

Extensional and Intensional Spatial Collectives

Antony Galton

School of Engineering, Mathematics and Physical Science
University of Exeter, UK

GIScience Centre, Zürich,
14th May 2013



Synopsis

1. The *De re/De dicto* distinction in Philosophy
2. Extensional and intensional collectives
3. Identification of spatial collectives
4. Tying it all together

The *De re/De dicto* distinction in Philosophy

De re About a thing

De dicto About what is said

De re/De dicto Example 1

Mary wants to marry a doctor

► De re reading

There is a doctor who Mary wants to marry:

$$\exists x(x \text{ is a doctor} \wedge \text{Mary wants}(\text{Mary marries } x))$$

► De dicto reading

Mary wants her future husband, whoever he may be, to be a doctor:

$$\text{Mary wants}(\exists x(x \text{ is a doctor} \wedge \text{Mary marries } x))$$

De re/De dicto Example 2

Jane will marry the richest man in town

► De re reading

As regards the richest man in town, it will be the case that Jane marries him:

$\exists x(x \text{ is the RMIT} \wedge \text{it will happen that}(\text{Mary marries } x))$

► De dicto reading

It will be the case that, as regards the richest man in town, Jane marries him:

It will happen that($\exists x(x \text{ is the RMIT} \wedge \text{Mary marries } x)$)

De re/De dicto Example 3

In five years' time, the poorest man in town will be a millionaire

► De re reading

The x such that x is the poorest man in town is such that it will be the case in five years' time that x is a millionaire.

[He makes his fortune!]

► De dicto reading

It will be the case in five years' time that the x such that x is the poorest man in town is such that x is a millionaire.

[The town becomes so prosperous that even the poorest man is a millionaire.]

Extensional and intensional collectives

Compare:

- (1) In five years' time all the committee will be dead.
- (2) In five years' time all the committee will be female.

More natural readings are:

- (1) The people who are now the members of the committee will, in five years time, all be dead. (*De re*)
- (2) In five year's time all the men currently on the committee will have been replaced by women. (*De dicto*)

Less natural readings are:

- (1) In five years' time the committee will consist entirely of corpses. (*De dicto*)
- (2) All the men currently on the committee will have a sex-change within the next five years. (*De re*)

[Example from Zena Wood and Antony Galton, 'A taxonomy of collective phenomena', *Applied Ontology*, 4:267–292, 2009]

► *De re* readings

- “The committee” picks out a certain plurality of people, and the sentence tracks the changes they undergo over the next five years.
- The sentence is not about the committee as such; it uses the phrase “the committee” as a convenient way of picking out a certain plurality of people.

► *De dicto* readings:

- “The committee” refers to the the committee *as such*, and the sentence tracks the changes it undergoes (as regards membership) over the next five years.
- For the purposes of these readings it is irrelevant who the current members of the committee are.

Two kinds of collective?

► Extensional collective

When a collective noun is read *de re*, it refers to a *bare plurality*, i.e., a particular collection of individuals. The history of the plurality is, essentially, the history of those individuals, and the plurality exists only so long as all its constituent individuals exist. It has fixed membership.

► Intensional collective

When a collective noun is read *de dicto*, it refers to a collection of individuals that satisfy some criterion for membership. At different times, different collections of individuals may satisfy the criterion, and hence the collective so designated has (potentially) variable membership.

Notation for collectives

$e\text{-coll}(S)$	The extensional collective whose members are precisely those of the set S .
$i\text{-coll}_{x,t}(\phi(x, t))$	The intensional collective whose members at time t are precisely the individuals x satisfying the condition $\phi(x, t)$.
$Member(a, X, t)$	Individual a is a member of collective X at time t .
$X \approx_t Y$	Collective X has the same members as collective Y at time t , i.e., $\forall x (Member(x, X, t) \leftrightarrow Member(x, Y, t))$.

We write $e\text{-coll}(a_1, \dots, a_n)$ instead of $e\text{-coll}(\{a_1, \dots, a_n\})$.

