Computer Based Porosity Design by Multi Phase Topology Optimization

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Abstract. A numerical simulation technique called Multi Phase Topology Optimization (MPTO) based on finite element method has been developed and refined by Fraunhofer IFAM during the last five years. MPTO is able to determine the optimum distribution of two or more different materials in components under thermal and mechanical loads. The objective of optimization is to minimize the component’s elastic energy. Conventional topology optimization methods which simulate adaptive bone mineralization have got the disadvantage that there is a continuous change of mass by growth processes. MPTO keeps all initial material concentrations and uses methods adapted from molecular dynamics to find energy minimum. Applying MPTO to mechanically loaded components with a high number of different material densities, the optimization results show graded and sometimes anisotropic porosity distributions which are very similar to natural bone structures. Now it is possible to design the macro- and microstructure of a mechanical component in one step. Computer based porosity design structures can be manufactured by new Rapid Prototyping technologies. Fraunhofer IFAM has applied successfully 3D-Printing and Selective Laser Sintering methods in order to produce very stiff light weight components with graded porosities calculated by MPTO.

Keywords: topology optimization, finite element method, multi phase distribution, graded porosity, cellular materials, bone structures

INTRODUCTION

The objective of conventional structure optimization methods is generally to produce mechanical components with light weight and high stiffness. Topology optimization methods simulate biological processes (e.g. adaptive bone mineralization) which soften or remove material at low stress locations. There are a lot of successful applications during the last twenty years [1-5].

Multi Phase Topology Optimization [6] does not change the topology or shape of the structure. It finds an optimal spatial distribution of several different materials in order to achieve highest stiffness. One application of Multi Phase Topology Optimization is to produce an optimal porosity design of a mechanically loaded structure similar to the porosity distribution of cancellous bone (Figure 1). Due to manufacturing restrictions in this work porosity design bases upon open-cell foams.
DESCRIPTION OF METHOD

In order to optimize the stiffness of a mechanically loaded structure by a spatial redistribution of several open-cell foams with different porosities \( p \) we assumed porosity dependent elastic properties which can be estimated by Gibson and Ashby [7] where \( E \) is the Young’s modulus, \( \rho \) is the density and \( \nu \) is the Poisson’s ratio of the solid (S) and porous material (*):

\[
\frac{E^*}{E_S} \approx \left( \frac{\rho^*}{\rho_S} \right)^2 = (1 - p)^2
\]

\[
\nu_S \approx \nu^* \approx \frac{1}{3}
\]

If you choose for instance aluminum as solid material you get following properties:

<table>
<thead>
<tr>
<th>Porosity ( p ) (%)</th>
<th>Density ( \rho^* ) (Mg/m³)</th>
<th>Young’s modulus ( E^* ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.70</td>
<td>69.0</td>
</tr>
<tr>
<td>10</td>
<td>2.43</td>
<td>55.9</td>
</tr>
<tr>
<td>20</td>
<td>2.16</td>
<td>44.2</td>
</tr>
<tr>
<td>30</td>
<td>1.89</td>
<td>33.8</td>
</tr>
<tr>
<td>40</td>
<td>1.62</td>
<td>24.8</td>
</tr>
<tr>
<td>50</td>
<td>1.35</td>
<td>17.3</td>
</tr>
<tr>
<td>60</td>
<td>1.08</td>
<td>11.0</td>
</tr>
<tr>
<td>70</td>
<td>0.81</td>
<td>6.2</td>
</tr>
<tr>
<td>80</td>
<td>0.54</td>
<td>2.8</td>
</tr>
<tr>
<td>90</td>
<td>0.27</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The total strain energy \( U \) of a mechanically loaded linear elastic structure is described by

\[
U = \frac{1}{2} \int \varepsilon^T \cdot \sigma \cdot dV = \frac{1}{2} \int \varepsilon^T \cdot D \cdot \varepsilon \cdot dV
\]
where $\varepsilon$ is the local strain tensor, $\sigma$ is the local stress tensor, $D(E, \nu)$ is the local elasticity matrix and $V$ is the volume of the structure [8]. If the mechanical problem is force controlled, the maximum stiffness can be assumed when the strain energy reaches a minimum state:

$$U \rightarrow \min$$  \hspace{1cm} (4)

