Lines of Sight

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Abstract

We develop a calculus of *line-of-sight* relations, that is, the different qualitative relations in which one object in a person's visual field can be positioned relative to another. Examples of such relations are 'partially hides' and 'just hidden by'. We treat the system of such relations in a way that is analogous to the treatment of temporal intervals by (Allen 1984) and (Freksa 1992*a*), and of spatial regions by (Randell, Cui & Cohn 1992). We give the conceptual neighbourhood diagram for line-of-sight relations and show how to generate the composition table from the composition tables for two independent simpler relations whose direct product gives rise to the complete set of line-of-sight relations. The line-of-sight calculus is presented as a testing ground for qualitative reasoning techniques developed in the temporal and spatial domains, and is also proposed as having potential application to the field of computer vision.

Line-of-sight relations

One way of describing the relative positions of objects is an *observer-centred* one: that is, we describe how the objects' positions look from a particular viewpoint. In the absence of depth cues, the three-dimensional scene appears as if it were laid out on the surface of a sphere centred on the observer (or in practice a hemisphere above the ground as diameter plane). The clearest case of this is the night sky, in which the real distance of the sun, moon, stars and planets is not manifest to the ordinary observer.

Depth cues in ordinary vision are provided by binocular parallax, by occlusion, by atmospheric effects, and by knowledge of the real size of objects in the field of view. In what follows we shall concentrate solely on occlusion. Consider two convex objects A and B freely moving within the field of view of an observer. We can distinguish the following 14 qualitatively distinct configurations, called *line-of-sight relations*:

C	A is clear of B
JC	A is just clear of B
PH	A partially hides B
PHI	A is partially hidden by B
JH	A just hides B
JHI	A is just hidden by B
H	A hides B
HI	A is hidden by B
EH	A exactly hides B
EHI	A is exactly hidden by B
F	A is in front of B
FI	A has B in front of it
JF	A is just in front of B
JFI	A has B just in front of it

(Note that the relation designated 'XI' is always the inverse of the relation designated 'X'.) The observer's view of 12 of these relations is portrayed in Figure 1. The two not portrayed are EH and EHI: in these cases the regions of the visual field subtended by A and B are identical; in the former case A is closer to the observer, in the latter B is.

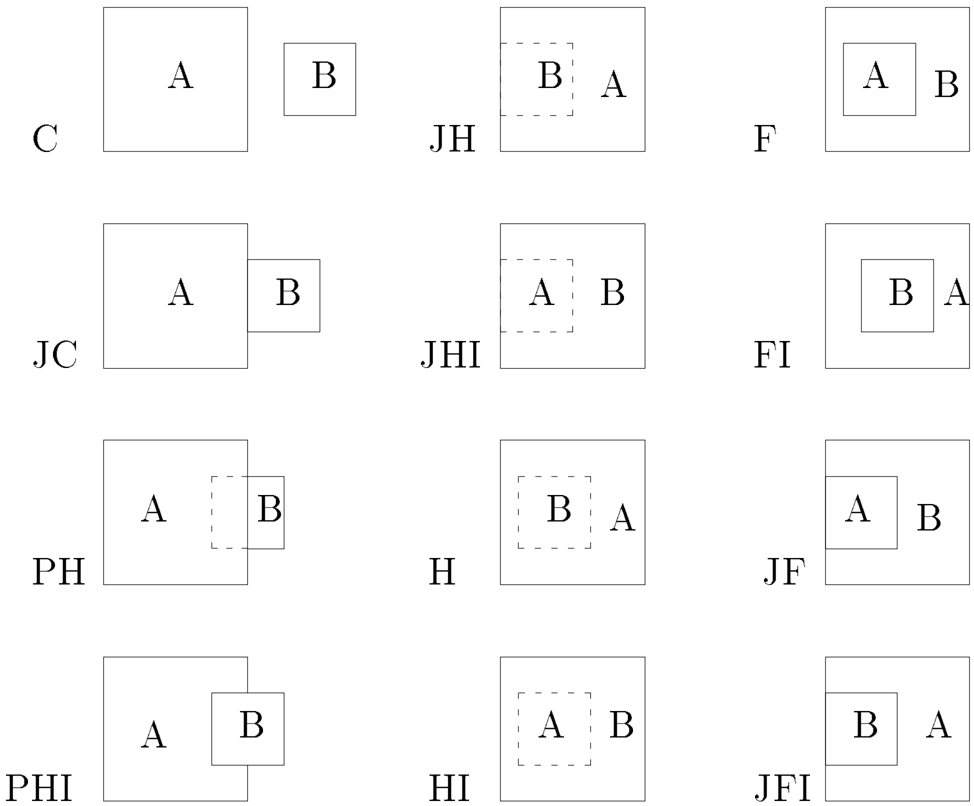

Figure 1: *Twelve of the 14 line-of-sight relations.*

An alternative way of portraying these relations is with A and B fixed and the observer free to move. This will give diagrams such as the one shown in Figure 2. Here regions, lines or points are labelled with the line-of-sight relation in which A stands to B when viewed from a point within the region, line or point so labelled.

