Towards an integrated logic of space, time, and motion

Antony Galton
Department of Computer Science
University of Exeter
Exeter, UK
Email: antony@dcs.exeter.ac.uk

ABSTRACT. In this paper we show how Allen's temporal logic, with the modifications suggested by Galton to enable it to accommodate continuity phenomena, can be combined with the spatial logic of Randall, Cui and Cohn to yield a useful framework for reasoning about the motion of a rigid body in space. The idea of a perturbation is introduced as the key to providing a qualitative account of continuity, and a set of axioms is given from which an important result called the Perturbation Principle is derivable. Finally it is shown how the system enables various types of events to be defined in terms of their conditions of occurrence.

1 Introduction

Over the last ten years logical systems for handling time have become commonplace in AI. The interval-based system of (Allen 1984) has become widely adopted in this area. A number of authors have suggested enhancements or modifications to it. My own (Galton 1990) is particularly relevant to the current paper, because it starts to take space, motion, and continuity seriously in a way that Allen did not. (Galton 1991) suggests a reworking of the syntax (unreification), which however I shall not follow here: the reified system here can in principle be unreified as suggested in that paper, but this leads to less wieldy formulae on the whole.

In contrast to time, space has received much less attention. A spatial reasoning system comparable to Allen's temporal logic has, however, been proposed recently by (Randell, Cui and Cohn 1992a) and (Randell, Cui and Cohn 1992b)—henceforth designated individually as RCCa and RCCb respectively, and collectively as RCC.

In the present paper I combine the spatial logic of RCC with my own modification of Allen's temporal logic to give a logic of space, time, and continuous motion.

2 Spatial Regions

RCC derive a set of relations between spatial regions, taking the notion of 'connection' as a primitive. Two regions are regarded as connected to each other if their topological closures are not disjoint. Since I shall have no use for the notion of topological closure, I prefer to say simply that connected regions are those which either overlap or abut one another. In the axiomatic theory, connectedness is stipulated to be reflexive and symmetric.

RCC go on to derive a set of other dyadic relations on regions, of which eight are singled out as constituting a mutually exhaustive and pairwise disjoint set (analogous to Allen's well-known 13 relations on temporal intervals). They are

DisConnected from (DC)
Externally Connected (EC)
Partially Overlaps (PO)
Equals (=)
Tangential Proper Part of (TPP)
Non-Tangential Proper Part of (NTPP)

and the inverses of TPP and NTPP (see Figure 1). For the formal definition of each of these in terms of Connection (C), see the RCC papers.

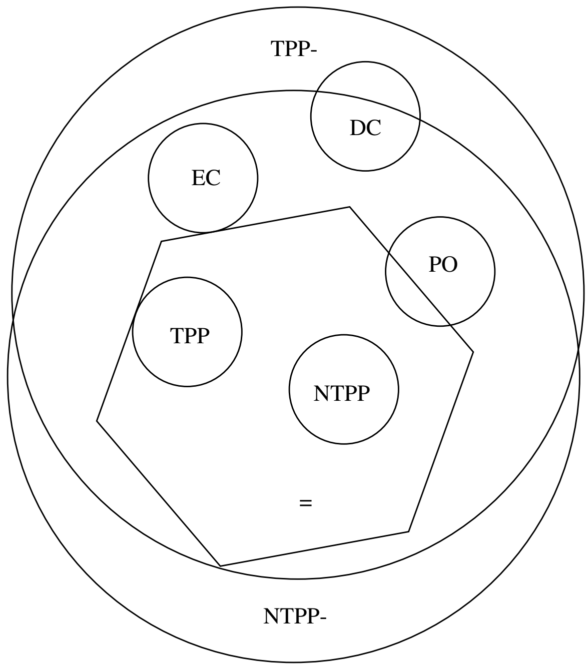

Figure 1: *The eight RCC relations. Each figure is related as stated to the hexagon.*

The exhaustiveness and exclusiveness of the eight relations above can be expressed using the axiom

$$\begin{aligned}&\forall x \forall y (Region(x) \wedge Region(y) \rightarrow \\ &\quad \text{DC}(x,y) \vee \text{EC}(x,y) \vee \text{PO}(x,y) \vee\ x = y \vee \\ &\quad \text{TPP}(x,y) \vee \text{NTPP}(x,y) \vee \\ &\quad \text{TPP}(y,x) \vee \text{NTPP}(y,x))\end{aligned}$$

together with 28 axioms of the form

$$\forall x \forall y \neg (R_1(x,y) \wedge R_2(x,y)),$$

where R_1 and R_2 are two (different) relations from the eight.

