0.1 $1/f$ noise and thermalized arithmetic quantum field theory

$1/f$ noise follows automatically from either p-adic or real thermodynamics applied to arithmetic quantum field theory with energies quantized as multiples of $log(p)$, p prime. There are small corrections to $1/f$ spectrum and these reflect directly the distribution of primes. Obviously this serves as a high precision test for the proposed explanation of $1/f$ noise.

0.1.1 Arithmetic quantum field theory with broken conformal symmetry describes critical systems

The hypothesis to be studied is that it that critical systems possses 'number theoretic' conformal invariance possibly broken to some sub-algebra. The generators of the full number-theoretic conformal symmetries are

$$L_q = q^z \frac{d}{dz} \ ,$$

where q is rational number. Commutators satisfy the commutation law

$$[L_{q_1}, L_{q_2}] = log(\frac{q_2}{q_1}) L_{q_1 q_2}$$

respecting multiplication of rationals.

Generators are eigenstates of $L_1 = d/dz$ under commutation. L_1 is analogous to energy (or momentum) since it generates translations. Energy eigenvalues are

$$E = log(q) = \sum_i k_i log(p_i)$$

where k_i are integers which can be also negative. If physical states correspond to integers for which energy is always positive, one has

$$E = \sum_i k_i log(p_i)$$

which is the energy spectrum of arithmetic quantum field theory. Positivity of the spectrum suggests that interpretation as energy rather than momentum is more appropriate. Energies correspond to frequencies by the standard $E = hf$ formula of quantum mechanics.

The generators L_p and $L_{1/p}$ generate the entire algebra by repeated commutations. What is remarkable, is that one obtains infinite hierarchy of symmetry breakings by dropping any subset of generators labelled by some subset of primes. An interesting hypothesis is that arithmetic quantum field theory with symmetry broken in this manner describes some critical systems.

If one assumes $p \simeq 2^k$, k prime, one obtains special kind of breaking of conformal symmetry. In this case the scaled generators

$$\hat{L}_p \equiv \frac{L_p}{log(2)}$$

have energies $\hat{L_1} = k$ so that a general generator $\hat{L}_q$ has integer valued energy. The algebra commutators satisfy in good approximation commutation relations for which

$$log(\frac{q_2}{q_1}) \simeq n_2 - n_1 \quad .$$

This means that one has in a good approximation standard conformal algebra of string models and statistical models of critical systems for which the commutators are proportional to the differences of the integer valued indices identifiable as conformal weights. This seems to provide a deeper symmetry based justification for p-adic length scale hypothesis.

0.1.2 $1/f$ spectrum from p-adic or ordinary thermodynamics

$1/f$ spectrum follows in straightforward manner by applying p-adic or ordinary thermodynamics to arithmetic quantum field theory.

a) The spectrum of frequencies in a mode p is harmonic oscillator spectrum:

$$f_n = nf_0 \times log(p) \quad ,$$

where n is integer identifiable as number of arithmetic bosons. $f_0 = 1/\tau$ is temporal infrared cutoff scale. τ could be interpreted as temporal duration of the cognitive spacetime sheet. Cognitive spacetime sheets give rise to $1/f$ noise which thus becomes direct signature of consciousness as speculated already earlier in the chapter "Biological realization of self-hierarchy" of [**?**]. Around human brain τ could be even of the order of lifetime: cognitive spacetime sheets with this duration make possible long term episodal memories.

b) The average number of particles in the mode p is calculable form p-adic thermodynamics one must assume that

$$H \equiv \frac{L_1}{log(p)}$$

having integer valued spectrum is in the role of Hamiltonian H. In p-adic thermodynamics Boltzmann weight $exp(-\beta H)$, $\beta = 1/kT$, does not exist as such p-adically and one must replace it by power of p which exists under certain constraints on the energy spectrum satisfied in conformally invariant theory:

$$exp(-\beta H) \rightarrow p^{\beta H} \quad .$$

This operator has eigenvalues $p^{\beta n)}$ Inverse temperature β must be positive-integer valued from the requirement that Boltzman weights exist p-adically:

$$\beta = m \ .$$

c) The partition function for mode p is nothing but standard harmonic oscillator partition function

$$Z = 1 + p^m + p^{2m} + ... = \frac{1}{1-p^m} \ .$$

The average value of particle number in mode p is given by

$$\langle n \rangle = \frac{\sum_n np^{nm}}{Z} = p^m + O(p^{2m}) \ .$$

The real counterpart of average particle number is obtained using canonical identification

$$\sum x_n p^n \rightarrow \sum_n x_n p^{-n}$$

mapping p-adic observables to real ones and one obtains

$$\langle n \rangle_R = p^{-m} + O(p^{-2m}) \ .$$

For large primes one obtains in excellent approximation $\langle n \rangle_R = 1/p^m$.

