

## 3-by-3 determinants

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

## 4-by-4 determinants

$$\begin{aligned} & \left| \begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array} \right| \\ = & a_{11}a_{22}a_{33}a_{44} + a_{11}a_{23}a_{34}a_{42} \\ & + a_{11}a_{24}a_{32}a_{43} + a_{12}a_{21}a_{34}a_{43} \\ & + a_{12}a_{23}a_{31}a_{44} + a_{12}a_{24}a_{33}a_{41} \\ & + a_{13}a_{21}a_{32}a_{44} + a_{13}a_{22}a_{34}a_{41} \\ & + a_{13}a_{24}a_{31}a_{42} + a_{14}a_{21}a_{33}a_{42} \\ & + a_{14}a_{22}a_{31}a_{43} + a_{14}a_{23}a_{32}a_{41} \\ & - a_{11}a_{22}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44} \\ & - a_{11}a_{24}a_{33}a_{42} - a_{12}a_{21}a_{33}a_{44} \\ & - a_{12}a_{23}a_{34}a_{41} - a_{12}a_{24}a_{31}a_{43} \\ & - a_{13}a_{21}a_{34}a_{42} - a_{13}a_{22}a_{31}a_{44} \\ & - a_{13}a_{24}a_{32}a_{41} - a_{14}a_{21}a_{32}a_{43} \\ & - a_{14}a_{22}a_{33}a_{41} - a_{14}a_{23}a_{31}a_{42} \end{aligned}$$