3 Spatial points

RCCa *only* admit regions into their class of spatial entities, just as Allen only admits intervals into his class of temporal entities. I argued in (Galton 1990) that he should have admitted points as well; here I argue analogously for the need for spatial points, a possibility explicitly considered, though not actually advocated, by RCCb. Points arise naturally in a number of ways, for example where two regions abut one another (e.g., two mutually tangential spheres or circles), at corners, or where two linear boundaries meet. As I have stressed in the temporal case, it is important not to think of regions as *sets* of points. If one does, one becomes embroiled in empirically meaningless distinctions such as that between topologically closed and topologically open regions. (In mathematics, we can distinguish clearly between the open disc $\{(x, y) \mid x^2 + y^2 < 1\}$ and the closed disc $\{(x, y) \mid x^2 + y^2 \leq 1\}$, but this distinction does not seem to have any application to physical space.)

There are just two relations between points comparable to the 8 region-region relations above: point x is either the same point as y, or a different one.

For relations between points and regions things are a little more interesting; we have, for point p and region r,

p is *inside* r
p *bounds* r
p is *outside* r.

We can take *inside* as primitive and define

$$\begin{aligned} Bounds(p, r) &\equiv_{def} \forall r'(Inside(p, r') \rightarrow \mathrm{PO}(r, r')) \\ Outside(p, r) &\equiv_{def} \neg Inside(p, r) \wedge \neg Bounds(p, r) \end{aligned}$$

4 Bodies in space

From the AI perspective, the point of talking about space is to enable us to talk about things *in* space: our space is populated by extended bodies. For some purposes, it might be convenient to refer to unextended bodies too, e.g., point masses. And in any case, we shall want to refer to things such as corners, which are distinguished points on the surfaces of extended bodies, or centres of gravity.

The RCC papers make no assumptions concerning the rigidity or otherwise of bodies. The resulting highly general system is applicable equally to rigid and to non-rigid bodies. Many bodies featuring in everyday discourse, such as people and animals, are of course non-rigid. But equally, many of them, such as bricks, tables, and houses, are to all intents and purposes rigid. And many objects which have internal moving parts, such as trolleys, toasters, and electric drills, are globally rigid in the sense that their overall shape, as viewed from outside, does not change. There is therefore good reason to investigate the effects of specialising the RCC treatment to rigid bodies. It turns out that the theory of the RCC relations exhibited by rigid bodies has a number of interesting features lacking from the more general account.

At any moment a body occupies a region of space, which we shall call its *position* at that time. By a position for a body I understand just the region of space it takes up, no more and no less[1]. The assumption of rigidity means that any two positions for a given body must be congruent. We shall

[1]For a symmetrical body, the position, in the sense introduced here, is ambiguous, since symmetry allows a body to occupy a position in more than one orientation. For simplicity we ignore this issue here, but acknowledge that a fuller treatment would need to take account of it.

write $pos(b)$ to denote the position of body b. This is a fluent in the sense of the Situation Calculus (McCarthy and Hayes 1969): that is, it can take different values at different times.

In normal discourse, we do not often have cause to refer to the position of a body in this sense; it is more typical to describe a body as being located within a larger region (e.g., John is in the kitchen). In fact, any language has a range of spatial prepositions such as the English *in*, *on*, *at*, etc., whose exact usage is hard to describe formally. For that reason we shall not try to define predicates such as $In(b, r)$ or $On(b, r)$. Instead, we shall describe the position of a body in relation to a region of space using the RCC relations DC, EC, PO, =, TPP, NTPP, TPP^{-1} and NTPP^{-1}.

This will also have the advantage that we can just as easily describe the spatial relationship between two bodies at a given time (as e.g., the ball is in the box).

Given a rigid body b and a region r, the possible positions of b in relation to r are constrained by the particular size and shape of each. There are six possibilities to consider:

1. They are *congruent*: that is, r is a possible value of $pos(b)$.

2. b can just fit inside r: that is, TPP($pos(b), r$) is possible, but not NTPP($pos(b), r$). An example would be a hemispherical body and a spherical region of the same radius.

3. b can just cover r: that is, TPP($r, pos(b)$) is possible but not NTPP($r, pos(b)$), e.g., a spherical body and a hemispherical region of the same radius.

