Instantaneous Events

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1 Introduction

Thinkers of an empiricist frame of mind have always tended to view with suspicion the notion of an instantaneous event (that is, an event strictly lacking in duration), claiming that those events which one might be inclined to regard as instantaneous do in fact last for some possibly very short positive time. Russell (1914, p.121), for example, states that 'events of which we are conscious do not last for a mathematical instant but always for some finite time, however short'. Newton-Smith (1980, p.135) endorses this view with the words 'we have direct experience of events that last for some interval of time and we have no direct experience of anything obtaining for only an instant'.

Arguments against this view have generally pointed to beginnings and endings as paradigm cases of truly instantaneous events, with at least a strong suggestion that these are the only such events that there are. Anscombe (1964), rebutting Russell, notes that 'though we cannot think of an instantaneous event falling within our experience that is not a terminus of something that takes time, we can think of plenty of events that are such termini; and we may perhaps reasonably take such events as what we mean by "instantaneous" ones'. Similarly, (Allen 1984) affirms that 'events that appear to refer to a point in time (e.g., finishing a race) are considered to be implicitly referring to another event's beginning or ending'.

In this paper I am not concerned to take a stand on the question of whether there really are instantaneous events, either in the physical world or in our experience. Rather I take as my starting point the inescapable fact that most systems that have been proposed for the logical analysis of temporal phenomena have the property that, whether or not their proposers acknowledged this, they admit—indeed require—the description of instantaneous events.

Consider, for example, a simple change of state, say a body's starting to move. In a scientific spirit, one might note that an extended body consists of trillions of atoms that are in a ceaseless state of agitation; the body's being at rest amounts to the fact that the vector sum of all these motions, averaged over time, is zero. The averaging over time makes it impossible to pinpoint, even in principle, an instant at which this condition ceases to obtain. Thus it might be claimed that no real meaning can be attached to the assertion that the body's beginning to move is an instantaneous event.

In the context of an exact qualitative logic of time, however, this kind of objection is beside the point. So long as one admits

(a) the notion of one time interval's meeting, or abutting, another, expressed by, say, the predicate $Meets(i, j)$;

(b) the notion of a state's holding over an interval, expressed by $Holds(s, i)$;

(c) the notion of a state's failing to hold within an interval, expressed by $Holds(not(s), i)$

then it is natural to allow formulae of the form

$$Meets(i, j) \wedge Holds(not(s), i) \wedge Holds(s, j)$$

which state, in effect, that the event of s's starting to obtain is instantaneous, occurring at the instant which forms the common boundary of intervals i and j.

To rule out this possibility, one might simply stipulate as an axiom the formula

$$\begin{aligned} i < j \wedge Holds(not(s), i) \wedge Holds(s, j) \rightarrow \\ \exists u(i < u < j \wedge \neg Holds(not(s), u) \wedge \neg Holds(s, u)), \end{aligned}$$

which postulates that between a period of s's wholly failing to hold and a later period of s's holding there must be a 'grey' interval during which s neither holds nor wholly fails to hold. Those who have set up logical systems for describing temporal phenomena have, however, conspicuously refrained from availing themselves of any such axiom. And indeed, even in they did, in most systems one would still end up with the situation expressed by the formula

$$Holds(s, j) \wedge \forall i(Meets(i, j) \rightarrow \neg Holds(s, i))$$

in which we can still recognise an event occurring at the initial instant of i, namely the onset of uninterrupted holding of s. (Before that instant, s could indeed hold, but only in infinitely fine alternation with $not(s)$—a situation that is explicitly ruled out by, for example, Hamblin (1971) and McDermott (1982).)

The kind of instantaneous event discussed in this section—Anscombe's 'termini'—is only one of a variety of phenomena which we shall need to consider under this heading. In the remainder of this paper I shall be taking a close look at a range of phenomena which have some claim to being called instantaneous events. My aim is to present an exhaustinve account of the phenomenon of instantaneity as this appears in those logical models of time that have found favour in the temporal reasoning community.

2 A brief look at events in general

Taken as a whole, events are a pretty heterogeneous bunch. In particular, most of the events which concern us in everyday life are, at least at one level of description, complex; that is, they can be broken down in thought into simpler events composed either sequentially, in parallel or according to a more complex set of ordering constraints. The event of my going for a swim, for example, involves going to the pool or the beach, getting changed, entering the water, spending a while in the water, leaving the water, then getting dried and dressed, and then leaving the pool or beach for wherever I am going next. This is typical in that it can be described as a sequence of simpler events each of which is either a *transition* or a *tenure*. A transition is an event-type that can be specified in terms of its start state and final state: the transition is the change from the former state's holding to the latter. For example, my getting changed is a transition from my being fully clothed to my being in my swimming things, my entering the water is a transition from my being out of the water to my being in it. A tenure is an event-type that can be specified in terms of a single state, the event itself consisting of that state's persisting over a period, e.g., my spending a while in the water.

