Vectors and matrices: Problem sheet 3

*Please answer all questions, and **check** your answers whenever possible*
Solutions should be submitted before or on Thursday 18 November 2004

1. In each case find the general solution and one particular solution of the system of linear equations, or show that the system is insoluble.

(i)
$$\begin{array}{rcrcr} 3x & + & 4y & = & 5 \\ 7x & + & 9y & = & 2 \end{array}$$

(ii)
$$\begin{array}{rcrcrcr} 2x & + & 2y & - & z & = & -2 \\ 3x & + & 2y & - & 3z & = & -7 \\ x & + & y & - & z & = & -3 \end{array}$$

(iii)
$$\begin{array}{rcrcrcr} 2x & - & 2y & + & 3z & = & 5 \\ 3x & - & 2y & + & 4z & = & 6 \\ 4x & + & y & + & 2z & = & 2 \end{array}$$

(iv)
$$\begin{array}{rcrcrcr} 3x & + & 2y & + & 13z & = & 2 \\ 4x & + & 2y & + & 16z & = & 1 \\ x & + & y & + & 5z & = & 1 \end{array}$$

(v)
$$\begin{array}{rcrcrcr} 2x & & & + & z & = & 3 \\ x & + & y & + & z & = & 2 \\ x & + & 3y & + & 2z & = & 3 \end{array}$$

(vi)
$$\begin{array}{rcrcrcrcr} 2x & + & 4y & + & z & - & 3t & = & 7 \\ 5x & + & 10y & + & 6z & - & 4t & = & 14 \\ 7x & + & 14y & + & 5z & - & 9t & = & 23 \end{array}$$

(vii)
$$\begin{array}{rcrcrcr} 3x & + & 4y & & & = & 1 \\ 2x & - & 4y & + & 2z & = & 4 \\ 3x & & & + & z & = & 2 \\ x & - & 2y & + & z & = & 2 \end{array}$$

(viii)
$$\begin{array}{rcrcrcrcr} 3x & - & y & + & 2z & + & t & = & 3 \\ x & + & 2y & - & z & + & t & = & -3 \\ 3x & + & y & + & z & + & t & = & 0 \\ 3x & + & 2y & & & + & t & = & -1 \end{array}$$

2. The following matrices are in echelon form. Using elementary row operations, transform each of them into reduced echelon form.

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \qquad \begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & -3 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 2 & 1 \end{pmatrix}.$$

3. In each of the following cases, find **wherever possible**, $A+B$, $A-B$, A^2, AB, BA, B^2, $(A+B)^2$ and $A^2+2AB+B^2$.

(i) $$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix};$$

(ii) $$A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ -3 & 0 \end{pmatrix};$$

(iii) $$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Practice matrix arithmetic with your own choices of matrices until you are completely confident with it.

4. Let A and B be square matrices of the same size. It is not necessarily the case that

$$(A+B)^2 = A^2 + 2AB + B^2. \qquad (*)$$

Give, and prove, a criterion in terms of the products of A and B decribing when equation $(*)$ does hold.

RJC 8/11/2004