4. b can fit right inside r: that is, NTPP($pos(b), r$) is possible. Example: a spherical body and a spherical region of greater radius.

5. b can more than cover r: that is, NTPP($r, pos(b)$) is possible. Example: a spherical body and a spherical region of smaller radius.

6. None of the above holds: we say b and r are *incommensurate*. Example: a pole 3 feet long and a cubical region measuring 1 foot along each side.

Each of these possibilities defines a subset of the eight RCC relations that can hold between $pos(b)$ and r, as follows:

1. DC, EC, PO, =.

2. DC, EC, PO, TPP.

3. DC, EC, PO, TPP^{-1}

4. DC, EC, PO, TPP, NTPP.

5. DC, EC, PO, TPP^{-1}, NTPP^{-1}.

6. DC, EC, PO.

We mentioned above that we may want to refer to unextended objects such as corners of extended bodies, or idealised point masses. These have positions too, the position $pos(p)$ of a point being a spatial point. The relevant relations here are the point/region relations $Inside$, $Outside$ and $Bounds$, and the point/point relations = and $\neq$.

5 Continuity

We are now in a position to present a qualitative characterisation of continuity. RCCb introduces the notion of a 'direct topological transition' between two RCC relations. No assumption is made concerning rigidity. They allow, for example, that a body's position can be a tangential proper part of a region at one time, and equal to it at a later time. This can only happen if the body increases in size. In the present paper, we restrict RCC's more general notion to rigid bodies. By doing so we are able to exploit an interesting connection with some earlier work on continuity.

We shall say that an RCC relation R' is a *perturbation* of RCC relation R with respect to a given body/region pair (b, r) if *either* it is possible to have $R(pos(b), r)$ holding at an instant t and $R'(pos(b), r)$ holding throughout an interval that is bounded by t, *or* it is possible to have $R(pos(b), r)$ holding throughout an interval i and $R'(pos(b), r)$ holding at an instant which bounds i. We now characterize continuity of position for a body by defining the possible perturbations for each of the RCC relations in which it can stand to a given region.

For example, suppose that TPP$(pos(b), r)$ holds at instant t. Then if position is a continuous function of time, it cannot be the case that over some interval (t, t') we have EC$(pos(b), r)$. Intuitively, in order to get from being inside the region r to being outside it, b has to pass through a position which is partly inside and partly outside r, i.e., if TPP$(pos(b), r)$ holds at t and EC$(pos(b), r)$ holds at a time t' later than t, then it must be that PO$(pos(b), r)$ holds at some time within the interval (t, t'). Here the point is that with respect to a body b which fits inside a region r, PO is a perturbation of both EC and TPP, but EC and TPP are not perturbations of each other.

Continuity demands that if a body/region pair stand in RCC relations $R_1, R_2, R_3, \ldots$ over time, then each R_{i+1} must be a perturbation of the corresponding R_i.

For each type of body/region pair we can construct a *perturbation diagram* which shows the perturbation relation on the set of RCC relations appropriate to that pair type, as follows.

1. DC $\longleftrightarrow$ EC $\longleftrightarrow$ PO $\longleftrightarrow$ $=$
2. DC $\longleftrightarrow$ EC $\longleftrightarrow$ PO $\longleftrightarrow$ TPP
3. DC $\longleftrightarrow$ EC $\longleftrightarrow$ PO $\longleftrightarrow$ TPP^{-1}
4. DC $\longleftrightarrow$ EC $\longleftrightarrow$ PO $\longleftrightarrow$ TPP $\longleftrightarrow$ NTPP
5. DC $\longleftrightarrow$ EC $\longleftrightarrow$ PO $\longleftrightarrow$ TPP^{-1} $\longleftrightarrow$ NTPP^{-1}
6. DC $\longleftrightarrow$ EC $\longleftrightarrow$ PO

For point bodies, the perturbation diagrams are much simpler:

1. p is a point body, q is a spatial point:
$$= \longleftrightarrow \neq$$
2. p is a point body, r is a region:
$$Outside \longleftrightarrow Bounds \longleftrightarrow Inside$$

6 States of Position and States of Motion

In (Galton 1990) I introduced a distinction between what I called states of position and states of motion. The defining characteristics of these two types of state are as follows:

- If s is a state of position, and s holds throughout the interval (t_1, t_2), then s holds at t_1 and at t_2.
- If s is a state of motion, and s holds at the instant t, then there is an interval i containing t such that s holds throughout i.

