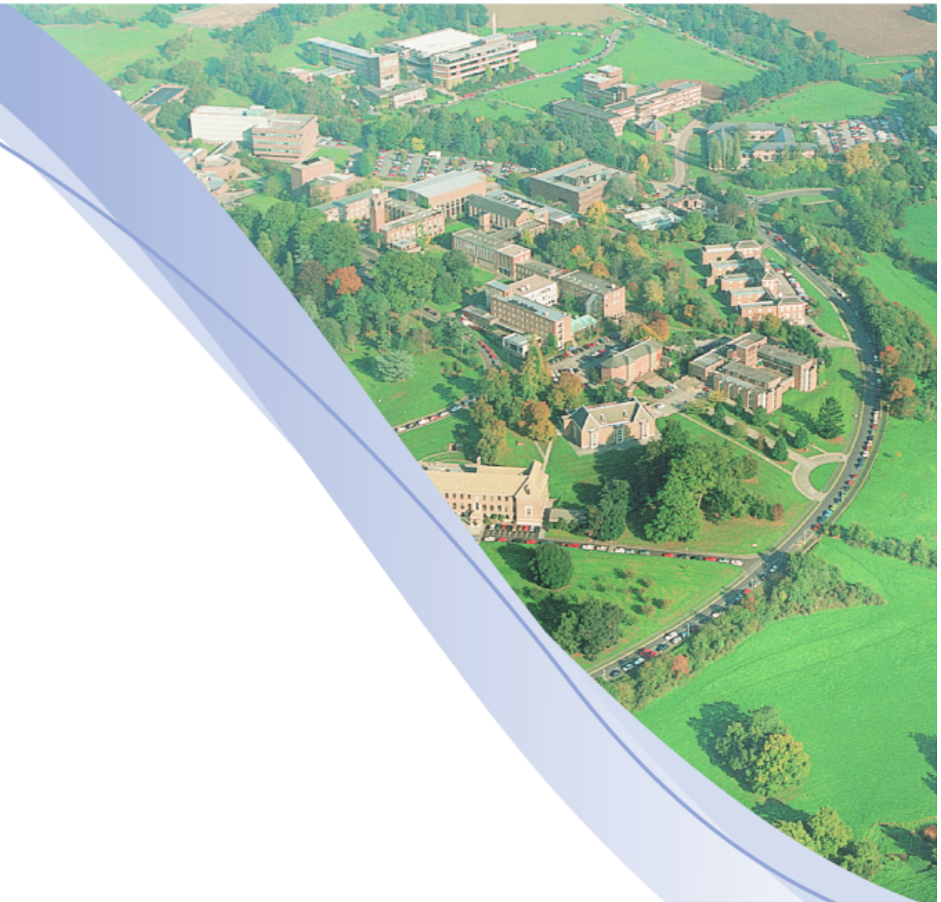


Convection in a porous medium

Tim Jupp

t.e.jupp@exeter.ac.uk

Harrison Room 273

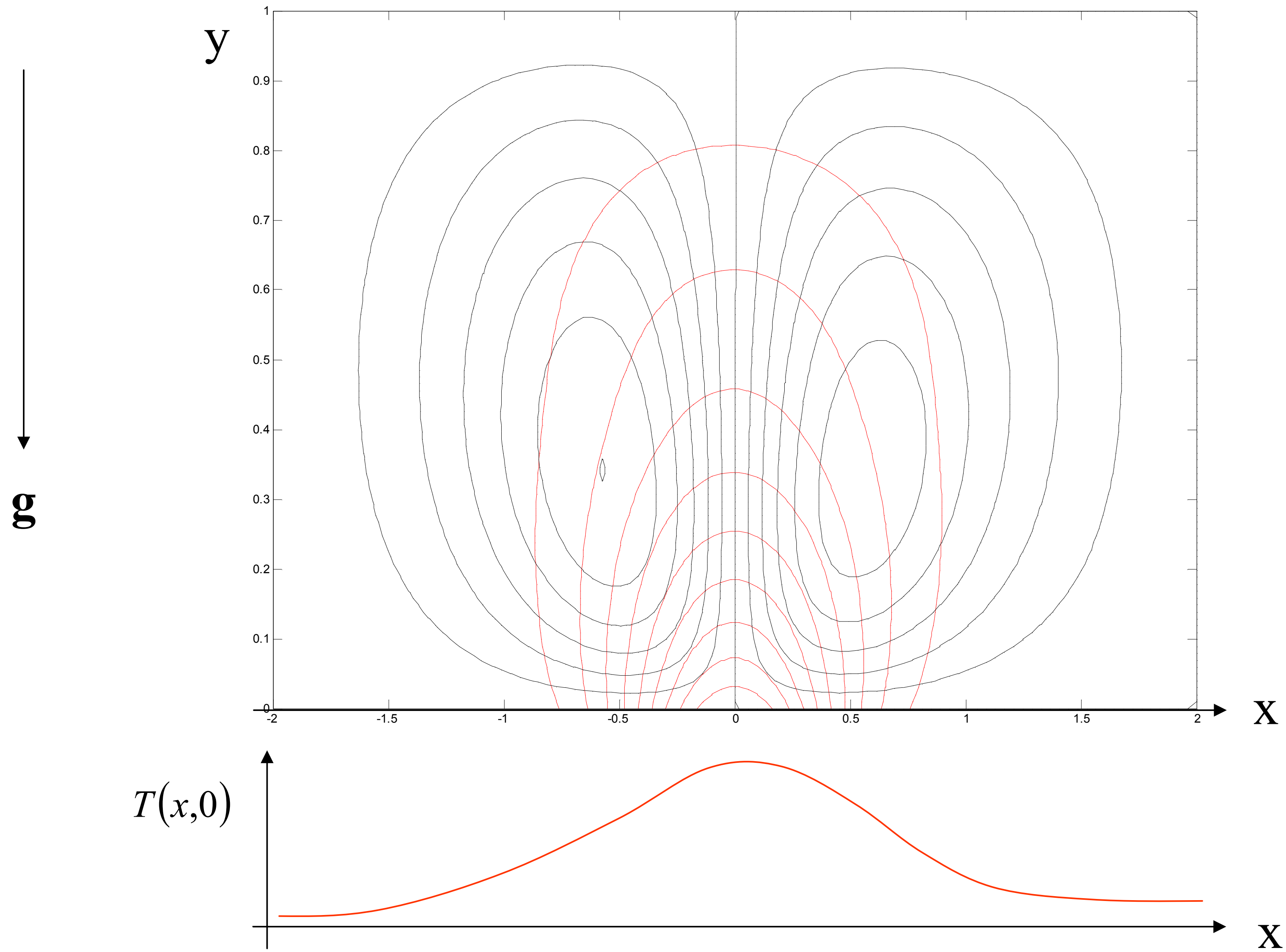
Overview

- Derive governing equations
- Nondimensionalise them
- Solve them numerically
- Investigate linear stability of conductive solution

- Porous medium convection on the web?

Sample matlab output

streamfunction $\psi(x,y)$
temperature $T(x,y)$



Notation (Roman letters)

Symbol	Description	S.I. units
c_p	specific heat capacity	$J \cdot kg^{-1} \cdot K^{-1}$
$D = \dfrac{\lambda}{\rho c_p}$	thermal diffusivity	$m^2 \cdot s^{-1}$
$\mathbf{g}$	gravitational acceleration	$m \cdot s^{-2}$
k	permeability	m^2
L	height of convection cell	m
p	fluid pressure	$Pa = N \cdot m^{-2} = kg \cdot m^{-1} \cdot s^{-2}$
T_0	reference (cold) temperature	K
T	temperature	K
t	time	s
$\mathbf{u}$	Darcy ("transport") velocity	$m \cdot s^{-1}$
x, y	horiz. / vert. coords	m

Notation

Symbol	Description	S.I. units
α	thermal expansivity	K^{-1}
λ	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