Coincidence of collectives

The membership conditions for collectives are given by:

$$\text{Member}(a, e\text{-coll}(S), t) \leftrightarrow a \in S \wedge \text{exists}(a, t).$$

$$\text{Member}(a, i\text{-coll}_{x,t}(\phi(x, t)), t) \leftrightarrow \phi(a, t).$$

If we assume that $\phi(a, t)$ implies $\text{exists}(a, t)$, then we have:

$$\begin{aligned} \text{Member}(a, i\text{-coll}_{x,t}(\phi(x, t)), t) &\leftrightarrow \phi(a, t) \\ &\leftrightarrow \phi(a, t) \wedge \text{exists}(a, t) \\ &\leftrightarrow a \in \{x \mid \phi(x, t)\} \wedge \text{exists}(a, t) \\ &\leftrightarrow \text{Member}(a, e\text{-coll}(\{x \mid \phi(x, t)\}), t) \end{aligned}$$

and hence $i\text{-coll}_{x,t}(\phi(x, t)) \approx_t e\text{-coll}(\{x \mid \phi(x, t)\})$.

Thus at any time, an intensional collective coincides with the extensional collective of its members at that time.

Are there really two sharply distinct kinds of collective?

- ▶ If we only recognise extensional collectives, then the things we would like to describe as “intensional collectives” can be specified as *partial functions* from times to extensional collectives:

$$i\text{-coll}_{x,t}(\phi(x, t)) : t \mapsto e\text{-coll}(\{x \mid \phi(x, t)\}).$$

- ▶ Conversely, if we only recognise intensional collectives, then the things we would like to describe as “extensional collectives” can be defined by setting their membership criteria as set membership:

$$e\text{-coll}(S) = i\text{-coll}_{x,t}(x \in S \wedge \forall y \in S(\text{exists}(y, t))).$$

Thus *all* collectives could be regarded as intensional, with some of them having fixed membership.

Degrees of intensionality

The sequence

- A = $i\text{-coll}_{x,t}(\text{At } t, x \text{ is the prime minister of an EU country})$
- B = $i\text{-coll}_{x,t}(\text{At } t, x \text{ is the prime minister of a country that was in the EU at } 1/1/2000)$
- C = $i\text{-coll}_{x,t}(\text{At } 1/1/2000, x \text{ is the prime minister of an EU country})$
- D = $e\text{-coll}(\{x \mid \text{At } 1/1/2000, x \text{ is the prime minister of an EU country}\})$

seems to run from most to least intensional. Only D is entirely extensional.

Note that all four collectives coincide at 1/1/2000.

Identifying Spatial Collectives

Reference: Zena Wood, *Detecting and Identifying Collective Phenomena within Movement Data*, Ph.D. thesis, University of Exeter, 2011.

Background Assumptions

The data consists of records of the form

$$\langle i, \vec{p}, t \rangle,$$

which states that the individual with identifier i is in position \vec{p} at time t .

Given a data-set of this form, the task is to identify evidence for collective behaviour — to detect the presence of collectives.

Note: Although not explicitly recorded in the data, velocities can be estimated by a formula such as

$$\vec{v}(i, t) = \frac{t^+ - t}{t^+ - t^-} \cdot \frac{\vec{p}(i, t) - \vec{p}(i, t^-)}{t - t^-} + \frac{t - t^-}{t^+ - t^-} \cdot \frac{\vec{p}(i, t^+) - \vec{p}(i, t)}{t^+ - t}.$$

where t^- and t^+ are respectively the latest time earlier than t and the earliest time later than t for which a position is recorded for i .

Spatial Collectives

A spatial collective reveals itself through the spatial properties of its members

The members of a spatial collective tend to exhibit some commonality with respect to their spatial behavior.

The simplest forms of commonality are **common position** and **common motion**.

The commonality can relate to **specific individuals** or **specific values**.

Spatial Collectivity Criteria

	Common position	Common motion
Individual based	PI A set of sufficiently many individuals are sufficiently close to each other sufficiently often.	MI The velocities of a set of sufficiently many individuals are sufficiently close to each other sufficiently often.
Value based	PV At sufficiently many times sufficiently many individuals are sufficiently close to a fixed position.	MV At sufficiently many times the velocities of sufficiently many individuals are sufficiently close to a fixed value.

Thresholds for “Sufficiently”

- ▶ **Cardinality threshold** θ_k , the number of individuals needed to count as “sufficiently many”.
- ▶ **Position proximity threshold** θ_p , such that individuals or positions will count as “sufficiently close” so long as the distance between them is not greater than θ_p .
- ▶ **Motion proximity threshold** θ_v , such that velocities will count as “sufficiently close” so long as they differ by not more than θ_v .
- ▶ **Frequency threshold** θ_f , the number of times a condition must recur in order to count as occurring “sufficiently often”.