For a given volume distribution of $n$ different linear elastic materials with properties defined by their elasticity matrices $D_j$ $(j=1,...,n)$ using a finite element model with $m=m_1+m_2+...+m_n$ elements the total strain energy is described by

$$U = \frac{1}{2} \left( \sum_{m_1} e^{T}_{1} \cdot D_{1} \cdot e_{1} \cdot V_{1} + \sum_{m_2} e^{T}_{2} \cdot D_{2} \cdot e_{2} \cdot V_{2} + ... + \sum_{m_n} e^{T}_{n} \cdot D_{n} \cdot e_{n} \cdot V_{n} \right)$$  \hspace{1cm} (5)

In order to minimize $U$ we used following algorithm:

1. Distribute materials $I$ to $n$ randomly to the element sets keeping the given volume fractions.
2. Perform a finite element analysis and calculate the element strain energies $U_1$, $U_2$, ..., $U_m$ by multiplying local strain energy density with element volume.
3. Sort element strain energies $U_i \geq U_j \geq ... \geq U_k$.
4. Redistribute materials $I$ to $n$ by moving the stiff materials to elements with high strain energies taking into account the given volume fractions.
5. Reject some movements by simulated annealing to achieve better convergence.
6. Control total strain energy and stop optimization after a given number of iterations.
7. Go to step 2 and continue.

This method was successfully applied to aluminum foam sandwich structures [6].

**APPLICATIONS**

Conventional topology optimization tools do not work if there is a homogenous stress distribution in the mechanically loaded structure. Figure 2 shows the two-dimensional multi phase topology optimization of a simple beam under uni-axial pressure using a uniform material distribution given by Table 1. Starting with a random spatial material distribution the optimization converges after 25 iterations to a graded structure showing a kind of fiber formation. The orientation of the fibers is parallel to the direction of applied forces.

Figure 3 shows the two-dimensional multi phase topology optimization of a bone structure under bending. The calculated optimal spatial material distribution is very similar to the distribution of real compact and cancellous bone (see Figure 1). Also anisotropy in the inner part of the bone is recognizable.
FIGURE 2. Multi Phase Topology Optimization of a Beam under Uni-axial Pressure
(Top = initial distribution, Bottom = final distribution, Red = 0 %, …, Dark Blue = 90 % Porosity)

FIGURE 3. Multi Phase Topology Optimization of a bone structure under Bending
(Top = initial distribution, Bottom = final distribution, Red = 0 %, …, Dark Blue = 90 % Porosity)
MANUFACTURING CONCEPTS

For two-dimensional porosity design it is possible to map the calculated density distribution (Figure 3) to equivalent pore sizes (Figure 4). Using standard triangulation language files (STL) such structures can be manufactured by conventional rapid prototyping (stereo lithography, selective laser sintering, 3D printing).

![Figure 4: Mapping of Computational Results to a Two-dimensional Porosity Design](image)

For three-dimensional porosity design convenient open cell structures have to be designed for each density level. A standard topology optimization has been used to design an optimal open cell under hydrostatic pressure and shear (Figure 5).

![Figure 5: Three-dimensional Design and Topology Optimization of an Open-cell Foam Structure](image)

Selective Laser Melting can be used to produce open-cell foam structures with a wall thickness less than 0.5 mm. Figure 6 shows titanium samples of computer designed cellular material (Figure 5) produced by Fraunhofer ILT (Aachen, Germany). For using rapid prototyping techniques it is very important to take into account that surplus material has to be removed from the cellular structure. So, not every open-cell porosity design is convenient for manufacturing.
CONCLUSIONS

Multi Phase Topology Optimization produces graded structures with very high stiffness. It shows fiber and anisotropy formation. Porosity design and manufacturing could be successfully applied to two-dimensional structures. Manufacturing concepts for three-dimensional graded structures are in work. First applications will be in the area of medical implants.

REFERENCES