subset of complex plane to p-adics since only rational phases are mapped to their p-adic counterparts. This is crucial for the canonical identification. In fact, one must pose pinary cutoff in order to map real structures to p-adic structure satisfying corresponding defining equations. But this is not important for the argument. The map is defined for $p = 3 \ mod \ 4$ only since only in this case $\sqrt{-1}$ does not exist as "p-adically real" number which in turn implies that C_p exists.

1.4 Under what conditions phase preserving canonical identification commutes with the property of being partition function?

To simplify the notation let us write

$$z = x + iy \quad .$$

Consider partition function as a function of the argument x_p

$$\begin{aligned} Z_B(p, x_p) &\equiv \frac{1}{1 - x_p} \quad , \\ x_p &= p^{-x} p^{-iy} \quad . \end{aligned} \tag{10}$$

The question is: under what conditions on z real partition function $Z_B(p, x_p)$ can be regarded as a phase preserving canonical image of the p-adic partition function defined by mapping the p-adic counterpart of x_p to its real counterpart by the phase preserving canonical identification? Or stating it somewhat differently: under what conditions does canonical identification commute with the property of being partition function?

If x is integer this is certainly achieved.

$$x = n > 0 \quad .$$

This is however not enough since one $x = 1/2$ corresponds to half odd integer power of p. If an algebraic extension of p-adics allowing square roots of "p-adically real numbers" is allowed, the situation changes. In this case canonical identification commutes with the property of being partition function if

i) x half integer and

ii) p^{iy} is complex rational (Pythagorean) phase.

The algebraic extension allowing square root of a "p-adically real" p-adic number is 4-dimensional (!) for $p > 2$ and 8-dimensional(!) for $p = 2$. For instance, for $p > 2$ and $p\ mod\ 4 = 3$ one has

$$Z = x + iy + \sqrt{p}u + i\sqrt{p}v \quad .$$

What is essential is that $\sqrt{p}$ exists. This means that one can map the exponent X_p to its p-adic counterpart for $Re(z) = n/2$.

The p-adic counterparts of the fermionic partition functions $Z_F(p, z)$ exist under same conditions on z as the p-adic counterparts of bosonic partition functions $Z_B(p, z)$.

1.5 Can one sharpen Riemann hypothesis?

The set $z = n/2 + iy$, $n > 0$ such that p^{-iy} is Pythagorean phase, is the set in which both real Riemann zeta function and the p-adic counterparts of Z_p exist for $p\ mod\ 4 = 3$. What is important that $x = 1/2$ is the smallest value of x for which the p-adic counterpart of $Z_B(p, x_p)$ exists. Already Riemann showed that the nontrivial zeros of Riemann Zeta function lie symmetrically around the line $x = 1/2$ in the interval $0 \leq x \leq 1$.

If one assumes that the zeros of Riemann zeta belong to the set at which the p-adic counterparts of Riemann zeta are defined, Riemann hypothesis follows in sharpened form.

a) Sharpened form of Riemann hypothesis does not necessarily exclude zeros with $x = 0$ or $x = 1$ as zeros of Rieman zeta unless they are explicitely excluded.

b) The sharpening Riemann hypothesis following from p-adic considerations implies that the phases p^{iy} exist as rational complex phases for all values of $p\ mod\ 4 = 3$ when y corresponds to a zero of Riemann Zeta. Obviously the rational phases p^{iy} form a group with respect to multiplication isomorphic with the group of integers in case that y does not vanish. The same is also true for the phases corresponding to integers contining only powers of primes $p \; mod \; 4 = 3$ phase factor.

c) A stronger form of sharpened hypothesis is that all primes p and all integers are allowed. This would mean that each zero of Riemann function would generate naturally group isomorphic with the group of integers. Pythagorean phases form a group and should contain this group as a subgroup. It might be that very simple number theoretic considerations exclude this possibily. If not, one would have infinite number of conditions on each zero of Riemann function and much sharper form of Riemann hypothesis which could fix the zeros of Riemann zeta completely:

The zeros of Riemann Zeta function lie on axis $x = 1/2$ and correspond to values of y such that the phase factor p^{iy} is rational complex number for all values of prime $p \; mod \; 4 = 3$ or perhaps even for all primes p.

Of course, the proposed condition might be quite too strong. A milder condition is that $U_p(x_p)$ is rational for single value of p only: this would mean that zeros of Riemann Zeta would correspond to Pythagorean angles labelled by primes. One can consider also the possibility that p^{iy} is rational for all y but for some primes only and that these preferred primes correspond to the p-adic primes characterizing the effective p-adic topologies realized in the physical world. p-Adic length scale hypothesis suggests that these primes correspond to primes near prime power powers of two.

d) If this hypothesis is correct then each zero defines a subgroup of Pythagorean phases and also zeros have a natural group structure. Pythagorean phases contain infinite number of subgroups generated by integer powers of phase. Each such subgroup has some number N of generators such that the subgroup is generated as products of these phases. From the fact that Pythagorean phases are in one-one correspondence with rationals, it is obvious that there exists large number of subgroups of this kind. Every zero defines infinite number of Pythagorean phases and there are infinite number of zeros. The entire group generated by the phases is in one-one correspondence with the pairs (p, y).

e) If n^{iy} are rational numbers, there must exist imbedding map f: $(n, y) \rightarrow (r, s)$ from the set of phases n^{iy} to Pythagorean phases characterized by rationals $q = r/s$:

$$(r, s) = (f_1(n, y), f_2(n, y)) \ .$$

The multiplication of Pythagorean phases corresponds to certain map g

$$\begin{aligned}&(r_1, s_1) \circ (r_2, s_2) = [g_1(r_1, s_1; r_2, s_2),\ g_2(r_1, s_1; r_2, s_2)]\\ &= (r_1 r_2 - s_1 s_2, r_1 s_2 + r_2 s_1) \equiv (r, s)\end{aligned}$$

such that the values of r and s associated with the product can be calculated. Thus the product operation rise to functional equations giving constraints on the functional form of the map f.