I pointed out that Allen's interval calculus and the associated *Holds* predicate relating states to intervals over which they hold only works for states of motion; and I proposed an enrichment of the calculus to make it work for states of position too. The enrichment consisted of introducing instants on the same footing as intervals, and three *Holds* predicates *Holds-on*, *Holds-in*, and *Holds-at*, the first two relating states to intervals, the third relating them to instants.

Using $\inf(i)$ and $\sup(i)$ to denote the instants which bound the interval i, and writing $\mathit{Div}(t, i)$ to mean that instant t falls within (and hence divides) the interval i, the characteristic properties of states of position and motion can be expressed as follows:

- For a state of position s,

$$\forall i(\mathit{Holds\text{-}on}(s, i) \rightarrow \\ \mathit{Holds\text{-}at}(s, \inf(i)) \wedge \mathit{Holds\text{-}at}(s, \sup(i))).$$

- For a state of motion s,

$$\forall t(\mathit{Holds\text{-}at}(s, t) \rightarrow \exists i(\mathit{Div}(t, i) \wedge \mathit{Holds\text{-}on}(s, i))).$$

The eight RCC relations neatly split into four states of position and four states of motion, as follows:

States of Position: EC, =, TPP, TPP^{-1}
States of Motion: DC, PO, NTPP, NTPP^{-1}

Comparing this with the perturbation diagrams above, we observe the following rule:

The Perturbation Principle. *Every RCC relation is a perturbation of itself, but otherwise a state of position can only be a perturbation of a state of motion, and vice versa.*

Note that the Perturbation Principle depends for its applicability on the fact that we have restricted our attention to rigid bodies. For non-rigid bodies the classification of RCC relations into states of position and states of motion itself becomes problematic. For example, TPP behaves like a state of position when it undergoes a perturbation to NTPP, but like a state of motion when it undergoes a perturbation to =, as it can in the non-rigid case.

A similar set of relationships hold for the point-body relations. The states of position are = and $Bounds$; the states of motion are $\neq$, $Outside$ and $Inside$, and the Perturbation Principle applies equally in these cases.

7 Axiomatizing perturbations

The perturbation relationships give rise to axioms as follows. Let R be one of the RCC relations, and let its possible perturbations be $R_1, R_2, \ldots, R_n$ (one of these will be R itself, and in any case $n = 2$ or $n = 3$). Then we have

$$\begin{array}{ll}(\text{A1}) & \textit{Holds-on}(R(pos(b), r), i) \rightarrow \\ & \quad \bigvee_{i=1}^{n} \textit{Holds-at}(R_i(pos(b), r), \sup(i)) \\ (\text{A2}) & \textit{Holds-on}(R(pos(b), r), i) \rightarrow \\ & \quad \bigvee_{i=1}^{n} \textit{Holds-at}(R_i(pos(b), r), \inf(i)).\end{array}$$

Note that these axioms are automatically satisfied if R is a state of position, since it is implied by the defining property of such states together with the fact that R is one of the R_i; for states of motion, though, they have to separately postulated.

A kind of converse of the above pair of axioms is

$$\begin{array}{ll}(\text{A3}) & \textit{Holds-at}(R(pos(b), r), t) \rightarrow \\ & \quad \exists t' \bigvee_{i=1}^{n} \textit{Holds-on}(R_i(pos(b), r), (t, t')) \\ (\text{A4}) & \textit{Holds-at}(R(pos(b), r), t) \rightarrow \\ & \quad \exists t' \bigvee_{i=1}^{n} \textit{Holds-on}(R_i(pos(b), r), (t', t)).\end{array}$$

These axioms are automatically satisfied if R is a state of motion, and only need to be separately postulated for states of position.

A third pair of axioms is required to ensure that the set of temporal instants has the right order type. It would be sufficient here simply to stipulate that they are ordered like the real numbers, but this seems a bit heavy-handed. For what we are going to do below, it is enough to lay down the axioms

$$\begin{array}{l}(\text{A5})\ \textit{Holds-on}(s, (t_1, t_2)) \wedge \neg \textit{Holds-at}(s, t_3) \wedge t_2 < t_3 \rightarrow \\ \quad \exists t(\textit{Holds-on}(s, (t_1, t)) \wedge \\ \qquad \forall t'(t < t' \rightarrow \neg \textit{Holds-on}(s, (t, t')))) \\ (\text{A6})\ \textit{Holds-on}(s, (t_2, t_3)) \wedge \neg \textit{Holds-at}(s, t_1) \wedge t_1 < t_2 \rightarrow \\ \quad \exists t(\textit{Holds-on}(s, (t, t_2)) \wedge \\ \qquad \forall t'(t' < t \rightarrow \neg \textit{Holds-on}(s, (t', t_2))))\end{array}$$

We now show that the Perturbation Principle follows from the axioms, together with the defining characteristics of states of motion and states of position.