I conjecture that any complex event can be broken down into simple events that are either transitions or tenures. Now if an instantaneous event is complex, its components must all be instantaneous, and they must occur simultaneously rather than in sequence. We are thus led to consider simple instantaneous transitions and tenures. This will be done in the next section.

Our aim is to specify an event-type by giving its *occurrence conditions*, i.e., necessary and sufficient conditions for the truth of a formula of the form $Occurs(e, t)$, whose intended meaning is that an occurrence of the event-type e takes place at time t—which may in general be either an instant or an interval. For a complete reductive account of a simple event type, we require the occurrence conditions to be composed only of formulae of the form $Holds(s, t)$ and relations on times, e.g., $Meets(i, j)$. (This is exemplified for a certain class of 'motion events' in (Galton 1993).) Our notation will largely follow that of (Allen 1984) as far as intervals are concerned, with natural extensions to instants as suggested in (Vilain 1982) and (Galton 1990).

As noted above, events in general may be divided into *transitions* and *tenures*. We shall refer to transitions using the notation $trans(s_1, s_2)$, where s_1 is a state that holds immediately before the transition and s_2 a state that holds immediately after, it being assumed in general that s_1 and s_2 cannot both hold together. A tenure is specified as the holding of some state, preceded and followed by times at which the state does not hold, and will be notated $ten(s)$, where s is the state whose holding characterises the tenure.

In an interval-based system such as that of Allen (1984), durative (non-instantaneous) forms of these event-types can be specified straightforwardly as follows:

$$\begin{aligned} Occurs(trans(s_1, s_2), i) \quad &\equiv \quad \exists i_1 \exists i_2 (Meets(i_1, i) \wedge Meets(i, i_2) \wedge \\ &\qquad Holds(s_1, i_1) \wedge Holds(s_2, i_2) \wedge \\ &\qquad Holds(not(s_1), i) \wedge Holds(not(s_2), i)) \\ Occurs(ten(s), i) \quad &\equiv \quad \exists i_1 \exists i_2 (Meets(i_1, i) \wedge Meets(i, i_2) \wedge \\ &\qquad Holds(s, i) \wedge Holds(not(s), i_1) \wedge Holds(not(s), i_2) \end{aligned}$$

3 Instantaneous Events

Since Allen's system does not contain instants, it is not possible similarly to specify instantaneous events of any kind, since the occurrence condition for such an event must take the form

$$Occurs(e, t) \equiv \cdots$$

where t denotes an instant. Nonetheless, some instantaneous events must occur in a world which is describable using Allen's system. For example, if we have

$$Meets(i_1, i_2) \wedge Holds(not(s), i_1) \wedge Holds(s, i_2)$$

then an event occurs which we may describe as the inception of state s. The event occurs within any interval which overlaps both i_1 and i_2, but it does not occur *on* any such interval. In fact the only possible candidate for the time at which the event occurs is the instant at which interval i_1 meets interval i_2. Allen & Hayes (1989) allow that this instant exists, in the sense that it can be constructed as a 'nest' of overlapping intervals, but they resolutely refuse to allow properties (what we here call states) to hold at instants, and have little to say about whether they allow events to occur at instants[1].

[1]But note the following: 'Since intervals are the periods during which a proposition is true, the places where they meet another interval are the times when truth-values change.'

Suppose we allow some events to occur at instants. We shall use the predicate $Occurs\text{-}at$ for this. Then we can specify an instantaneous transition by the occurrence condition

$$\begin{aligned} Occurs\text{-}at(trans(s_1, s_2), t) \quad \equiv \quad & \exists i_1 \exists i_2 (Meets\text{-}at(i_1, i_2, t) \wedge \\ & Holds(s_1, i_1) \wedge Holds(not(s_2), i_1) \\ & Holds(s_2, i_2) \wedge Holds(not(s_1), i_2)) \end{aligned}$$

Note that Allen and Hayes should be quite happy with this, since the definiens does not contain any notions absent from their system.