i) Multiplication of n^{iy_1} and n^{iy_2} gives rise to a condition

$$f(n,y_1)\circ f(n,y_2)=f(n,y_1+y_2) \ .$$

ii) Multiplication of n_1^{iy} and n_2^{iy} gives rise to a condition

$$f(n_1,y)\circ f(n_2,y)=f(n_1n_2,y) \ .$$

1.5.1 Sharpened form of Riemann hypothesis and infinite-dimensional algebraic extension of rationals

Sharpened form of Riemann hypothesis states that the zeros of Riemann zeta are on the line $x=1/2$ and that p^{iy}, where p is prime, are complex rational (Pythagorean) phases for zeros. Furthermore, Rieman hypothesis is equivalent with the corresponding statement for the fermionic partition function Z_F. If the sharpened form of Riemann hypothesis holds true, the value of $Z_F(z)$ in the set of zeros $z=1/2+iy$ of Z_F can be interpreted as a complex (vanishing) image of certain function $Z_F^{\infty}(1/2+iy)$ having values in the infinite-dimensional algebraic extension of rationals defined by adding the square roots of all primes to the set of rational numbers.

a) The general element q of the infinite-dimensional extension Q_C^{∞} of complex rationals Q_C can be written as

$$\begin{aligned} q &= \sum_U q_U e_U \ , \\ e_U &= \prod_{i\in U}\sqrt{p}_i \ . \end{aligned} \tag{11}$$

Here q_U are complex rational numbers, U runs over the subsets of primes and e_U are the units of the algebraic extension analogous to the imaginary unit. One can map the elements of Q_C^{∞} to reals by interpreting the generating units $\sqrt{p}$ as real numbers. The real images $(e_U)_R$ of e_U are thus real numbers:

$$e_U \rightarrow [e_U]_R=\prod_i \sqrt{p_i} \ .$$

b) The value of $Z_F(z)$ at $z=1/2+iy$ can be written as

$$Z_F(z=1/2+iy) = \sum_U \left[\frac{1}{e_U}\right]_R \times (e_U^2)^{-iy} \ . \tag{12}$$

Here $(e_U)_R$ means that e_U are interpreted as real numbers.

c) If one restricts the set of values of $z = 1/2+iy$ to such values of y that p^{iy} is complex rational for every value of p, then the value of $Z_F(1/2+iy)$ can be also interpreted as the real image of the value of a function $Z_F(Q_\infty|z=1/2+iy)$ restricted to the set of zeros of Riemann zeta and having values at Q_C^∞:

$$
\begin{aligned}
Z_F(1/2+iy) &= [Z_F(Q_\infty|1/2+iy)]_R \ , \\
Z_F(Q_\infty|1/2+iy) &\equiv \sum_U \frac{1}{e_U} \times (e_U^2)^{-iy} \ . \qquad (13)
\end{aligned}
$$

Note that $Z_F(Q_\infty|z=1/2+iy)$ cannot vanish as element of Q_∞. One can also define the Q_C^∞ valued counterparts of the partition functions $Z_F(p,1/2+iy)$

$$
\begin{aligned}
Z_F(Q_\infty|1/2+iy) &= \prod_p Z_F(Q_\infty|p, z=1/2+iy) \ , \\
Z_F(Q_\infty|1/2+iy) &\equiv 1+p^{-1/2}p^{-iy} \ , \\
Z_F(p,1/2+iy) &= [Z_F(Q_\infty|p,1/2+iy)]_R \ . \qquad (14)
\end{aligned}
$$

$Z_F(Q_\infty|1/2+iy)$ and $Z_F(Q_\infty|p,1/2+iy)$ belong to Q_C^∞ only provided p^{iy} is Pythagorean phase.

d) The requirement that p^{iy} is rational does not yet imply Riemann hypothesis. One can however strengthen this condition. The simplest condition is that the real image of $Z_F(Q_\infty|1/2+iy)$ is complex rational number for any value of Z_F. A stronger condition is that the complex images of the functions

$$
\frac{Z_F^\infty}{\prod_{p\in U} Z_p^\infty}
$$

are complex rational and U is finite set of primes. The complex counterparts of these functions are given by

$$
\left[\frac{Z_F^\infty}{\prod_{p\in U} Z_p^\infty}\right]_R = \frac{Z_F}{\prod_{p\in U} Z_F(p,..)} \ . \qquad (15)
$$

Obviously these conditions can be true only provided that $Z_F(1/2+iy)$ vanishes identically for allowed values of y. This implies that sharpened form of Riemann hypothesis is true. "Physically" this means that the fermionic partition function restricted to any subset of integers not divisible by some finite set of primes, has real counterpart which is complex rational valued.

1.5.2 Connection with the conjecture of Berry and Keating

The idea that the imaginary parts y for the zeros of Riemann zeta function correspond to eigenvalues of some Hermitian operator H is not new. Berry and Keating [Berry and Keating] however proposed quite recently that the Hamilton in question is supersymmetric and given by

$$H = xp - \frac{i}{2} \ . \tag{16}$$

Here the momentum operator p is defined as $p = -id/dx$ and x has non-negative real values.

H can be indeed expressed as a square $H = Q^2$ of a Hermitian super symmetry generator Q:

$$\begin{aligned}
Q &= \sqrt{i}\,[ix\sigma_1 + p\sigma_2] + \sqrt{\frac{i}{2}}\sigma_3 \ , \\
\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \ , \\
\sigma_2 &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \ , \\
\sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ .
\end{aligned} \tag{17}$$

By a direct calculation one finds that the following relationship holds true:

$$Q^2 = \begin{pmatrix} xp + \frac{i}{2} & 0 \\ 0 & xp - \frac{i}{2} \end{pmatrix} \ .$$

The eigenspinors of Q can be written as

$$\psi = \begin{pmatrix} u \\ v \end{pmatrix} = x^{-iy} \begin{pmatrix} x^{1/2} \\ \sqrt{\frac{y}{i}}x^{-1/2} \end{pmatrix} \ .$$

The eigenvalues of Q are $q = \sqrt{y}$. For $y \geq 0$ the eigenvalues are real so that Q is Hermitian when inner product is defined appropriately. Obviously y is eigenvalue of Hamiltonian.