μ	dynamic viscosity	$Pa \cdot s$
ρ	fluid density	$kg \cdot m^{-3}$
ρ_0	reference (cold) fluid density	$kg \cdot m^{-3}$
ψ	streamfunction	$m^2 \cdot s^{-1}$

Dimensional governing equations

$$\mathbf{u} = \begin{pmatrix} \dfrac{\partial \psi}{\partial y} \\ -\dfrac{\partial \psi}{\partial x} \end{pmatrix} = \begin{pmatrix} \psi_y \\ -\psi_x \end{pmatrix}$$

conservation of mass

$$\mathbf{u} = -\frac{k}{\mu}\left(\nabla p - \rho \mathbf{g}\right)$$

conservation of momentum

$$\frac{\partial T}{\partial t} = -\mathbf{u} \cdot \nabla T + D \nabla^2 T$$

conservation of energy

$$\rho = \rho_0 \left[1 - \alpha\left(T - T_0\right)\right]$$

constitutive relation

Nondimensionalisation

Dimensional variable	Scaling factor	Dimensionless variable
x, y	L	$\hat{x} = x / L, \hat{y} = y / L$
ψ	$\dfrac{kg\alpha\rho_0 L(\Delta T)}{\mu}$	$\hat{\psi} = \left(\dfrac{\mu}{kg\alpha\rho_0 L(\Delta T)}\right)\psi$
T	$(\Delta T) = T_{\max} - T_0$	$\hat{T} = T / (\Delta T)$
t	$\dfrac{L^2}{D}$	$\hat{t} = \left(\dfrac{D}{L^2}\right)t$