The above occurrence condition does not, of course, say anything about whether or not a particular type of instantaneous transition can occur. We will want to allow that it can occur for some choices of s_1 and s_2, but not for others. For example, if r_1 and r_2 are two regions of space which do not touch, and b is a body small enough to fit inside either region, then the transition

$$trans(in(b, r_1), in(b, r_2))$$

cannot be instantaneous, since in order to pass from the interior of r_1 to the interior of r_2, the body b has to cross over the space between the two regions. Later we will look in more detail at which pairs of states give rise to the possibility of instantaneous transitions.

Given their refusal to allow states to hold at instants, it would seem to be impossible, in the system of Allen and Hayes, to specify instantaneous tenures. Their refusal is admittedly not absolute, for they allow that a proposition might be said to be true at an instant so long as it is true over some interval within which that instant falls. If we use $Holds\text{-}at(s, t)$ to say that a state s holds at an instant t, this may be expressed by the equivalence

$$Holds\text{-}at(s, t) \equiv \exists i(Div(t, i) \wedge Holds(s, i)).$$

Here I use $Div(t, i)$ to mean that the instant t divides or falls within the interval i.

I argued in (Galton 1990) that we must also allow a state to hold instantaneously, that is, to hold at an instant without holding over any interval bounded by that instant. If we do not allow this then we cannot have continuous change. This is because a continuously changing object passes through each state only instantaneously. Thus what we require is the possibility of

$$Holds\text{-}at(s, t) \wedge \forall i(Div(t, i) \rightarrow \neg Holds(s, i)),$$

directly contradicting the Allen and Hayes position.

This will allow us to specify an instantaneous tenure by means of the occurrence condition

$$\begin{aligned} Occurs\text{-}at(ten(s), t) \quad \equiv \quad & Holds\text{-}at(s, t) \wedge \exists i_1 \exists i_2 (Meets\text{-}at(i_1, i_2, t) \wedge \\ & Holds(not(s), i_1) \wedge Holds(not(s), i_2)). \end{aligned}$$

Not all states give rise to the possibility of instantaneous tenure. We will investigate further below the circumstances under which any given state may or may not hold instantaneously.

Instantaneous tenure would appear to be possible in all and only those worlds where continuous change is possible. In fact, I would advocate the following view, which may be regarded as in some ways analogous to the view expressed by Allen and Hayes. They insist that a state may hold at an instant only by virtue of holding over an interval divided by that instant; thereby they reduce $Holds\text{-}at$ to $Holds$. My view, which is motivated by the definition of the derivative in the differential calculus, is that a state may only hold at an isolated instant by virtue of states arbitrarily close to it holding during arbitrarily small intervals divided by that instant.

More exactly, a state can have instantaneous tenure only if it belongs to a continuum of states endowed with the notion of a *neighbourhood*. A neighbourhood of a state s is a state n such that, first, whenever s holds, so does n, and second, any transition from s to $not(n)$ or vice versa must involve n & $not(s)$. Then for $ten(s)$ to occur at t it is necessary that for any neighbourhood n of s, however small, there is an interval i divided by t such that n holds over i.

The paradigmatic case of instantaneous tenure involves a continuous real-valued variable v. Let s be the state $v = v_0$ in which v takes some particular value; for this to have instantaneous tenure at t, it is necessary that for any $\epsilon_1, \epsilon_2 > 0$ there is an interval i divided by t throughout which the state $v_0 - \epsilon_1 < v < v_0 + \epsilon_2$ holds. (This is simply a version of the standard mathematical definition of continuity.) The neighbourhoods of the state $v = v_0$ are all the states $v_0 - \epsilon_1 < v < v_0 + \epsilon_2$, where $\epsilon_1, \epsilon_2 > 0$. It may be that *any* world in which continuous change is possible must admit description in these terms; at any rate this has proved to be a most fruitful paradigm for mathematical physics, which essentially made no progress until mathematics had developed to the point where it could support such a paradigm[2].

An example of a world where continuous change (and hence instantaneous tenure) is not possible is a world where the facts of temporal incidence can be adequately described using Allen's logic. In such a world, whenever a state holds, it holds over an interval. When a state holds over an interval, during that interval the world is unchanging with respect to that state. If on a neighbouring interval a different, incompatible state holds, then the transition from the first state to the second must happen discontinuously. This is the world of Zeno's arrow, which is at rest at each position it finds itself in; if one believes that motion must by nature be continuous, then in such a world motion is impossible, as Zeno stated.