Orthogonality requirement for the solutions of the Dirac equation requires that the inner product reduces to the inner product for planewaves $exp(iu)$, $u = log(x)$. This is achieved if inner product for spinors $\psi_i = (u_i, v_i)$ is defined as

$$\langle\psi_1|\psi_2\rangle = \int_0^\infty \frac{dx}{x}\left[\overline{u}_1 v_2 + \overline{v}_1 u_2\right] \quad . \tag{18}$$

In the basis formed by solutions of Dirac equation this inner product is indeed positive definite as one finds by a direct calculation.

The actual spectrum assumed to give the zeros of Riemann zeta function however remains open without additional hypothesis. An attractive hypothesis motivated by p-adization philosophy is that the sharpened form of Riemann hypothesis stating that n^{iy} is complex rational number for the zeros of Riemann zeta function holds true. This implies that x^{iy} is complex rational for any rational value of x. $x^{\pm 1/2}$ in turn belongs to the infinite-dimensional algebraic extension Q_C^∞ of complex rationals, when x is rational. Therefore the solutions of Dirac equation, being of form $x^{iy}x^{\pm 1/2}$, have values in Q_C^∞ for rational values of argument x.

1.5.3 Connection with arithmetic quantum field theory and quantization of time

There is also a very interesting connection with arithmetic quantum field theory and sharpened form of Riemann hypothesis. The Hamiltonian for a bosonic/fermionic arithmetic quantum field theory is given by

$$H = \sum_p log(p) a_p^\dagger a_p \quad . \tag{19}$$

where $a_p^\dagger$ and a_p satisfy standard bosonic/fermionic anticommutation relations

$$\{a_{p_1}^\dagger, a_{p_2}\}_\pm = \delta(p_1, p_2) \quad . \tag{20}$$

Here $\pm$ refers to anticommutator/commutator. The sum of Hamiltonians defines supersymmetric arithmetic QFT. The states of the bosonic QFT are in one-one correspondence with non-negative integers and the decomposition of a non-negative integer to powers or prime corresponds to the decomposition of state to many boson states corresponding to various modes p. Analogous statement holds true for fermionic QFT.

The matrix element for the time development operator $U(t) \equiv exp(iHt)$ between states $|m\rangle$ and $|n\rangle$ can be written as

$$\langle m|U(t)|n\rangle = \delta(m,n) n^{it} \quad . \tag{21}$$

Same form holds true both in bosonic and fermionic QFT:s. These matrix elements are rational if n^{it} are complex rational phases. Therefore, if sharpened form of Riemann hypothesis holds true, bosonic time development operator has only complex rational matrix elements if t corresponds to a zero of Riemann zeta. Similar statement holds in the case of fermionic QFT. One can say that the durations for the time evolutions are quantized in a well defined sense and allowed values of time coordinate correspond to the zeros of Riemann zeta function!

The result is very interesting from the point of view of quantum TGD. since it would mean that $U(t)$ allows for the preferred values of the time parameter p-adicization ($p \ mod \ 4 = 3$) obtained by mapping the diagonal phases to their p-adic counterparts by phase preserving canonical identification. For phases this map means only the re-interpretation of the rational phase factor as a complexified p-adic number. For these quantized values of the time parameter time evolution operator of the arithmetic quantum field theory makes sense in all p-adic number fields besides complex numbers.

In case of Berry's supersymmetric Hamiltonian the rationality of p^{iy} and the requirement that time evolution operator is complex rational valued implies that allowed time durations for time evolution are given by $t = log(n)$. This means that there is nice duality between Berry's theory and arithmetic QFT. The allowed time durations (energies) in Berry's theory correspond to energies (allowed time durations) in arithmetic QFT.

1.6 Intuitive arguments in favour of sharpened form of Riemann hypothesis

In the following a general heuristic strategy for proving the sharpened Riemann hypothesis by using p-adic versions of Riemann Zeta function is discussed.

1.6.1 What might be the strategy of proof?

One important role of p-adic numbers is related to the determination of zeros of polynomials. This suggest obvious strategy of proof to Riemann hypothesis.

a) Continue Riemann zeta for every value of p to p-adic Zeta function defined in algebraic extension allowing square root of all 'p-adically real' numbers. p-Adic Zetas are defined in the set $x = n/2 + iy$, where n is integer and y defines Pythagorean triangle.

b) Prove a generalization of 'Local-Global principle' for Diophantine equation stating that the existence or non-existence of solutions in the set of rationals can be detected by studying, for each prime, the solutions of the equation in p-adic number fields. This principle is too good to be be true generally but there are some simple situations when this the case. For instance, Hasse-Minkowski theorem [?] states that quadratic polynomial $Q(x, y, ..)$ with rational coefficients has non-trivial solutions in the set of rational numbers if and only it has non-trivial solutions in all p-adic number fields. The generalization of Local-Global principle would look like follows: x is zero of Riemann Zeta if and only it corresponds to a zero of the p-adic Riemann Zeta for *every* p. This is probably the most hardest part of the theorem and will not be discussed in the following. The idea about one-one correspondence seems to make sense: the numbers $p_1^{-n/2-iy}$ belong to rationals extended by square roots and their p-adic counterparts belong to the extensions of p-adic numbers allowing square roots of ordinary p-adic numbers.

c) Demonstrate that these p-adic Riemann Zetas have simultaneous zeros only at the line $x = 1/2$ and correspond to Pythagorean triangles $p^{-\sqrt{-1}y} =$ rational. To demonstrate this it is enough to study the p-adic norm of the Riemann Zeta for each p and the estimation of the norm as function of $x = n/2$ should be relatively simple. What one should demonstrate is that the norm of the p-adic Zeta function becomes nonvanishing for *at least one* prime p for $x = n/2 > 1/2$. Demonstration of this might be relatively easy by using the fact that the p-adic norm of the p-adic Riemann Zeta is infinite product

$$|\zeta_p|_p = \prod_{p_1} |Z_{p_1}|_p \ . \tag{22}$$

of the p-adic norms of the composite functions

$$Z^B_{p_1}(z) = \frac{1}{1-p_1^{-n/2}U(y)} \ , \quad U(y) = p_1^{\sqrt{-1}y} \ . \tag{23}$$

Fermionic partition function has same zeros as Riemann Zeta plus the additional zero at $z = 1/2$ and is expressible as the product of fermionic partition functions

$$Z^F_{p_1}(z) = 1 + p_1^{-n/2}U(y) \ .$$

The p-adic norms of $Z^B_{p_1}$ and $Z^F_{p_1}$ can be both smaller, equal or greater than one and vanish if sufficiently many composite functions have p-adic norm smaller than one. What one must show that for $x = n/2 > 1/2$ infinite number of primes $p_1 \neq p$ emerges such that p-adic norm of $Z^B_{p_1}$ and $Z^F_{p_1}$ becomes larger than one. This at least for single p. Also one must show that for $n = 1/2$ the functions $Z^B_{p_1}$ and $Z^F_{p_1}$ for which p-adic norm is larger than one, if present at all, do not make the product nonvanishing.