Note that, in general, for a given A and B not all 14 relations will be possible. For example, F, JF and EH are only possible if A is smaller than B, while FI, JFI and EHI are only possible if A is bigger than B. It is the former case that is portrayed in Figure 2. The relations EH and EHI are only possible if there is a viewpoint from which A and B have the same shape. Thus a motor-car can never exactly hide or be hidden by an elephant, though either of them can hide or just hide the other. If A and B are the same size, the 14 relations are reduced to 8. The only way in which all 14 relations can occur for two given objects is for at least one of the objects to be non-rigid.

It should also be noted that the set of 14 relations is only complete on the assumption that the bodies in question are convex. For two concave bodies, further relations are possible in general: in particular, it is possible for each body to partially hide the other. This suggests a natural further development of the present work, but we are not at present able to report on this.

Conceptual neighbourhood

Freksa (1992*a*) defines two relations between pairs of intervals or events to be *conceptual neighbours* if they can be directly transformed into one another by continuously deforming the events in a topological sense. This concept generalises readily to many other kinds of relations, in particular to our line-of-sight relations. With the help of Figure 2, we can construct the *neighbourhood diagram* for the 14 relations. This is shown in Figure 3. This diagram shows, for instance, that to move from a position in which H holds to one in which F holds, one can either go directly through EH or take a more roundabout path such as H–JH–PH–JF–F.

Not all paths in the diagram can be followed if one assumes the objects are fixed and only the observer is free to move. For example, the path PH–JC–PHI requires A to move away from the observer, actually touching B at one point in order to get behind it without at any point moving clear of it from the observer's point of view. For fixed objects (as in Figure 2), the only way of getting from PH to PHI is along the path PH–JC–C–JC–PHI.

The arrows in the neighbourhood diagram refer to the *dominance* relations, which help to determine the temporal nature of transitions between the relations (Galton 1993, Galton 1994). Suppose relation R holds throughout some interval of time i but not necessarily at the endpoints of the interval. Then another relation R' can only hold at one of the endpoints of i if R' dominates R. In particular, we may wish to consider the case of a relation holding for only an isolated instant: say interval i_1 meets interval i_2 at the instant t (which is therefore a common endpoint of i_1 and i_2), and suppose R_1 holds over i_1, R_2 over i_2, and R' just at the instant t. Then R' must dominate both R_1 and R_2.

From the neighbourhood diagram we see, for example, that JH dominates both H and PH.