Another kind of world in which there is no continuous change is one in which time is discrete. Any interval can be divided up into a finite collection of *atomic intervals*, that is, intervals which cannot themselves be divided any further. During an atomic interval no change can occur. As with Allen's world, in discrete time instantaneous tenure is not possible. In both worlds, however, instantaneous transitions occur whenever two incompatible states hold over neighbouring intervals.

In the discrete-time world, there is another phenomenon which might be mistaken for an instantaneous tenure but in fact is quite different. This is tenure over an atomic interval. It may be specified by the occurrence condition

$$Occurs(ten(s), i_n) \equiv Holds(not(s), i_{n-1}) \wedge Holds(s, i_n) \wedge Holds(not(s), i_{n+1}),$$

where the atomic intervals are designated $\ldots, i_{-2}, i_{-1}, i_0, i_1, i_2, \ldots$. Allen and Hayes call atomic intervals 'moments'. A tenure over an atomic interval might therefore be called *momentary* rather than instantaneous.

Discrete time could fittingly be described as *cinematographic time*, each frame of a cine-film corresponding to an atomic interval. If the atomic intervals are short enough, humans are unable to distinguish discrete time from continuous time. This is a function of the human nervous system, and it is not necessarily obvious that we should duplicate this inability to discriminate in our logical theory. (But see below, section 7.)

[2]Although an explicit, rigorous formulation of the paradigm did not occur for more than a century after its first appearance—the interval between Newton and Cauchy.

4 Examples of Instantaneous Events

4.1 Instantaneous events in continuous time

We shall illustrate the above ideas by considering the continuous motion of bodies in space. We shall denote by $b, b', b'', \ldots$ various extended physical bodies, and by $r, r', r'', \ldots$ various regions of space. At each instant the body b has a position $pos(b)$, which is exactly the region of space occupied by b at that instant.

Qualitative relations between regions can be can be expressed using the spatial calculus of Randell, Cui & Cohn (1992). We shall use the following predicates taken from their system:

- $DC(r, r')$: r and r' are *disconnected*, i.e., they do not overlap and they do not share any boundary points.
- $EC(r, r')$: r and r' are *externally connected*, i.e., they do not ovelap but they share at least one boundary point.
- $PO(r, r')$: r and r' are *partially overlapping*, i.e., they have a common subregion but neither is wholly contained in the other.
- $r = r'$: r and r' are the same region.
- $TPP(r, r')$: r is a *tangential proper part* of r', i.e., r is wholly contained in r' and shares at least one boundary point with it.
- $NTPP(r, r')$: r is a *non-tangential proper part* of r', i.e., r is wholly contained in r' and shares no boundary points with it.

An example of an event which is always instantaneous is

$$trans(DC(pos(b), pos(b')), EC(pos(b), pos(b'))).$$

This is the event of bodies' b and b' coming into contact; it is expressed by saying that their positions change from being disconnected to being externally connected.

The event $ten(EC(pos(b), pos(b')))$ may be instantaneous but doesn't have to be. If the bodies move apart again as soon as they have touched then it is instantaneous; but if they stay in contact for a while then it is not.

Similarly, the event

$$trans(DC(pos(b), r), PO(pos(b), r)),$$

by which body b moves from being disconnected from a region to being partly inside it, may be instantaneous or not. The only way to get continuously from $DC(pos(b), r)$ to $PO(pos(b), r)$ is by passing through $EC(pos(b), r)$. But this intermediate state may hold instantaneously or not: b may pass straight through this position, or it may stop there before proceeding.

The event

$$trans(EC(pos(b), r), TPP(pos(b), r))$$

cannot be instantaneous. This is the event of a body's entering a region: initially it is just outside the region, and after the event it is just inside it. Since the body is extended it takes time for it to cross the boundary of the region. While the event is in progress the relation $PO(pos(b), r)$ holds.

Similarly, the event

$$ten(DC(pos(b), pos(b'))$$

cannot be instantaneous. This is the event of two bodies' spending a while not touching. Immediately before the event, and immediately after it, the relation $EC(pos(b), pos(b'))$ must hold, i.e., the bodies are touching. The event thus consists of the bodies' breaking contact and moving apart from each other, then moving together again. This cannot happen instantaneously.

Thus we see that we can describe three kinds of events: those which *must* be instantaneous, those which *may* be instantaneous but need not be, and those which *cannot* be instantaneous. We have exhibited both tenures and transitions of the second and third kind, but only a transition of the first; intuitively it seems plausible that no tenure event *has* to be instantaneous, though some may be. In general, we have used our intuitive understanding of continuity to determine which category each of our examples belongs to; what we should like to do is to replace intuition by a systematic method for deriving these results.