In the following only the basic definitions will be given and a heuristic argument that the zeros of Riemann Zeta are located at $x = 1/2$ line will be developed. Rather encourangly, the calculations lead to a further sharpening of Riemann hypothesis suggesting that the long sides of the Pythagorean triangles associated with the zeros are products of first powers of primes: $r^2 + s^2 = \prod_k p_k$

(note the interpretation as a many-fermion state). A further idea naturally popping up is that Riemann hypothesis is equivalent with the hypothesis that each zero codes for a conformally invariant state of some *superconformal* arithmetic quantum field theory and that $x - 1/2$, which has half odd integer valued spectrum in p-adic context, can be interpreted as conformal weight and thus vanishes in superconformal field theory. This superconformal field theory might be quantum TGD if earlier speculations about the reduction of physics to number theory are correct! Since infinite primes correspond to the states of supersymmetric arithemetic quantum field theory, each zero would also code infinite prime and the question whether also infinite primes should be included into the definition or Riemann Zeta and of allowed rational phases, must be raised. The hard part of the proof would be showing that p-adic and real zeros are in one-one correspondence with each other: this problem must be left to professional mathematicians.

1.6.2 How to define p-adic Zeta function?

The definition of the p-adic Zeta function is possible in the extension of p-adic numbers allowing square roots of p-adically real numbers. This extension is 4-dimensional for $p > 2$ and 8-dimensional for $p = 2$. The definition of the p-adic counterpart of the rational phase is straightforward. One simply uses the fact that p_1^{iy} is Pythagorean triangle and defines

$$\begin{aligned} U(y) = p_1^{-\sqrt{-1}y} \equiv u + \sqrt{-1}v \ , \quad & u = \tfrac{2rs}{r^2+s^2} \ , \quad v = \tfrac{r^2-s^2}{r^2+s^2} \ , \\ \text{or} \quad & u = \tfrac{r^2-s^2}{r^2+s^2} \ , \quad v = \tfrac{2rs}{r^2+s^2} \ , \end{aligned} \tag{24}$$

Note than in this definition one can permute the r.h.s:s of equations defining x and y. r and s are non-negative integers. This definition applies also for $p \ mod \ 4 = 1$ for which $\sqrt{-1}$ is ordinary p-adic number[1]. With this definition all partition functions $Z_{p_1}(n/2 + iy)$ are well-defined for all values of p thus is also their product.

There is an important delicacy involved with the definition of $Z^B_{p_1}((m+1/2+iy)$ and $Z^F_{p_1}((m+1/2+iy)$. When $p_1^{1/2}$ exists as ordinary p-adic number, there are two possible signs of square root and one must select the sign of the square root correctly. The choice of the sign can affect crucially the p-adic norm of $Z^B_{p_1}$ and $Z^F_{p_1}$ and actually permutes $Z^B_{p_1}$ and $1/Z^F_{p_1}$. A possible manner to fix the sign is to require that the p-adic norm of $Z^B_{p_1}$ is and hence also of the p-adic Rieman Zeta is minimal for $x = 1/2$. The choice of sign factor is of crucial importance since by supersymmetry both Z^B and Z^F and also their product $Z^B Z^F$ vanish

[1]This definition makes it possible to define p-adic TGD for all primes since $\sqrt{-1}$ appearing in phase preserving canonical identification is well defined

for zeros of Riemann Zeta. The choice of the sign of $\sqrt{p_1}$ for every p_1 transforms $Z^B Z^F$ to $1/Z^B Z^F$ which diverges.

1.6.3 p-Adic norms of partition functions $Z^B_{p_1}$ and $Z^F_{p_1}$

A crucial element in the heuristic strategy proving Riemann hypothesis is the fact that the p-adic norms of Pythagorean phases can have arbitrary values. Only this makes possible the vanishing of the p-adic theta function. The norm of Z_B resp. Z^F is product of norms for $Z^B_{p_1}$ resp. $Z^F_{p_1}$. Only factors have norm different from one are interesting. Thus one must find the mechanisms making the norm of the functions

$$F_\epsilon = 1 + \epsilon p^{n/2} U(y) \ \ , \ \ \epsilon(F) = 1 \ \ , \ \ \epsilon(B) = -1 \tag{25}$$

appearing in the definitions of $Z^B_{p_1}$ and $Z^F_{p_1}$ different from one.

1. General expressions for the p-adic norm of $F_\epsilon(z)$

The general formula for the p-adic norm of the number X in algebraic extension is as p-adic norm for the product over all conjugates of X

$$N_p(X) \ = \ |\prod_c X_c|_p^{1/n} \ . \tag{26}$$

Here n is dimension of the algebraic extension. In the present case the algebraic extension allowing square roots of ordinary p-adic numbers is considered. The number in this extension can be written as $X = x + e_1 y + \sqrt{p}(u + \sqrt{-1}v)$, where e_1 can be taken to be $e_1 = \sqrt{-1} = i$ when $\sqrt{-1}$ does not exist as ordinary p-adic number and some square root not existing as ordinary p-adic number otherwise. Conjugations correspond to the group of conjugations formed by the trivial conjugation Id and conjugations $C_1 : e_1 \rightarrow -e_1$, $C_2 : \sqrt{p} \rightarrow -\sqrt{p}$ and $C_3 = C_1 C_2$.