Suppose we have $\textit{Holds-at}(R(pos(b), r), t)$, where R is a state of motion. Then there is an interval (t, t') such that $\textit{Holds-on}(R(pos(b), r), (t, t'))$. If $R(pos(b), r)$ ever fails to hold after t', then by (A5) there is a latest time t' such that $\textit{Holds-on}(R(pos(b), r), (t, t'))$, in which case we must have, from (A1), $\textit{Holds-at}(R'(pos(b), r), t')$, where R' is a perturbation of R. We cannot have $R' = R$, since this would imply that there is a still later time t'' such that $\textit{Holds-on}(R(pos(b), r), (t, t''))$, contradicting the maximality of t'. So we see that if an RCC state of motion ceases to hold, it must be immediately followed by one of its perturbations.

Now suppose $\textit{Holds-at}(R(pos(b), r), t)$, where R is a state of position. By (A3), there is an interval (t, t') such that $\textit{Holds-on}(R'(pos(b), r), (t, t'))$, where R' is a perturbation of R. If $R' \neq R$ then when R ceases to hold it is immediately followed by one of its perturbations. If $R' = R$, then by (A5), if R ever ceases to hold there is a latest time t' such that $\textit{Holds-on}(R(pos(b), r), (t, t'))$. Since R is a state of position, this implies $\textit{Holds-at}(R(pos(b), r), t')$, and by (A3) there is a time t'' and a perturbation R' of R such that

$$\textit{Holds-on}(R'(pos(b), r), (t', t'')).$$

From the maximality of t', we know that in this case $R' \neq R$, and so once again we have that when an RCC state ceases to hold, it is immediately followed by one of its perturbations.

8 Reasoning with Perturbations

In principle we could use the axioms (A1)–(A6) to perform actual reasoning. However, it is much simpler to use the Perturbation Principle, which as we have seen is a consequence of the axioms.

For example, suppose we know that

$$\mathit{Holds\text{-}at}(\mathrm{DC}(pos(b), r), t_1) \wedge \mathit{Holds\text{-}at}(\mathrm{PO}(pos(b), r), t_2),$$

where $t_1 < t_2$.

Since PO is not a perturbation of DC, to get from DC to PO we have to pass through a sequence of states $\mathrm{DC} \equiv R_0, R_1, \ldots, R_n \equiv \mathrm{PO}$, where each R_{i+1} is a perturbation of R_i. A suitable chain is $\mathrm{DC}, \mathrm{EC}, \mathrm{PO}$, and it is easy to see that any other possible chain must contain this one as a subchain. Hence we know that

$$\exists t(\mathit{Div}(t, (t_1, t_2)) \wedge \mathit{Holds\text{-}at}(\mathrm{EC}(pos(b), r), t)).$$

9 Events

So far, we have only talked about states. Many events can be defined in terms of state changes. This is especially obvious in the case of motion in space. As suggested in (Galton 1990), we shall replace Allen's predicate $Occurs$ by three predicates *Occurs-on*, *Occurs-in*, and *Occurs-at*. If e is an event type then *Occurs-in*(e, i) says that an event of type e occurs during the interval i. If the event takes time, and exactly takes up the whole interval i, then we can write *Occurs-on*(e, i). If, on the other hand, the event is instantaneous (a possibility not countenanced by Allen), and occurs at the instant t, we write *Occurs-at*(e, t).