4.2 Instantaneous and momentary events in discrete time

We shall take as our model for discrete time the sequence of moves in a game of chess (here we have discrete space as well). We should distinguish at the outset between a game of chess as a mathematical abstraction and its physical realisation as the motion of wooden pieces on a board. Any chess-player knows that the physical pieces and board are unnecessary, and it is perfectly possible for two competent players to conduct a game entirely verbally, while walking down the street, say.

Considered *in abstracto*, the game consists of a discrete sequence of states

$$s_1, s_2, s_3, \ldots, s_n,$$

each state being characterised by the distribution of pieces on the board together with such information as which player has the next move, whether each king has moved (determining whether castling is possible), and so on. Nothing that happens outside this sequence of states has any reality as far as the game is concerned. In particular, states of the game may be considered to follow on from one another without a gap. A move in the game is an instantaneous transition $trans(s_i, s_{i+1})$ from one state to the next in the sequence.

If s and s' are arbitrary game-states, then the event $trans(s, s')$ may or may not be possible, and if it is possible then it may or may not be instantaneous. It will be impossible if, for example, there are more pieces in play in state s' than in state s (pieces cannot be "untaken"). Given that the transition is possible, then it can be instantaneous if it is possible to get from s to s' in a single move, but not otherwise. But even if it *can* be instantaneous, it doesn't have to be: it may be possible to get from s to s' indirectly by a sequence of moves.

As we have already noted, instantaneous tenure is not possible in discrete time; instead we have momentary tenure, when a state persists for just one atomic period. In chess, a player *must* move when it is his turn, and therefore every occurrence of $ten(s)$, where s is a game-state, must be momentary.

We need not consider only complete game-states, however. We can introduce a predicate at which gives the location of a piece on a square. Thus $at(black\text{-}king, C5)$ is the state in which the black king is on square C5. The event $ten(at(black\text{-}king, C5))$ occurs if the black king is moved onto C5 and subsequently moved to a different square. Because of the nature of chess as a two-player game, it happens that events of this kind *cannot* be momentary; if black moves his king to C5 then two moments must elapse before he can move it away again, since it is white's turn to move next. Other kinds of tenure can be momentary, though: for example, the rules of the game dictate that $ten(in\text{-}check(black))$ cannot be other than momentary.

The transition

$$trans(at(black\text{-}king, C5), at(black\text{-}king, D4))$$

can be instantaneous, since it can be accomplished by a single diagonal move of the king; but it need not be, since the king can get from C5 to D4 by way of, for example, C4. On the other hand, the transition

$$trans(at(black\text{-}king, C5), at(black\text{-}king, C2))$$

cannot be instantaneous since it requires at least three moves to accomplish it.

5 Criteria for instantaneity

The general problem confronting us may be stated as follows: given a description of an event-type e in a form such as $trans(s, s')$ or $ten(s)$, how can we determine whether the occurrences of e may, must, or cannot be instantaneous? Let us call this the *instantaneity problem*.

5.1 States of position and states of motion

In (Galton 1990) I proposed a division of states into two categories, called *states of position* and *states of motion*. A state of position holds at the instants which limit any interval throughout which it holds, and it can hold at isolated instants (i.e., it admits of instantaneous tenure); a state of motion, on the other hand, can only hold at an instant by virtue of holding throughout some interval divided by that instant. I pointed out that Allen's interval calculus is adequate for handling states of motion but not for handling states of position.

In (Galton 1993) I used states of position and states of motion to provide a solution to the instantaneity problem for the motion of a rigid body, insofar as this can be described in terms of transitions using the spatial relation predicates of Randell, Cui, and Cohn. The analysis makes use of the notion of *perturbation*: a state s' is a perturbation of state s so long as either of the following cases is possible:

$$Holds(s, i) \wedge Lim(t, i) \wedge Holds\text{-}at(s', t)$$
$$Holds(s', i) \wedge Lim(t, i) \wedge Holds\text{-}at(s, t)$$

where $Lim(t, i)$ says that the instant t is one of the limits (end-points) of interval i. The *perturbation principle* states that for a rigid body b and a region r, and distinct spatial relation predicates R and R', the state $R(pos(b), r)$ can only be a perturbation of state $R'(pos(b), r)$ if one of the two states is a state of motion and the other a state of position.