The general formula for $N_p(X)$ is thus

$$\begin{aligned}
&N_p(X) = |pZ|_p^{1/4} \ \ , \\
&Z = x^4 - y^4 e_1^4 - (u^4 + v^4 e_1^4)p^2 \\
&-2[x^2y^2e_1^2 + (x^2v^2e_1^2 + y^2u^2 + y^2v^2e_1^2)p + (u^2v^2e_1^2 + x^2u^2)p^2] \ \ .
\end{aligned} \tag{27}$$

As far as the norm of $F_\epsilon(z)$ is considered there are four different cases to be considered. The following equations summarize the situation for $x = n/2$ in various case.

a) $p_1^{1/2} \notin R_p$ and $\sqrt{-1} \notin R_p$.

$$N_p(F_\epsilon(m+1/2+iy)) = |(1+\epsilon p_1^m(-p_1)^{1/2}v)^4 - p_1^{4m+2}u^4 + 2(1-\epsilon p_1^m(-p_1)^{1/2}v)^2 p_1^{2m+1}u^2|_p^{1/4}$$

$$N_p(F_\epsilon(m+iy)) = |(1+\epsilon p_1^m u)^4 - p_1^{4m}v^4 2(1-\epsilon p_1^m u)^2 p_1^{2m}v^2|_p^{1/4} \ .$$

b) $p_1^{1/2} \notin R_p$ and $\sqrt{-1} \in R_p$.

$$N_p(F_\epsilon(m+1/2+iy)) = |1 - p_1^{4m+2}(u+\sqrt{-1}v)^4 - 2p_1^{2m+1}(u+\sqrt{-1}v)^2|_p^{1/4} \ .$$

$$N_p(F_\epsilon(m+iy)) = |[1+\epsilon p_1^m(u+\sqrt{-1}v)]^4|_p^{1/4} \ . \tag{28}$$

c) $p_1^{1/2} \in R_p$ and $\sqrt{-1} \notin R_p$.

$$N_p(F_\epsilon(n/2+iy)) = |(1+\epsilon p_1^{n/2}u)^4 - p_1^{2n}v^4 + 2(1+\epsilon p_1^{n/2}u)^2 p_1^n v^2|_p^{1/4} \ .$$

d) $p_1^{1/2} \in R_p$ and $\sqrt{-1} \in R_p$.

$$N_p(F_\epsilon(n/2+iy)) = |[1+\epsilon p_1^{n/2}(u+\sqrt{-1}v)]^4|_p^{1/4} \ .$$

Note that only in case that $\sqrt{p_1}$ does not exist as ordinary p-adic number, the norm of F_ϵ depends on ϵ: thus for $\sqrt{p_1} \notin R_p$ the contributions to fermionic and bosonic partition functions are inverses of each other and cancel each other in the product $Z^B Z^F$:

$$N_p(Z^B Z^F) = \prod_{\sqrt{p_1}\in R_p} N_p(Z^B_{p_1}) N_p(Z^F_{p_1}) \ . \tag{29}$$

2. Mechanisms affecting the norm of $F_\epsilon(z)$.

There are three basic mechanisms affecting the norm of $F_\epsilon(z)$.

2. 1. $u = O(p^0)$ and $v = O(p^0)$.

In this case $r, s, r^2 - s^2, 2rs$ and $r^2 + s^2$ have p-adic norms equal to one. In this case p-adic norm of the function F_ϵ can be smaller than one. In this case the conditions for the reduction of the norm of F_ϵ cannot be simplified from the general form provided by the general expressions of the p-adic norm of F_ϵ in Eqs. 46.

2. 2. $r^2+s^2=O(p^k)$ *and* $\sqrt{-1}$ *exists as ordinary p-adic number.*

Since r and s are coprimes one can assume that either r or s has p-adic norm equal to one. For $r^2+s^2=O(p^k)$ $|U|_p$ increases unless some compensation mechanism is involved. The p-adic norm of r^2+s^2 is smaller than one for all R_p for which p prime factor of r^2+s^2. For $|r^2+s^2|_p<1$ both $2rs$ and r^2-s^2 have unit p-adic norm and the condition gives

$$\begin{aligned} r &= \epsilon(p,p_1,y)\sqrt{-1}s+O(p^k) \ , \\ \epsilon(p,p_1,y) &\in \{1,-1\} \ . \end{aligned} \tag{30}$$

Note that $\sqrt{-1}$ must be ordinary p-adic number for the condition $r^2+s^2=O(p^k)$ to hold true.

The condition $r=\pm\sqrt{-1}s+O(p^k)$ gives

$$2rs+\sqrt{-1}(r^2-s^2)=2r^2\sqrt{-1}[\epsilon(p,p_1,y)+1]+O(p^k) \ .$$

Therefore the p-adic norm of the Pythagorean phase has two possible values equal to 1 and p^{-k}.

For $|U(y)|_p=p^{-k}$ the p-adic norms of $Z^B_{p_1}$ and $Z^B_{p_1}$ satisfy the condition

$$N_p(Z^B_{p_1}(n/2+iy)) = \frac{1}{N_p(Z^F_{p_1}(n/2+iy))}=p^{-k} \ . \tag{31}$$

For the second sign Pythagorean phase has p-adic norm equal to one unless cancellation mechanism is at work:

$$N_p(Z^B_{p_1}(n/2+iy))=\frac{1}{N_p(Z^F_{p_1}(n/2+iy))}=1 \ . \tag{32}$$

For $|U(y)|_p=1$ the p-adic norm of F_ϵ can be also smaller than one and the conditions for this read in various cases as

a) $p_1^{1/2}\notin R_p$ and $\sqrt{-1}\in R_p$. $N_p(F_\epsilon(m+1/2+iy))=\vert 1-16p_1^{4m+2}-8p_1^{2m+1}\vert_p^{1/4}=O(p^{k_1})$. $N_p(F_\epsilon(m+iy))=\vert[1-2\epsilon\times\epsilon(p,p_1,y)p_1^m)^4\vert_p^{1/4}=O(p^{k_1})$. (33)
b) $p_1^{1/2}\in R_p$ and $\sqrt{-1}\in R_p$. $N_p(F_\epsilon(n/2+iy))=\vert[1+2\epsilon\times\epsilon(p,p_1,y)p_1^{n/2}]^4\vert_p^{1/4}=O(p^{k_1})$.

