PROOF OF WHY ALL THE ZEROS OF THE RIEMANN ZETA FUNCTION ARE ON THE CRITICAL LINE RE(*P*)=1/2

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Abstract:

This paper tries to describe a proof about the Riemann (hypo)Thesis.

Index:

1. HYPOTHESIS BECOMES THE THESIS OF RIEMANN

It is known that any non-trivial zero lies in the open strip {*p*∈C: 0<Re(p)<1, with p=a+jb complex number}, which is called the critical strip. The Riemann hypothesis, asserts that any non-trivial zero *p* has Re(*p*)=1/2 or that has a=1/2.
We want to prove this with a reductio ad absurdum since the zeros can never be symmetrical with respect to the critical line Re(*p*)=1/2.

If we have the following four p zeros in the critical strip:

1)a+jb
2)1–a+jb
3)1–a-jb
4)a-jb

(see Figure 1 below at page 7)

From the functional equation:

$$Z(p) = 2^{p}\,\pi^{p-1}\,\sin(\pi p/2)\,\Gamma(1-p)\,Z(1-p)$$

Let's assume, for hypothesis from the functional equation, Z(1-p) = 0.
It should also be that Z(p) = 0.
If it was true we have that:

1)and 3) are related zeros
2)and 4) are related zeros

We must observe that the zeros are NOT symmetrical with respect to the critical line Re(*p*)=1/2, but they are only with respect to the *POINT P (1/2, 0)*.
These four zeros are arranged on the "diagonals" of a rectangle and not its horizontal sides (see Figure 1 below).

We know that another fundamental property of the zeta function is the position of symmetry with respect to the real axis (Im(p) = 0 for the complex plane) of its values and as a result of its zeros.
If we choose a value z = a + jb its value complex conjugate is z* = a-jb.
It is therefore important the equality:

Z(a+jb)=Z*(a-jb)

or also

Z(a-jb)=Z*(a+jb)

This also means that:

Z(a+jb)=c+jd

then it is also

Z(a-jb)=c-jd

If we consider that a + jb is one of the zeros of the zeta function, with c=d=0, we have

Z(a+jb)=0

and then we also have

Z(a-jb)=0

and at the end

Z(a+jb)=Z(a-jb)=0

so that "automatically" if z = a+jb is a zero we also have that z = a-jb is a zero.

If the zeta function has a zero for some value of the variable z, there is also zero z* complex conjugate.
So we have a symmetry of the Riemann zeta function with respect to the real axis Im(p) = 0.

The zeta function has thus always 2 and only 2 two zeros to the complex variable z or equivalently has always double zeros.

We should admit that there are also related zeros 1) and 2) and also related zeros 3) and 4) but this is not possible for the functional equation because:

Z(a+jb) ≠ Z(1-a+jb) and even then Z(a+jb) ≠ Z(1-a+jb) ≠ 0
Z(1-a-jb) ≠ Z(a-jb) and even then Z(1-a-jb) ≠ Z(a-jb) ≠ 0
but not for a=1/2.

because:

Z(a+jb) = c+jd
Z(1-a+jb) = e+jf

If it were that
Z(a+jb) = 0 and then c=d=0 (as seen above)
we cannot even have that
Z(1-a+jb) = 0 because if it were that e=f=0 the zeros 1) and 2) are symmetrical with respect to the line Re(*p*)=1/2, but they are only with respect to the *POINT P (1/2, 0)*.
This depends on the functional equation because i**t is never symmetrical with respect to the critical line Re(*p*)=1/2.**

See figure 2) at page 8 with a numerical example.

Only in the case of complex coniugate:

If it were that
Z(a+jb) = c+jd =0 **and then** c=d=0
we have also that
Z(a-jb)=c-jd = 0

Of course the zeros are always symmetrical respect to the line Im(p) = 0. Consequently, for every non-trivial zero a + ib there is another in a - ib.

The zeta function cannot simultaneously have four zeros of which two complex conjugate pairs as to form the horizontal sides of the rectangle (see Figure 1 below).

We have a contradiction - reductio ad absurdum - and this implies that ONLY for Re (p) = 1/2 is valid that:

Z(1/2+jb)=Z(1/2-jb)=0

In this case all the zeros are symmetrical with respect to the POINT P (1/2, 0) when Re(p) = 1/2 and are symmetrical with respect to the line Im(p) = 0 and it is satisfied in full the properties of the functional equation.
For the construction of the functional equation made by Riemann or simply by its "nature" it will never be symmetrical with respect to critical line Re(p)=1/2.
This applies to all other representations of the Riemann zeta function and from the Dirichlet series which proofs its symmetry one and only with respect to the line Im(p) = 0.

This condition is the only acceptable and is a necessary and a sufficient condition and proves unequivocally that all the infinite zeros of the zeta function can only stay on the critical line Re(*p*)=1/2.

The BIG mistake is made by considering that the zeros are symmetrical with respect to the line Re(*p*)=1/2, but they are symmetrical ONLY with respect to the *POINT P (1/2, 0.)*. They are never symmetrical to the critical line Re(*p*)=1/2.
Four zeros simultaneously is so impossible.