It is then possible to classify events of the form $trans(R(pos(b), r), R'(pos(b), r))$ with respect to instantaneity as follows:

1. *Necessarily instantaneous events*. R and R' are perturbations of each other (and therefore one is a state of position and the other a state of motion).

2. *Possibly instantaneous events*. R and R' are distinct states of motion, having a common perturbation R'' (which must therefore be a state of position).

3. *Necessarily durative events*. All other cases, i.e., distinct states of motion without a common perturbation, a state of motion and a state of position which are not perturbations of each other, distinct states of position.

5.2 A new classification based on 'dominance'

It was noted in (Galton 1993) that for non-rigid bodies the two-fold distinction between states of position and states of motion is inadequate when we consider the motion of a non-rigid body, the example cited being that $TPP(pos(b), r)$ seems to be a state of position when considered as a perturbation of $NTPP(pos(b), r)$ but a state of motion when considered as a perturbation of $pos(b) = r$. The latter perturbation does not occur, of course, if b is rigid.

I have since found a wide range of examples in which the distinction between states of motion and states of position breaks down. In all these cases, the simple binary division must be replaced by a more complex hierarchical classification based on a special relation of *dominance*. Roughly speaking, state s dominates state s' if s is a perturbation of s' and behaves like a state of position when so considered. A more exact definition is given below.

We shall call state s a *bounding state* of state s' so long as it is possible for s to hold at an instant which bounds an interval throughout which s' holds, i.e., it is possible to have

$$Holds\text{-}at(s, t) \wedge Lim(t, i) \wedge Holds(s', i).$$

Clearly any state is trivially a bounding state of itself so long as it can hold over an interval (which presumably all states can). Our earlier notion of perturbation is simply the symmetric closure of the bounding-state relation, i.e., s and s' are perturbations of each other just if each is a bounding state of the other.

We now say that state s *dominates* state s' so long as s is a bounding state for s' but s' is *not* a bounding state for s. Thus dominance is both irreflexive and asymmetric. In addition, if s dominates s' then s is a perturbation of s', though the reverse implication does not hold.

To illustrate these ideas, we consider a world in which we have various bodies $b_1, b_2, b_3, \ldots$ which are free to move about in space. We shall be interested in which bodies are touching one another at any given time. We shall abbreviate the states $EC(pos(b_i), pos(b_j))$ and $DC(pos(b_i), pos(b_j))$ as $e(i, j)$ and $d(i, j)$ respectively. Assuming the bodies cannot interpenetrate, each of these states is equivalent to the negation of the other.

The case of two bodies is simple: $e(i, j)$ dominates $d(i, j)$. For three bodies, things become more complex. There are altogether 8 distinct states to consider, namely

$$\begin{array}{c}
e(i,j)\ \&\ e(j,k)\ \&\ e(k,l) \\
e(i,j)\ \&\ e(j,k)\ \&\ d(k,l) \\
e(i,j)\ \&\ d(j,k)\ \&\ e(k,l) \\
e(i,j)\ \&\ d(j,k)\ \&\ d(k,l) \\
d(i,j)\ \&\ e(j,k)\ \&\ e(k,l) \\
d(i,j)\ \&\ e(j,k)\ \&\ d(k,l) \\
d(i,j)\ \&\ d(j,k)\ \&\ e(k,l) \\
d(i,j)\ \&\ d(j,k)\ \&\ d(k,l)
\end{array}$$

For brevity, we may refer to these as eee, eed, etc. Consider eed. In this state, b_i is touching b_j, which is touching b_k, but b_i is not touching b_k. Possible pertubations from this state are

- Body b_i comes into contact with body b_k, giving state eee. If eed holds at instant t, then some time must elapse before eee holds, during which b_i and b_k are moving towards each other. This time is limited by the first instant at which eee holds. Thus eee is a bounding state of eed, but not vice versa; so eee dominates eed.

- Body b_j loses contact with b_k, giving state edd. When this happens, there is a last moment at which eed holds, and then edd holds over an interval during which b_j moves away from b_k (and possibly for a longer period thereafter). Hence eed dominates edd.
- Body b_i loses contact with b_j, giving state ded. As above, eed dominates ded.
- Body b_j simultaneously loses contact with both b_i and b_k, giving state ddd. Here eed dominates ddd.

One might suppose that further perturbations are possible, such as 'b_i simultaneously loses contact with b_j and comes into contact with b_k, leading to state dee'. But a little reflection will show that at the instant this happens the state eee must hold: a case of instantaneous tenure. So dee is not a perturbation of eed; rather each is a perturbation of eee, which dominates them both.