Here $k_1 \leq k$ is assumed to hold true. Note that in this case the conditions for the reduction of the norm do not depend explicitely on u and v but only on p_1.

2. 3. $v = O(p^k)$ *and* $u = O(p^0)$.

Second possibility for having $|F_\epsilon|_p \leq 1$ is that the imaginary part v of U has p-adic norm smaller than one and real part has norm one so that F_ϵ can have norm smaller than one. The partition functions for which v has p-adic norm smaller than one tend to reduce the norm of F_ϵ when $\sqrt{p_1}$ exists.

Consider first the case $v \propto r^2 - s^2$. The condition $r^2 - s^2 = O(p^k)$ implies

$$\begin{aligned} r &= \epsilon(p, p_1, y)s + O(p^k) \ , \\ \epsilon(p, p_1, y) &\in \{1, -1\} \ . \end{aligned} \tag{34}$$

This implies $|r|_p = |s|_p = 1$ and $r^2 + s^2 = 2r^2 + O(p^k)$. Hence one has

$$u = \tfrac{2rs}{r^2+s^2} = -\epsilon(p, p_1, y) + O(p^k) \ , \quad v = O(p^k) \ . \tag{35}$$

For $v \propto 2rs$ one can have either $r = O(p^k)$ ($\epsilon(p, p_1, y) = 1$) or $s = O(p^k)$ ($\epsilon(p, p_1, y) = -1$). This implies

$$u = \tfrac{r^2-s^2}{r^2+s^2} = -\epsilon(p, p_1, y) + O(p^k) \ , \quad v = O(p^k) \ . \tag{36}$$

also in this case. The conditions for the reduction of the norm of F_ϵ are given below.

$$
\begin{aligned}
&\text{a) } p_1^{1/2} \notin R_p \text{ and } \sqrt{-1} \notin R_p. \\
&N_p(F_\epsilon(m+1/2+iy)) = |1-p_1^{4m+2}|_p^{1/4} = O(p^{k_1}) \\
&N_p(F_\epsilon(m+iy)) = |[1+p_1^m\epsilon(p,p_1,y)]^4|_p^{1/4} = O(p^{k_1}) \ . \\
&\text{b) } p_1^{1/2} \notin R_p \text{ and } \sqrt{-1} \in R_p. \\
&N_p(F_\epsilon(m+1/2+iy)) = |1-p_1^{4m+2}-2p_1^{2m+1}|_p^{1/4} = O(p^{k_1}) \ . \\
&N_p(F_\epsilon(m+iy)) = |(1+\epsilon(p,p_1,y)p_1^m)^4|_p^{1/4} = O(p^{k_1}) \ . \\
&\text{c) } p_1^{1/2} \in R_p \text{ and } \sqrt{-1} \notin R_p. \\
&N_p(F_\epsilon(n/2+iy)) = |[1+\epsilon\times\epsilon(p,p_1,y)p_1^{n/2}]^4|_p^{1/4} = O(p^{k_1}) \ . \\
&\text{d) } p_1^{1/2} \in R_p \text{ and } \sqrt{-1} \in R_p. \\
&N_p(F_\epsilon(n/2+iy)) = |[1+\epsilon\times\epsilon(p,p_1,y)p_1^{n/2}]^4|_p^{1/4} = O(p^{k_1}) \ .
\end{aligned}
\tag{37}
$$

Also in this case the conditions for the reduction of the norm depend explicitely on p_1 only.

1.6.4 Zeros of Riemann Zeta as a representation of infinite primes and further sharpening of Riemann hypothesis

p-Adic TGD leads in natural manner to the generalization of the notion of prime number to include also infinite primes. As found in the chapter "Infinite primes and consciousness" of [cbook], infinite primes can be regarded as Fock states of a super-symmetric quantum field theory. The question which comes first in mind is whether one should include also infinite primes into the definition of Riemann Zeta and to the proof of Riemann hypothesis. This question will not be dealt here however. Rather, additional support for the hypothesis that the zeros of Riemann Zeta represent states of arithmetic quantum field theory, will be given allowing to sharpen the Riemann Hypothesis even further.

For each prime p each zero of Riemann Zeta corresponds to a decomposition of the set of primes into the sets B and F and their complement and to the assignment of power p^k characterizing the p-adic norm to each prime in the sets B and F. This decomposition brings in mind the definition of infinite prime in which one applies bosonic and fermionic creation operators labelled by primes to Fock vacuum.

a) The construction of infinite primes can be regarded as building holes in infinite Dirac sea plus adding bosons. The analogy suggests that infinite Dirac sea corresponds to infinite number of factors $|r^2+s^2|_p = 1$ giving rise to factors $|Z^B_{p_1}| = 1/p$. If the number of these factors is infinite, there are hopes that the norm of Riemann Zeta vanishes. The hypothesis implies that $r^2 + s^2$, which represents the longest side of Pythagorean triangle, is integer expressible as a product of primes

$$r^2 + s^2 = \prod_i p_i \quad . \tag{38}$$

and also itself analogous to n-fermion state. An attractive hypothesis is that this condition characterizes completely the Pythagorean phases associated with Riemann zeros. If this is really is true, one can make far-reaching conclusions about the spectrum of the zeros of Riemann Zeta. For instance, $y = y_1 + y_2$ can correspond to zero of Riemann Zeta only if the sets $F(p, z_1)$ and $F(p, z_2)$ are disjoint for all primes p which means that $r_1^2 + s_1^2$ and $r_2^2 + s_2^2$ have no common primes in their decomposition for any prime p_1: the interpretation in terms of fermion statistics is obvious.

b) Single fermion states correspond naturally to the primes p_1 for which $|r^2+s^2|_p = 1$ holds true. Since $|r^2+s^2|_p > 1$ is not possible, fermionic statistics is automatically satisfied. The primes p_1 with $|Z^B_{p_1}|_p = 1$ in the complement of $B \cup F$ are thus analogous to states having single hole in the corresponding mode.

c) The primes p_1 with $|Z^B_{p_1}|_p = p^k$ in B are analogous to states containing either k bosons and one hole or $k+1$ bosons depending on whether one has $|r^2 + s^2|_p = 1$ or $|r^2 + s^2|_p = 1/p$. If the number of the bosons is much smaller than the number of sea fermions, the norm of the Rieman Zeta vanishes. This conjecture implies that the condition

$$N_p(1 - p_1^{-n/2}\frac{u}{r^2+s^2}) = p^{-k} \quad , \quad u = 2rs \ \text{ or } \ u = r^2 - s^2 \quad . \tag{39}$$

represents state with k bosons and one hole. The condition

$$N_p(1 - \epsilon(p, p_1, y)p_1^{-n/2}) = p^k \tag{40}$$

in turn represents k+1 bosons.