We can specify event types in terms of changes in respect of the RCC relations. Consider for example the event of one body's *coming into contact with* another. Initially, the positions of the two bodies are disconnected (DC) from each other; after contact they are externally connected (EC). The event of making contact is instantaneous, so we define an instantaneous event type $contact(b_1, b_2)$ by the rule

$$\begin{aligned}&\mathit{Occurs\text{-}at}(contact(b_1, b_2), t) \equiv\\ &\quad \mathit{Holds\text{-}at}(\mathrm{EC}(pos(b_1), pos(b_2)), t) \wedge\\ &\quad \exists t'(\mathit{Holds\text{-}on}(\mathrm{DC}(pos(b_1), pos(b_2)), (t', t)).\end{aligned}$$

Another example is the event of a body's entering a region. There are a number of different events that may be being referred to here, depending on what we regard as the beginning and end points of the event. An obvious one to go for is that for a body to enter a region is for it to undergo a transition between being just outside (EC) it to being just inside (TPP) it; call this event-type $enter(b, r)$. While the event is actually happening, the body is partly in and partly out of the region (PO). Thus we have

$$\begin{aligned}&\mathit{Occurs\text{-}on}(enter(b, r), i) \equiv\\ &\quad \mathit{Holds\text{-}at}(\mathrm{EC}(pos(b), r), \mathrm{inf}(i)) \wedge\\ &\quad \mathit{Holds\text{-}at}(\mathrm{TPP}(pos(b), r), \mathrm{sup}(i)) \wedge\\ &\quad \mathit{Holds\text{-}on}(\mathrm{PO}(pos(b), r), i).\end{aligned}$$

Both of the event-types we have considered are examples of *transitions*[2]. If R_1 and R_2 are RCC relations involving the same body/region pair, we define $trans(R_1, R_2)$ as the type of events in which initially R_1 holds, and finally R_2 holds. The detailed occurrence conditions depend on whether R_1 and R_2 are states of motion or position, and whether R_2 is a perturbation of R_1. These also determine whether the event is durative or instantaneous. An exhaustive list of possibilities is

1. R_1 is a state of position, R_2 is a state of motion, R_2 is a perturbation of R_1.
2. R_1 is a state of motion, R_2 is a state of position, R_2 is a perturbation of R_1.
3. R_1 and R_2 are distinct states of motion, and both have a common perturbation R_3 (which must of course be a state of position).
4. R_1 and R_2 are distinct states of motion without a common perturbation.
5. R_1 and R_2 are distinct states of position.
6. R_1 is a state of position, R_2 is a state of motion, and neither is a perturbation of the other.
7. R_1 is a state of motion, R_2 is a state of position, and neither is a perturbation of the other.

The occurrence conditions for each of these cases are as follows.

1. The event is instantaneous, with occurrence condition

$$\begin{aligned}&\textit{Occurs-at}(trans(R_1, R_2), t) \equiv \\ &\qquad \textit{Holds-at}(R_1, t) \wedge \exists t' \textit{Holds-on}(R_2, (t, t')).\end{aligned}$$

2. This case is just the time reversal of Case 1; the occurrence condition is

$$\begin{aligned}&\textit{Occurs-at}(trans(R_1, R_2), t) \equiv \\ &\qquad \exists t' \textit{Holds-on}(R_1, (t', t)) \wedge \textit{Holds-at}(R_2, t).\end{aligned}$$

3. The event may be instantaneous or durative, depending on whether the body spends more than an instant in the intermediate state of position. This gives us two occurrence conditions

$$\begin{aligned}&\textit{Occurs-at}(trans(R_1, R_2), t) \equiv \\ &\quad \exists t' \exists t'' (\textit{Holds-on}(R_1, (t', t)) \wedge \\ &\qquad\qquad \textit{Holds-on}(R_2, (t, t''))). \\ &\textit{Occurs-on}(trans(R_1, R_2), i) \equiv \\ &\quad \exists i' \exists i'' (\textit{Meets}(i', i) \wedge \textit{Meets}(i, i'') \wedge \\ &\qquad\qquad \textit{Holds-on}(R_1, i') \wedge \textit{Holds-on}(R_3, i) \wedge \\ &\qquad\qquad \textit{Holds-on}(R_2, i''))\end{aligned}$$

   Note that in the first condition we need not include a conjunct $\textit{Holds-at}(R_3, t)$ since this follows from the other conjuncts by the perturbation principle (since a given pair of RCC relations has at most one common perturbation). In the second condition we do need the extra conjunct, though by the perturbation principle we could replace it by $\neg\textit{Holds-in}(R_1, i) \wedge \neg\textit{Holds-in}(R_2, i)$, thus avoiding any explicit mention of R_3.

[2]See (Gooday and Galton September 1992), where however a somewhat different treatment is given of transitions in general.