The three-body system $\{b_i, b_j, b_k\}$ can be thought of as a combination of three independent two-body systems, namely $\{b_i, b_j\}$, $\{b_j, b_k\}$, and $\{b_i, b_k\}$. These systems are independent because whether two bodies are in mutual contact does not in general depend on their state of contact with other bodies. (In particular cases, of course, dependencies may arise as a result of extra factors not present in the general case: e.g., it is not possible for a person to be simultaneously touching both Nelson's Column and the Eiffel Tower, assuming those monuments to be fixed in their present positions.)

A general rule may be formulated for determining the dominance relations in complex systems which can be decomposed into simpler independent ones. We shall use the notation $s \preceq s'$ to mean that either s dominates s' or $s = s'$. Then the rule is

> Let the system M consist of n subsystems M_1, M_2, ..., M_3 acting independently. For $i = 1, \ldots, n$ let s_i and s'_i be two possible states of M_i. Then state s_1 & s_2 & $\cdots$ & s_n of M is a perturbation of s'_1 & s'_2 & $\cdots$ & s'_n if and only if either $s_i \preceq s'_i$ for $i = 1, \ldots, n$, or $s'_i \preceq s_i$ for $i = 1, \ldots, n$; and in the former case s_1 & s_2 & $\cdots$ & $s_n \preceq s_i \preceq s'_i$, while in the latter case it is the other way about.

The three-body example given above illustrates this rule in operation.

6 Dominance and instantaneity

We are now in a position to state precisely what instantaneous events are possible in continuous time.

The event $trans(s, s')$, where s and s' are incompatible states, is possible only if there exists a sequence $s = s_0, s_1, \ldots, s_n = s'$ such that, for $i = 1, \ldots, n$, s_i is a perturbation of s_{i-1} and incompatible with it. Call this a *transition sequence* from s to s'. Note that in a sense this observation is entirely trivial since an arbitrary transition from s to s' must be characterisable by one of the transition sequences s, s' and $s, -s\ \&\ -s', s'$.

If s is a perturbation of s' we get the case $n = 1$, and a direct transition is possible. For some choices of s and s' no other case occurs, that is, there are no sequences with $n > 1$. In this case $trans(s, s')$ is *necessarily instantaneous*. An example is two bodies coming into contact, i.e., $trans(d(b_i, b_j), e(b_i, b_j))$ in the notation of the previous section.

If s is a perturbation of s' and there also exists a transition sequence with $n > 1$, then the event $trans(s, s')$ is *possibly instantaneous*. An occurrence of the event which involves a direct transition from s to s' will be instantaneous, but one which involves traversing a transition sequence with $n > 1$ will not be. As an example in the three-body world, $trans(eee, ddd)$ can occur instantaneously, if all three bodies lose contact simultaneously, but it can also occur duratively, e.g., by means of the transition sequence eee, eed, edd, ddd.

If s' is not a perturbation of s then $trans(s, s')$ can be instantaneous if and only if there is a transition sequence s, s_1, s', where s_1 dominates both s and s'; and even in this case instantaneity is merely possible and not necessary. An example from the three-body world is $trans(edd, ded)$, for which there exist three minimal transition sequences, namely (1) edd, ddd, ded, (2) edd, eed, ded, and (3) edd, eee, ded. The middle term dominates the outer terms in cases (2) and (3); thus the transition can occur instantaneously. In both cases what happens is that b_i loses contact with b_j at the same instant that it comes into contact with b_k; if sequence (3) is followed then in addition b_k makes fleeting contact with b_j just at the same instant. Note that the transition associated with sequence (1) must be durative, since it involve b_i separating from b_j and subsequently making contact with b_k.

All other transitions must be durative.

We now turn to tenure events. The rule is simple to state. The tenure $ten(s)$ can be instantaneous if and only if there are states s', s'', distinct from s though not necessarily distinct from each other, such that s dominates both s' and s''. Then the transition sequence s', s, s'' can occur, and it is possible (though not necessary) for state s to hold only for an instant: in which case we have an instantaneous occurrence of $ten(s)$. For example, in the two-body world, $ten(e(b_i, b_j))$ can occur instantaneously, but $ten(d(b_i, b_j))$ cannot. In the three-body world, the sequence ddd, edd, ddd affords the possibility of an instantaneous tenure $ten(edd)$; the sequence eed, edd, eed is also possible, but the corresponding occurrence of $ten(edd)$ is necessarily durative.