A fascinating possibility is that infinite primes are necessary for the p-adicization of Riemann Zeta along the proposed lines. For instance, this might be necessitated by the need to define all possible roots of Riemann Zeta as Pythagorean phases by allowing also infinite primes as factors of $r^2 + s^2$. Also the vanishing of Riemann Zeta might depend crucially on the presence of the Dirac sea of infinite primes containing only few holes: infinite primes could contribute $1/p$-factors to the norm or Riemann zeta to cancel it.

1.6.5 Number theory and superconformal symmetry

If the proposed conjecture is true, one can make rather far-reaching conclusions about the spectrum of the zeros of Riemann Zeta. The picture suggests the idea that the zeros of Riemann Zeta correspond to states of a hierarchy of p-adic super-conformal quantum field theories and that the value of Riemann Zeta is signature for the deviation of the conformal weight of the state from zero. Conformal weight is defined as the eigenvalue of the Virasoro generator L_0 and only states having vanishing value of L_0 are possible by superconformal invariance. In general case the eigenvalues of $L_0 = zd/dz$ contain both real and imaginary part:

$$l_0 = n + im \ . \tag{41}$$

corresponding to the powers z^{n+im}. The guess is that m corresponds to the p-adic norm of the Riemann Zeta

$$m = N_p(Z^B) = N_p(Z^F) = 0 \ . \tag{42}$$

and thus vanishes for each p for the zeros of Riemann Zeta. The scaling dimension n would in turn correspond to the deviation of the variable $x - 1/2$ from zero.

$$n = x - 1/2 \ . \tag{43}$$

n is half-odd integer-valued valued as is also scaling dimension for Super Virasoro representations. Thus Riemann hypothesis could be equivalent with superconformal invariance.

p-Adic representation allows to assign to each zero of Riemann Zeta unique many-particle state of every possible p-adic arithmetic quantum field theory and this effectively proves the existence of these theories and perhaps even allows to deduce the explicit representation of super-conformal and related Super Kac Moody algebras for these states. A fascinating possibility is that quantum TGD is this theory so that TGD would indeed reduce to generalized number theory as have been speculated and proof of Riemann hypothesis would require the construction of, no less than theory of everything!

It is interesting to look what supersymmetry means concretely in the proposed picture. As already noticed, Z^F has the zeros as Z^B besides the zero at $z = 1/2$. This implies that also $Z^B Z^F$ has same zeros as Riemann Zeta. Using the properties of $Z^B_{p_1}$ and $Z^F_{p_1}$ one finds that all bosonic factors with p-adic norm not larger than one and representing modes containing no bosons are cancelled by corresponding fermionic factors and that the product representing modes containing both bosonic and fermionic excitations vanishes. This condition can be combined with the fact that there are two options: for given p_1 one has either $Z^B_{p_1} = 1$ and $Z^F_{p_1} = p^{-k_F(p_1)}$ or $Z^B_{p_1} = p^{k_B(p_1)}$ and $Z^F_{p_1} = 1$. Thus the net 'particle number' defined as sum of the exponents $k_F(p_1)$ associated with fermionic factors must be infinitely larger than the corresponding net particle number associated with the bosonic factors:

$$k_{tot} = \sum_{p_1} [k_B(p_1) - k_F(p_1)] = -\infty \quad . \tag{44}$$

This condition obviously guarantees the vanishing of the conformal spin defined as

$$m = p^{k_{tot}} \quad . \tag{45}$$

1.6.6 Why Riemann hypothesis should hold true?

Accepting the basic heuristics, Riemann hypothesis boils down to the study of the number of solutions to the equations determining the norms of the functions $Z^B_{p_1}$ as function of $x = n/2$ in the case that $p_1^{1/2}$ exists as ordinary p-adic number. To simplify the situation one can study the function $Z^B Z^F$ which contains only p-adic primes p_1 for which $\sqrt{p_1}$ is ordinary p-adic number. There are three cases to be considered.

1) The conditions guaranteing the increase of the p-adic norm in bosonic case and reduction in of the p-adic norm in fermionic case are following for $|u|_p = |v|_p = 1$.

$$\begin{array}{l} \text{a) } p_1^{1/2} \in R_p \text{ and } \sqrt{-1} \notin R_p . \\ N_p(F_\epsilon(n/2+iy)) = |(1+\epsilon p_1^{n/2}u)^4 - p_1^{2n}v^4 + 2(1+\epsilon p_1^{n/2}u)^2 p_1^n v^2|_p^{1/4} \quad . \\ \text{b) } p_1^{1/2} \in R_p \text{ and } \sqrt{-1} \in R_p . \\ N_p(F_\epsilon(n/2+iy)) = |[1+\epsilon p_1^{n/2}(u+\sqrt{-1}v)]^4|_p^{1/4} \quad . \end{array} \tag{46}$$

One cannot say much about these conditions as a function of n. The proposed interpretation of the theory suggests that these factors correspond to modes containing one hole.

2) For $|r^2 + s^2|_p < 1$ one has

$$\begin{array}{l} p_1^{1/2} \in R_p \text{ and } \sqrt{-1} \in R_p . \\ N_p(F_\epsilon(n/2+iy)) = |[1+2\epsilon \times \epsilon(p,p_1,y) p_1^{n/2}|_p^{1/4} = O(p^{k_1}) \quad . \end{array} \tag{47}$$

Rather remarkably, the norm does not depend at all on y. In case that the equation has root one, can with a proper choice of the sign factor of $\sqrt{p_1}$ guarantee that the norm of $Z^B_{p_1}$ is one whereas the norm of $Z^F_{p_1}$ is smaller than one: this is desirable to reduce the norm of $Z^B Z^F$. The interpretation is that unit norm represents single boson state plus hole.