d) This argument applies with minor modification also in real context. In this case one has

$$\langle n \rangle = \frac{\sum_n np^{-\beta n}}{Z} \ , \quad Z = \sum_n p^{-\beta n} \ ,$$

where one has $\beta = f_0/T$. In good approximation one has for small values of temperature

$$\langle n \rangle \simeq p^{-\beta} \ .$$

e) Let us calculate the average number of states $N(f)$ with frequency smaller than f using the approzimate expresssion

$$\frac{dp}{dx} \simeq \frac{1}{u} \ , \quad u = log(x) \ .$$

for the density of primes in the set of reals. One has

$$N(f) = \sum_p \frac{1}{p^\beta} \simeq \int \frac{dx}{x^\beta} \times \frac{1}{u}$$

$$= \int_{log(2)}^{log(p)} du \times exp\left[(1-\beta)u\right] \times \frac{1}{u} \ .$$

For $T = 1$ (hf_0 is the unit of energy) one has

$$N(f) = log\,[log(n)] + \frac{1}{log(n)} = log(\frac{f}{f_0}) \ .$$

From this the average density of states is

$$\frac{dN}{df} = \frac{1}{f} \ .$$

or $\beta \neq 1$ one does not obtain analytic formula. For small deviations rom $T = 1$ one has

$$\frac{dN}{df} = \beta - 1 + \frac{\beta}{f} \ .$$

It is interesting to look how the situation changes when the allowed primes satisfy the constraint given by p-adic lengt scale hypothesis. A natural assumption is that k-adic thermodynamics applies for mode $p \simeq 2^k$, k prime, is the case the previous consideration applies with minor modifications. For $T = 1$ the spectrum becomes

$$\frac{dN}{df} = \frac{1}{flog(\frac{f}{f_0})} \ .$$

There are only logarithmic deviation from $1/f$ spectrum.

To sum up, thermalized arithmetic QFT implies $1/f$ spectrum and deviations from precise $1/f$ form reflect the properties of the distribution of primes. p-Adic thermodynamics implies that $T = 1/m$ are the only allowed temperatures and $T = 1$ corresponds to the highest possible p-adic temperature: note that the calculation of elementary particle masses using p-adic thermodynamics assumes also $T = 1$. A natural interpretation for arithmetic QFT could be as a description of cognitive spacetime sheets.

0.1.3 A possible connection with hydrodynamic turbulence

$1/fnoise$ is present also in atmospheric hydrodynamics. This is one mystery. Second mystery is how macroscopic coherent structures are possible. For instance, El Nino is real mystery from the view point of ordinary hydrodynamics.

In TGD vortices are accompanied by Z^0 magnetic fields whose flux tubes flow parallel to the spiral vortex cores. At the tip of the vortex flux must go somewhere since it is conserved. The only possibility seems to be that it goes to second spacetime sheet. Thus spiral vortices might be associated with what I call wormhole magnetic fields, double-sheeted structures carrying opposite magnetic fields created by the wormholes rotating on the boundaries of second sheet of the double sheeted structure. If second sheet has negative time orientation, its energy is negative and entire structure can have finite time duration. Cognitive spacetime sheet is in question. Wormhole magnetic fields form a fractal hierarchy: spacetime sheets glued to spacetime sheets.

This inspires the following hypothesis: it is wormhole magnetic fields which are described by arithemetic quantum field theory. The mode with energy $log(p)$ corresponds to a definite structure, perhaps smaller spacetime sheet carrying magnetic field glued to the larger sheet. This leads to Selvam's claim. There are structures labelled by primes and thus the distribution of primes is somehow reflected in the dynamics of the system. Wormhole magnetic fields might provide universal description of spiral waves associated with excitable systems. For instance, those appearing at cellular level and in heart.