Let's consider again the functional equation:

$$Z(p) = 2^{p} \pi^{p-1} \sin(\pi p/2)\, \Gamma(1-p)\, Z(1-p)$$

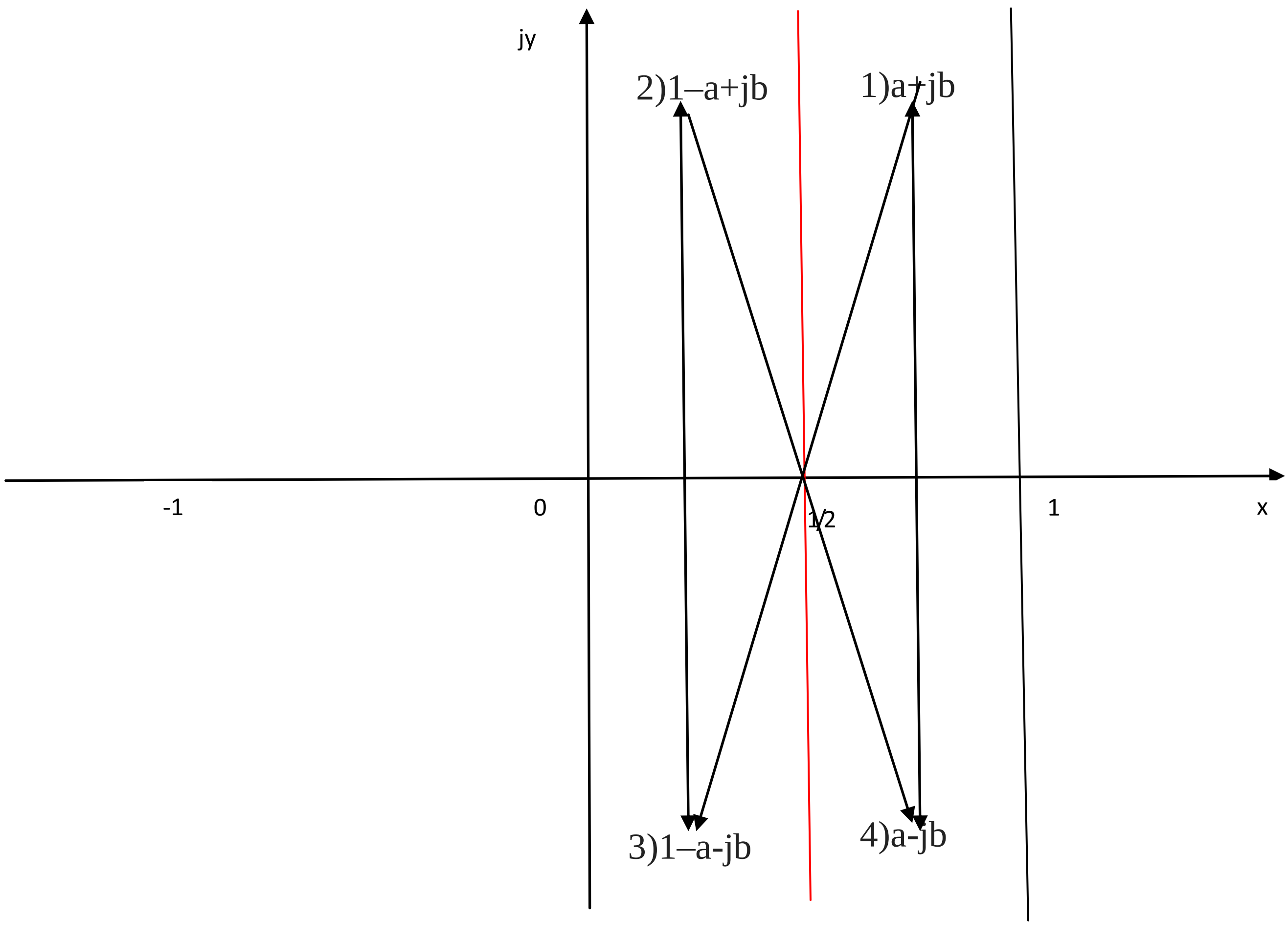
It follows that the Riemann zeta function has only trivial zeros even negative integers and zeros of the form 1/2 ± jb.

Z(-2)=Z(-4)=Z(-6)=Z(-8)=....=0
Z(1/2 ± jb)=0 for certain values of b

So there cannot be zeros p and 1-p, that it was suggested at the beginning of the proof but they do not have to be considered valid.
It applies and it’s true only if Re(*p*)=1/2.