Computable formulations of the collectivity criteria

PI There is a set I of individuals and a set T of times such that

1. $|I| \geq \theta_k$
2. $|T| \geq \theta_f$
3. $\forall t \in T \forall x, y \in I (|\vec{p}(x, t) - \vec{p}(y, t)| \leq \theta_p)$

MI There is a set I of individuals and a set T of times such that

1. $|I| \geq \theta_k$
2. $|T| \geq \theta_f$
3. $\forall t \in T \forall x, y \in I (|\vec{v}(x, t) - \vec{v}(y, t)| \leq \theta_v)$

PV There is a position \vec{p} and a set T of times such that

1. $|T| \geq \theta_f$
2. $\forall t \in T (|\{x : |\vec{p}(x, t) - \vec{p}| \leq \theta_p\}| \geq \theta_k)$

MV There is a velocity \vec{v} and a set T of times such that

1. $|T| \geq \theta_f$
2. $\forall t \in T (|\{x : |\vec{v}(x, t) - \vec{v}| \leq \theta_v\}| \geq \theta_k)$

Intermittent vs Continuous Commonality

An additional dimension of variation concerns the distribution of the times T at which commonality holds:

- ▶ **Continuous commonality** occurs when the elements of T form a convex set in the sense that any time represented in the dataset that falls between two members of T must itself be in T .
- ▶ **Intermittent commonality** covers all other cases (but clearly there are degrees of intermittency).

Examples of collectives detectable using the criteria

- PI “Common position, individual-based”. *A choir on tour.*
- MI “Common motion, individual-based”.
A group of associated stars revealed by their common proper motion (e.g., five of the bright stars in Ursa Major form such a group)
- PV “Common position, value-based”. *Tourists crowding round the “Mona Lisa”. (Intermittent example: Crowd gathering to watch Glockenspiel display.)*
- MV “Common motion, value-based”. *Shower of rain (common velocity of the raindrops).*

Other forms of common motion

Some more sophisticated variants of the “common motion” criteria include:

- ▶ Common *pattern* of motion — e.g., traffic on a roundabout.
- ▶ Common *description* of motion — e.g., convergence, divergence.

Example

A school, where there are several classes and several classrooms. Each classroom belongs to one of the teachers, and during the day the classes move from room to room for lessons with different teachers.

An individual class is detectable as a PI collective (and perhaps also MI) whereas a classroom is detectable as the location for a PV collective.

Tying It All Together

In twenty minutes' time the children in room 1 will be in room 2

► De re reading

The children who are in room 1 are such that in twenty minutes' time it will be the case that they are in room 2.

► De dicto reading

In twenty minutes' time it will be the case that the children who are in room 1 are such that they are in room 2.

[Necessarily false!]

In twenty minutes' time the children in room 1 will be two years younger than the ones there now.

► De re reading

The children who are in room 1 are such that in twenty minutes' time it will be the case that they are two years younger than the ones there now. [Necessarily false!]

► De dicto reading

In twenty minutes' time it will be the case that the children who are in room 1 are such that they are two years younger than the ones there now.

What does “The children in room 1” denote?

- ▶ In “In twenty minutes’ time the children in room 1 will be in room 2” the phrase “The children in room 1” has a *de re* reading, and denotes an **extensional** collective, which could be identified from the movement patterns of the children as **type PI**. Ontologically, the collective is **specifically dependent** on its members and the members are **essential parts** of the collective.
- ▶ In “In twenty minutes’ time the children in room 1 will be two years younger than the ones there now” the phrase “The children in room 1” has a *de dicto* reading, and denotes an **intensional** collective, which could be identified from the movement patterns of the children as **type PV**. Ontologically, the collective is **generically dependent** on its members and the members are **inessential parts** of the collective.

Correspondences

READING OF DESCRIPTOR	TYPE OF COLLECTIVE	DETECTION CRITERIA
<i>De re</i>	Extensional	PI, MI
<i>De dicto</i>	Intensional	PV, MV

Some loose ends

- ▶ What about an intensional collective which meets at different locations with replacement of members? Not detectable by any of PI, MI, PV, or MV.
(Not a problem if no natural examples?)
- ▶ What about a collective that is intensional with respect to its members but extensional with respect to the roles they play?
(E.g., a string quartet)
- ▶ Are extensional collectives really collectives anyway? Maybe better to use the word *collections* for these. (But many things we call collections are intensional — a stamp collection, an art collection.)