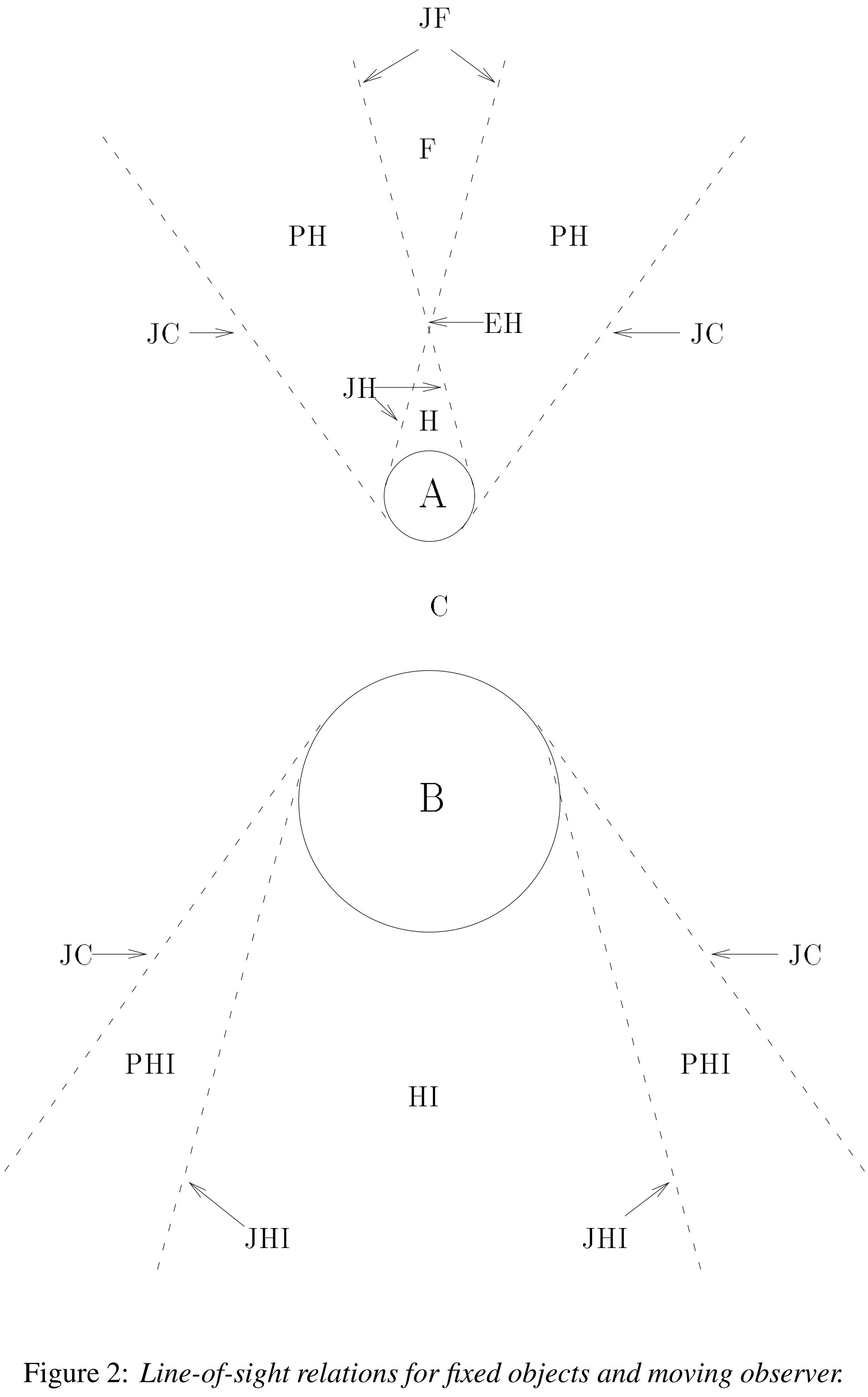

Figure 2: *Line-of-sight relations for fixed objects and moving observer.*

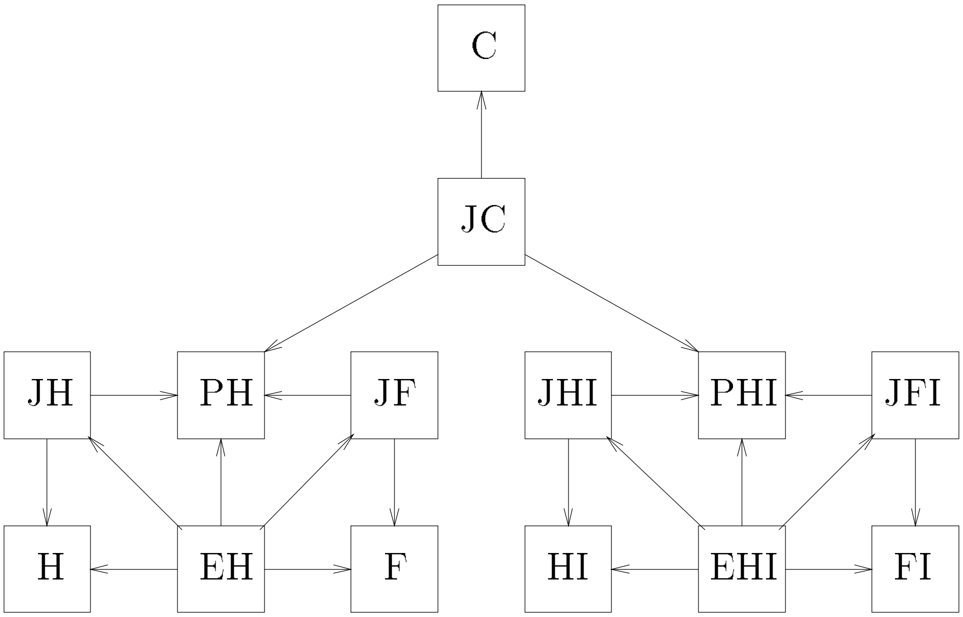

Figure 3: *Neighbourhood diagram for the 14 line-of-sight relations.*

It should therefore be possible for the relation JH to hold instantaneously, in the course of a transition from H to PH. A glance at Figure 2 shows that this must indeed be so, the instantaneous occurrence of JC corresponding to the observer's crossing of the boundary between the H and PH regions. On the other hand JH cannot hold instantaneously in the course of a transition from EH to PH.

Composition of line-of-sight relations

The system outlined above will no doubt remind the reader of the Interval Calculus of Allen (1984) and the spatial calculus of Randell, Cui & Cohn (1992). It is indeed in very much the same mould. As with those calculi, the line-of-sight relations can be composed, giving rise to a *composition table*, which effectively reduces many reasoning tasks over the particular domain to table look-up[1].