4. The event must be durative; this case resembles the durative part of the previous case, except that now we cannot explicitly mention the intermediate states, since we do not know how many there are or how often each occurs (for example, to get from NTPP to DC, any of the sequences

   NTPP, TPP, PO, EC, DC
   NTPP, TPP, PO, EC, PO, EC, DC
   NTPP, TPP, PO, TPP, PO, EC, DC
   NTPP, TPP, PO, TPP, PO, EC, PO, EC, DC
   ⋮

   would do). The occurrence condition must therefore be given as

$$\begin{aligned} &\textit{Occurs-on}(trans(R_1, R_2), i) \equiv \\ &\quad \neg\textit{Holds-in}(R_1, i) \wedge \neg\textit{Holds-in}(R_2, i) \wedge \\ &\quad \exists i' \exists i'' (\textit{Meets}(i', i) \wedge \textit{Meets}(i, i'') \wedge \\ &\qquad\qquad \textit{Holds-on}(R_1, i') \wedge \textit{Holds-on}(R_2, i'')). \end{aligned}$$

5. The event must be durative, since to get from R_1 to R_2 we have to pass through at least one intermediate state of motion, which cannot hold just for an isolated instant. The occurrence condition is

$$\begin{aligned} &\textit{Occurs-on}(trans(R_1, R_2), i) \equiv \\ &\quad \textit{Holds-at}(R_1, \mathrm{inf}(i)) \wedge \neg\textit{Holds-in}(R_1, i) \wedge \\ &\quad \neg\textit{Holds-in}(R_2, i) \wedge \textit{Holds-at}(R_2, \mathrm{sup}(i)). \end{aligned}$$

6. The event has to be durative, with occurrence condition

$$\begin{aligned} &\textit{Occurs-on}(trans(R_1, R_2), i) \equiv \\ &\quad \textit{Holds-at}(R_1, \mathrm{inf}(i)) \wedge \\ &\quad \neg\textit{Holds-on}(R_1, i) \wedge \neg\textit{Holds-on}(R_2, i) \wedge \\ &\quad \exists i' (\textit{Meets}(i, i') \wedge \textit{Holds-on}(R_2, i')). \end{aligned}$$

7. This is just a time-reversal of the previous case.

$$\begin{aligned} &\textit{Occurs-on}(trans(R_1, R_2), i) \equiv \\ &\quad \textit{Holds-at}(R_2, \mathrm{sup}(i)) \wedge \\ &\quad \neg\textit{Holds-on}(R_1, i) \wedge \neg\textit{Holds-on}(R_2, i) \wedge \\ &\quad \exists i' (\textit{Meets}(i', i) \wedge \textit{Holds-on}(R_1, i')). \end{aligned}$$

The above list does not exhaust all the kinds of events we will want to refer to; it only handles changes of position of one body with respect to one region. A common kind of event which does not fall under this description is when a body moves from one region to another, say b moves from r_1 to r_2. Assuming that b is extended and fits inside both r_1 and r_2, and that r_1 and r_2 are disconnected from each other, the occurrence condition for this event-type could be given by

$$\begin{aligned} &\textit{Occurs-on}(move(b, r_1, r_2), i) \equiv \\ &\quad \textit{Holds-at}(\mathrm{TPP}(pos(b), r_1), \mathrm{inf}(i)) \wedge \\ &\quad \textit{Holds-at}(\mathrm{TPP}(pos(b), r_2), \mathrm{sup}(i)) \wedge \\ &\quad \neg\textit{Holds-in}(\mathrm{TPP}(pos(b), r_1), i) \wedge \\ &\quad \neg\textit{Holds-in}(\mathrm{TPP}(pos(b), r_2), i). \end{aligned}$$

Here the assumption is that for the event to occur, b must start inside r_1 and end up inside r_2; the relation TPP is required since this will hold when b has *just* entered or is just about to leave a region. One might instead suggest that the relation we should be considering is PO, or EC. For a point body the relation could be either $Bounds$ or $Inside$. Only experience with applying the formalism to particular reasoning tasks will tell us what the most suitable events to define are: what matters is that our system provides a good basis for a wide range of possible definitions.