QED

Figure 1:



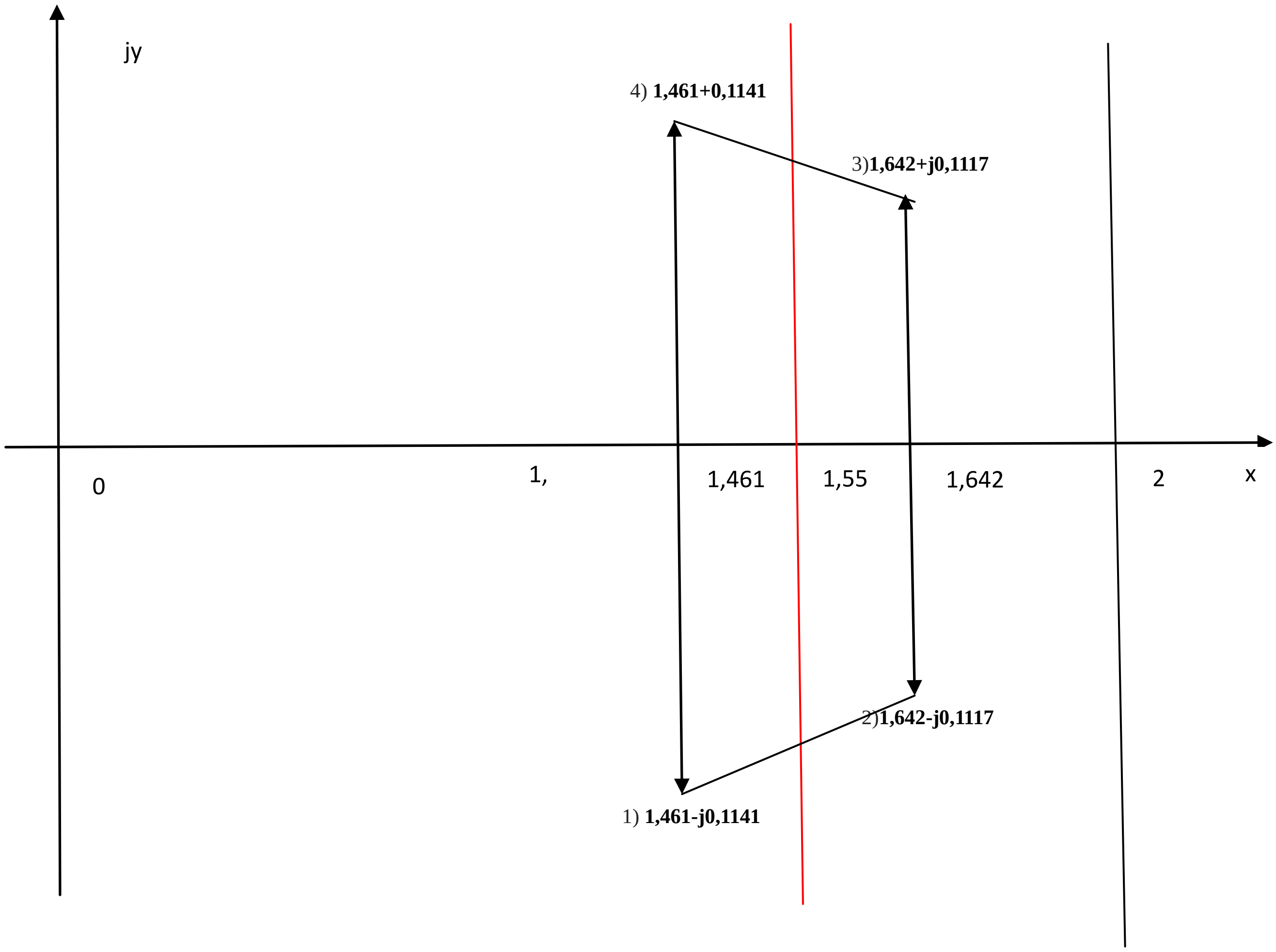
The segments joining the 1)-2) and 3)-4) points are not to be possible, so the only value possible is to have a=1/2 and then all zeros are on the line x=1/2 so that the zeros are symmetrical with respect to the *POINT P (1/2, 0)* when Re(*p*)=1/2 and are symmetrical with respect to the line Im (p) = 0.

Example for a=3/4:

for 1) p=(3/4+j10)

1) **Z(3/4+j10)=1,4614…-j0,1141…**
2) **Z(1/4+j10)=1,6425…-j0,1117…**
3) **Z(1/4-j10)=1,6425…+j0,1117…**
4) **Z(3/4-j10)=1,4614…+j0,1141…**

Figure 2:



The rectangle of Figure 1) always for a=3/4, is projected in an isosceles trapezium. Then the values of the four zeros of Z(p) are never symmetrical with respect to a vertical line. In this case to the line x= **Re(Z(*p*))=** 1,55…
Obviously this is true for any value of a that we choose **because:**

Z(a+jb) ≠ Z(1-a+jb) and even then Z(a+jb) ≠ Z(1-a+jb) ≠ 0
Z(1-a-jb) ≠ Z(a-jb) and even then Z(1-a-jb) ≠ Z(a-jb) ≠ 0

but not for a=1/2 where

Z(1/2+jb)=Z(1/2-jb)=0

for certain values of b, the only valid non-trivial zeros.

2. REFERENCES

1) Bernhard Riemann - (1859) - *Über die Anzahl der Primzahlen unter einer gegebenen Größe*