Dimensionless governing equations

$$\hat{\nabla}^2 \hat{\psi} = -\hat{T}_{\hat{x}}$$

mass, momentum

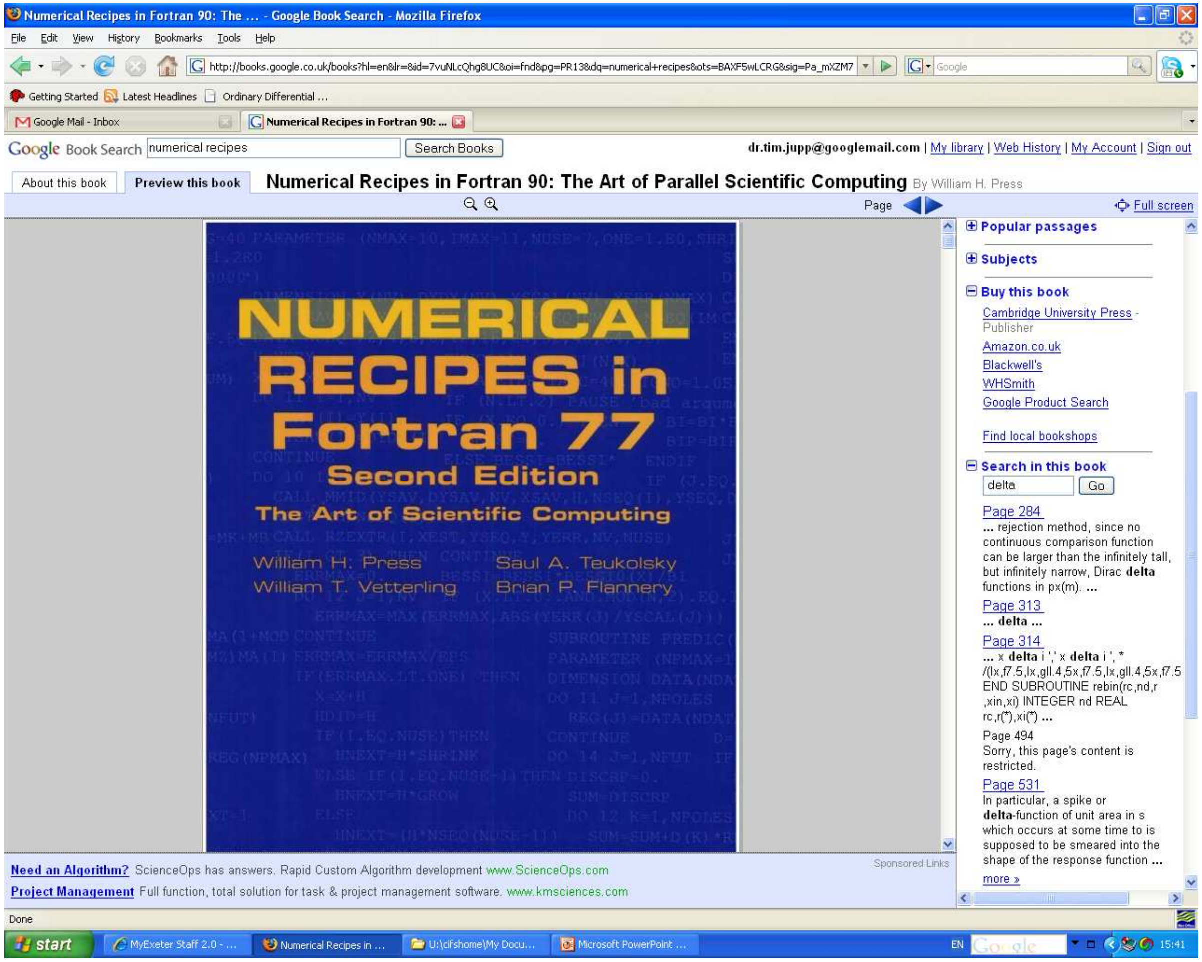
$$\hat{T}_{\hat{t}} = Ra \cdot \left(-\hat{\psi}_{\hat{y}} \hat{T}_{\hat{x}} + \hat{\psi}_{\hat{x}} \hat{T}_{\hat{y}} \right) + \hat{\nabla}^2 \hat{T}$$

Where we have defined the "porous medium Rayleigh number"

$$Ra = \frac{k g \alpha \rho_0 L (\Delta T)}{D \mu}$$

For simplicity, we may from now on omit the "hats" if it is clear that we are referring to dimensionless variables!

Numerical Analysis reference



Numerical Recipes (Press et al.)

University Library <u>or</u> online at

http://www.nrbook.com/a/bookfpdf.php

(in particular chapter 19 on PDEs)

Tasks for next week

- read Lapwood, Horton + Rogers (you are not expected to understand every last detail)
- try to translate between their notation(s) and ours
- (hint 1 darcy = 10^{-12} m^2)
- describe (in words – and not in detail) what they do beyond what we have done