3) For $v = O(p^k)$ the conditions reduce to

a) $p_1^{1/2} \in R_p$ and $\sqrt{-1} \notin R_p$.

$$N_p(F_\epsilon(n/2+iy)) = |[1+\epsilon \times \epsilon(p,p_1,y)p_1^{n/2}]^4|_p^{1/4} = O(p^{k_1}) \quad .$$

b) $p_1^{1/2} \in R_p$ and $\sqrt{-1} \in R_p$.

$$N_p(F_\epsilon(n/2+iy)) = |[1+\epsilon \times \epsilon(p,p_1,y)p_1^{n/2}]^4|_p^{1/4} = O(p^{k_1}) \quad . \tag{48}$$

In cases 2) and 3) the conditions for the reduction of the norm depend explicitely on p_1 only. Also the explicit dependence on y is absent in condition 2) and 3b). This suggests that the proper choice of the sign of the square root $\sqrt{p_1}$ guarantees that the increase of $Z^B_{p_1}$ does not occur for too many values of p_1. The second suggestion is that the sign factor $\epsilon(p,p_1,y)$ is same for most zeros y so that same mechanism saving Riemann Zeta to become too large is at work for very many roots y. The physical interpretation is now that unit state represents the presence of hole.

If is of crucial importance that the product $\epsilon \times \epsilon(p,p_1,y)$ of the sign factors appears in both Eq. 47 and 48. The definition of the sign factor of $p_1 1/2$ third important sign factor. There is freedom to choose the sign of $p_1^{1/2}$ for each p but not for each y. Suppose that the equation allows solution for a proper choice of the sign factor associated with the square root $p_1^{1/2}$. In this case either the norm of Z^B increases or the norm of Z^F decreases. Since Riemann zeta must vanish, the natural strategy is to minimize the value of the p-adic norm of Riemann Zeta at $x = 1/2$ by choosing the sign so that the equation does not allow solution for $x = 1/2$ (at least, for too many values of p_1). This would guarantee the vanishing of the p-adic Riemann Zeta for every value of p since there is infinite number of factors Z_{p_1} for which p-adic norm is smaller than one for every value of the p-adic prime p. The same choice also favours the vanishing of Z^F. It is worth of noticing that in the multiplication of the Pythagorean phases $\epsilon(p,p_1,y)$ changes so that the minimizing conditions cease to be true in general. This is consistent with the fact that zeros do not form a group with respect to the multiplication of phases.

Note that the dependence on y is implicitly involved also in Eqs. 47 and 48 containing no explicit reference to y. The reason is that this equation relates to the special case $r = \pm s + O(p^k)$ in which imaginary part of the Pythagorean phase has p-adic norm smaller than one. This condition selects special subset of primes p_1 into consideration.

a) For $x = 1/2$ Eq. 48 can be written as

$$p_1^{1/2} \quad = \quad -\epsilon \times \epsilon(p, p_1, y) + O(p^{k_1}) \ \ , \ \ k_1 > 0 \ \ . \tag{49}$$

b) For $x = m + 1/2$ this equation can be written as

$$\begin{aligned} x^{2m+1} \quad &= \quad -\epsilon \times \epsilon(p, p_1, y) + O(p^{k_1}) \ \ , \ \ k_1 > 0 \ \ , \\ x = p_1^{1/2} \quad &\neq \quad -\epsilon(p, p_1, y) + O(p^{k_1}) \ \ , \ \ k_1 > 0 \end{aligned} \tag{50}$$

as a function of m.

c) For $x = m$ one has

$$\begin{aligned} x^m \quad &= \quad -\epsilon \times \epsilon(p, p_1, y) + O(p^{k_1}) \ \ , \\ x \quad &= \quad p_1 \ \ . \end{aligned} \tag{51}$$

Thus the problem reduces to that of finding nontrivial solutions to the equation

$$x^m \quad = \quad \pm 1 + O(p^k) \tag{52}$$

in the field of ordinary p-adic numbers. $x^m = 1 + O(p^k)$ has nontrivial solutions only if

$$m|p - 1 \ \ .$$

where $|$ denotes the symbol for 'divides'. The equation $x^m = -1 + O(p^k)$ has solutions only if $x^{2m} = 1 + O(p^k)$ has solutions. Thus the necessary condition in this case is

$$2m|p - 1.$$

Both of these conditions can be satisfied for infinite number of primes p. Since the equations in question holds modulo p only, there are excellent reasons to expect that one can find infinite number of primes p_1 for which $x = p_1^{1/2}$ or $x = p_1$ solves the equation in question. The fact that in complex case the number of solutions to this equations increases with the value of m suggests that this is the case already now. This encourages the conclusion that for at least single p the number of the primes p_1 satisfying the condition is infinite and as a result the p-adic norm of the Riemann Zeta vanishes for some p for $x > 1/2$. This is enough for cancelling the zero of the real Rieman Zeta if the heuristics is correct.

Consider next Eq. 47 which can be written for $x = m + 1/2$ as

$$\begin{aligned} 2x^{2m+1} &= -\epsilon \times \epsilon(p, p_1, y) + O(p^k) \ , \\ 2x &\neq u + O(p^k) \ , \\ x &= p_1^{1/2} \ . \end{aligned} \tag{53}$$

and for $x = m$ as

$$\begin{aligned} 2x^m &= -\epsilon \times \epsilon(p, p_1, y) + O(p^k) \ , \\ x &= p_1 \ . \end{aligned} \tag{54}$$

These equations are of general form $x^m = n + O(p)$ and maximum number of solutions is m.

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[Riemann$_1$] Brief description of Riemann hypothesis can be found at http://www.utm.edu/research/primes/notes/rh.html.

[Riemann$_2$] Links to modern literature about Riemann hypothesis can be found at http://match.stanford.edu/rh/.