7 Some other models

I believe that the foregoing considerations exhaust the topic of instantaneous events so long as the model for time is either continuous or discrete. Not all models of time can easily be categorised in this way, however. In particular, many researchers have been attracted to models of time which are not discrete but in which there nonetheless exist atomic intervals. The system of (Allen & Hayes 1989) is of this kind. They define a *moment* as an interval with no internal structure; the motivation is to provide times of occurrence for events such as flashes and bangs which do not possess perceptible duration but which are not instantaneous in the strict sense. What makes their system different from the discrete-time system discussed above is their axiom

$$\forall m \forall n(Moment(m) \wedge Moment(n) \rightarrow \neg Meets(m, n)).$$

Thus moments cannot be contiguous, and time cannot be regarded as discrete. The motivation for this is to allow for different levels of granularity; although I must confess that I find their detailed explanation of how this is achieved in their system rather obscure.

A momentary tenure in this system could be defined using the occurrence condition

$$\begin{aligned} Moment(m) \rightarrow (Occurs(ten(s), m) \quad \leftrightarrow \quad & \exists i \exists j(Meets(i, m) \wedge Meets(m, j) \wedge \\ & Holds(not(s), i) \wedge Holds(s, m) \wedge \\ & Holds(not(s), j))) \end{aligned}$$

Likewise, a momentary transition would be characterised by

$$\begin{aligned} Moment(m) \rightarrow (Occurs(trans(s, s'), m) \quad \leftrightarrow \quad & \exists i \exists j(Meets(i, m) \wedge Meets(m, j) \wedge \\ & Holds(s, i) \wedge Holds(s', j) \wedge \\ & Holds(not(s), m) \wedge Holds(not(s'), m))) \end{aligned}$$

Truly instantaneous events will be defined as in the other systems we have discussed, though it is not at this stage clear how easy it will be to reconcile this particular model for time with the continuity requirements for instantaneous tenure.

An alternative construction in which time is not discrete but there nonetheless exists the notion of a minimal interval is suggested by a psychological account of the subjective present. Many considerations lead to the idea that our experience of the present is not instantaneous but occupies a short interval, such that everything that happens in that interval is present to our consciousness together. This can explain how it is that we perceive motion, so long as it is neither too slow nor too fast, as a phenomenal quality *sui generis* rather than simply as a succession of different positions. On the other hand, it seems implausible to suggest that these subjective presents form a discrete series; one rather thinks of a "window" moving smoothly along a continuum[3].

One way we might attempt to model this is by picking out certain small, but not necessarily minimal, intervals as *moments*. We allow moments to meet or even overlap, and add a density requirement to the effect that given any two moments, there is a third whose beginning falls strictly between the beginning of the earlier moment and the beginning of the later moment. We also ensure that moments are all of essentially the same duration by insisting that whenever moment a begins before moment b begins then it also ends before moment b ends (i.e., the mapping from moment-beginnings to moment-endings is order-preserving).

The resulting model is not a model of the subjective experience of time alone; rather it models the way in which this is superimposed on an underlying physical continuum which contains intervals of arbitrarily short duration. This is very much in the spirit of Russell's attempt to reconcile the psychological account of time with the physico-mathematical one. We can now characterise an event as momentary so long as it falls entirely within one moment. Subjectively, we would perceive two momentary events as simultaneous so long as there is a single moment within which they both occur; this is compatible with their in fact occurring successively.

8 Conclusions

In this paper I distinguished strictly *instantaneous* events, which have zero duration, from *momentary* events which have a positive duration that is in some sense minimal. I further classified events, insofar as their occurrence conditions can be given in terms of the holding or not holding of states, into *transitions*, which are characterised in terms of the states holding immediately before and after the event, and *tenures*, which are characterised in terms of a state holding when the event actually happens, but neither immediately before nor after it. These categories can be considered in relation to both continuous and discrete models of time, resulting in a multiplication of cases to consider.

Precise occurrence conditions were given for all events arising from these considerations, using Allen's well-known interval calculus, extended to allow reference to instants. In addition, the concept of *dominance* was introduced to furnish a criterion for whether or not a given qualitative event type admits instantaneous occurrences.

I further considered, though in less detail, the possibility of a temporal model that is strictly speaking neither continuous nor discrete, instancing as examples Allen and Hayes' *moments* and Russell's account of the relation between the psychological subjective present and an underlying physical time continuum.

[3] Cf. Russell (1914, p.139).

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