Vectors and matrices: Coursework sheet 2

*Please **check** your answers whenever possible*
This sheet counts as 5% of your assessment
Each question counts as 20 marks
Solutions should be submitted to module tutors before or on Monday 10 January 2005

1. Find the general solution of the following system of linear equations.

$$\begin{array}{rcrcrcrcl} 2x & + & 2y & + & 3z & + & t & = & 1 \\ 3x & + & 4y & + & 5z & + & 4t & = & 3 \\ 4x & - & 2y & + & 3z & - & 2t & = & 4 \\ 3x & + & 2y & + & 4z & + & t & = & 2 \end{array}$$

2. Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

   Calculate A^{-1}, A^2, A^3, and A^4 and find a general formula for A^n.

3. Find the inverse of the following matrix.

$$\begin{pmatrix} 2 & 1 & -1 & 2 \\ 3 & 0 & 4 & 5 \\ -5 & -2 & 0 & -5 \\ 1 & 0 & 2 & 3 \end{pmatrix}$$

4. Find the matrix representing an anticlockwise rotation about the origin through $\pi/4$ radians. Hence find the images of

   (a) the ellipse with equation $x^2 + 2y^2 = 1$,
   (b) the parabola with equation $y = x^2$, and
   (c) the hyperbola with equation $x^2 - 2y^2 = 1$

   under this rotation.

5. Calculate, and factorize, the following determinant.

$$\begin{vmatrix} 1 & -1 & 1 & t \\ -1 & 1 & t & 1 \\ 1 & t & 1 & -1 \\ t & 1 & -1 & 1 \end{vmatrix}$$

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