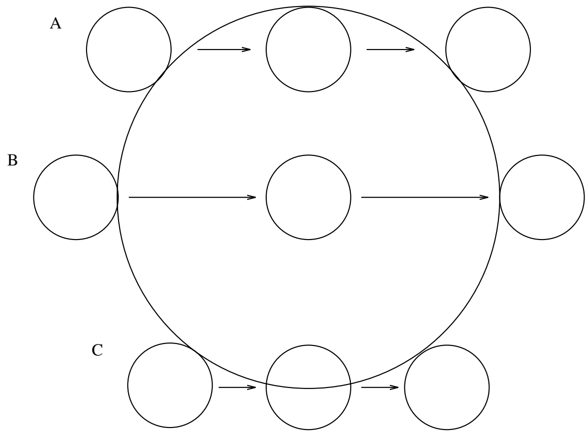

Figure 2: *An extended body passing through a region: three possible trajectories.*

A final example is the event of a body b's *passing through* a region r. Our formalism allows us to consider various different ways of describing this event and to choose the one that best suits our purposes. In figure 2, three possible trajectories are shown. An obvious choice for 'passing through' is to allow trajectories of types A and B, but not of type C. This would give us the occurrence condition

$$\begin{aligned}
&\textit{Occurs-on}(transit(b, r), i) \equiv \\
&\quad \textit{Holds-at}(\text{EC}(pos(b), r), \text{inf}(i)) \wedge \\
&\quad \textit{Holds-at}(\text{EC}(pos(b), r), \text{sup}(i)) \wedge \\
&\quad \neg\textit{Holds-in}(\text{EC}(pos(b), r), i) \wedge \\
&\quad \textit{Holds-in}(\text{TPP}(pos(b), r), i)
\end{aligned}$$

This covers type B as well as type A since by the perturbation principle a body must pass through TPP in order to get from EC to NTPP. If we want to allow trajectories of type C too, then all we need to do is to replace the last conjunct of the occurrence condition by $\neg\textit{Holds-in}(\text{DC}(pos(b), r), i)$.

For a point body p passing through a region r only one type of trajectory is possible, corresponding to the occurrence condition

$$\begin{aligned}
&\textit{Occurs-on}(transit(p, r), i) \equiv \\
&\quad \textit{Holds-at}(Bounds(pos(p), r), \text{inf}(i)) \wedge \\
&\quad \textit{Holds-at}(Bounds(pos(p), r), \text{sup}(i)) \wedge \\
&\quad \textit{Holds-on}(Inside(pos(p), r), i)
\end{aligned}$$

In general, events involving point bodies are simpler to describe than those involving extended bodies since there are fewer RCC-type relations to consider.

10 Conclusion

We took the temporal logic of (Allen 1984), as modified by (Galton 1990), and combined it with the spatial logic of (Randell et al. 1992a, Randell et al. 1992b) to produce a simple but expressive logic of space, time, and continuous motion for rigid bodies.

Continuity was ensured by means of the Perturbation Principle, which makes use of the distinction between states of position and states of motion introduced in (Galton 1990). A set of axioms was given from which the Perturbation Principle follows.

In addition we showed how we could use our system to define a variety of event-types by specifying their occurrence conditions in terms of the elementary positional relations on bodies and regions.

Acknowledgments

I am grateful to John Gooday for critically reading several drafts of this paper, and to the anonymous referees for a number of useful suggestions for improving it.

References

Allen, J. (1984). Towards a general theory of action and time, *Artificial Intelligence* **23**: 123–154.

Galton, A. P. (1990). A critical examination of Allen's theory of action and time, *Artificial Intelligence* **42**: 159–188.

Galton, A. P. (1991). Reified temporal theories and how to unreify them, *Proceedings of the 12th International Joint Conference on Artificial Intelligence*, International Joint Conferences on Artificial Intelligence, Inc., pp. 1177–1182.

Gooday, J. M. and Galton, A. P. (September 1992). A calculus of transitions, *Proceedings of the GWAI-92 Logic and Change Workshop*, Bonn, Germany.

McCarthy, J. and Hayes, P. J. (1969). Some philosophical problems from the standpoint of artificial intelligence, *in* B. Melzer and D. Michie (eds), *Machine Intelligence 4*, Edinburgh University Press.

Randell, D. A., Cui, Z. and Cohn, A. G. (1992a). An interval logic for space based on 'connection', *Proceedings of the Tenth European Conference on Artificial Intelligence*, John Wiley and Sons, Chichester, UK, pp. 394–398.

Randell, D. A., Cui, Z. and Cohn, A. G. (1992b). A spatial logic based on regions and connection, *Proceedings of the Third International Conference on Knowledge Representation and Reasoning*, Morgan Kaufmann, San Mateo, California, pp. 165–176. Cambridge, Massachusetts, October 1992.