For example, suppose that from a particular viewpoint, A hides B, which partially hides C. What are the possible relations between A and C? (The supposition here is that B has now been removed, enabling us to consider how just A and C look from the viewpoint in question.) In Figure 4 we see that there are three possibilities, labelled C1, C2, and C3, corresponding to the relations PH, JH and H respectively. We can write

$$\mathrm{H} \circ \mathrm{PH} = (\mathrm{PH}, \mathrm{JH}, \mathrm{H}).$$

This set of relations constitutes a *conceptual neighbourhood*—a connected subset of the

[1]For a complexity analysis, see van Beek, Kautz & Vilain (1986). For the utility of approaches to qualitative reasoning using composition tables, see for example Allen, Kautz, Pelavin & Tenenberg (1991), Randell, Cohn & Cui (1992*b*), and Freksa (1992*b*).

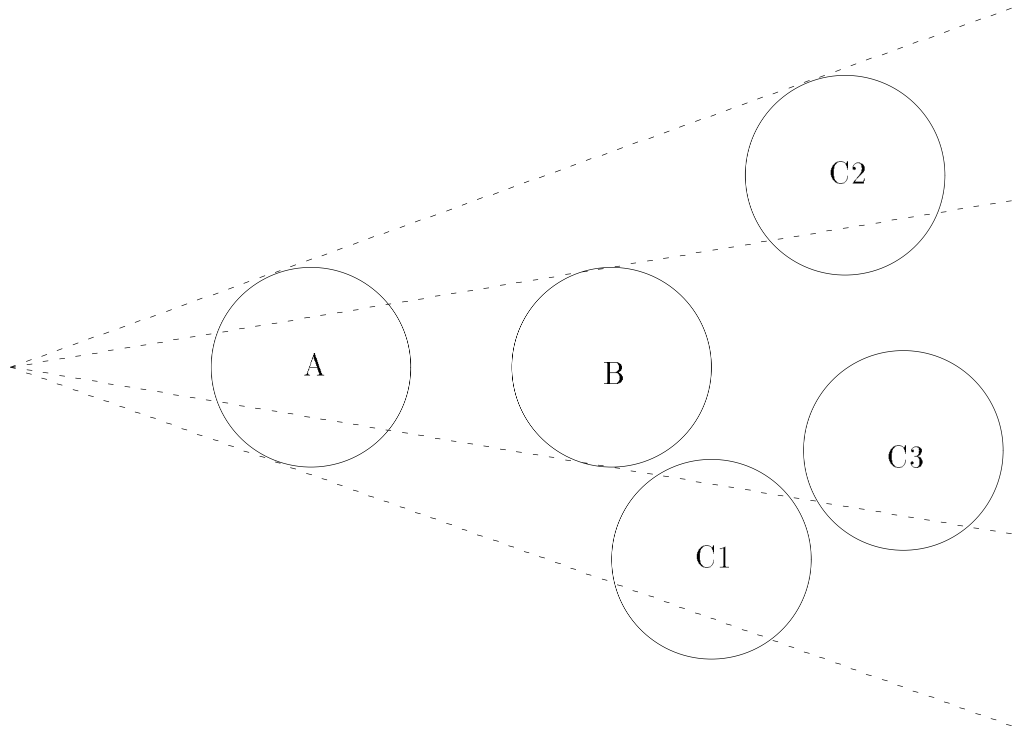

Figure 4: *Composition of H with PH.*

neighbourhood diagram. In the Interval Calculus (Allen 1984), as also in the spatial calculus of (Randell, Cui & Cohn 1992), the result of composing any two simple relations is always a conceptual neighbourhood. This was first pointed out for the Interval Calculus by Freksa (1992*a*). Cohn, Gooday & Bennett (1994) state their belief that, for arbitrary domains, "every composition table entry forms a conceptual neighbourhood". Our next example shows that this is not the case for the line-of-sight calculus.

Figure 5 illustrates the situation in which A is hidden by B and B in turn exactly hides C. Since C may be either nearer to the observer than A, or further than it, there are two possible relations between A and C, namely HI and F (corresponding to C1 and C2 in the diagram):

$$\mathrm{HI} \circ \mathrm{EH} = (\mathrm{F}, \mathrm{HI}).$$

These two relations do not together constitute a conceptual neighbourhood.

One might at first glance suppose that the reason why composite relations need not be conceptual neighbourhoods is that some of the simple relations themselves are not conceptual neighbourhoods. For example, in Figure 2, the relation PHI is represented by two regions that are disconnected from each other, while the relation JC corresponds to no fewer than four disconnected lines. In the former case, this is an artefact of the two-dimensional nature of the diagram; one should think of the diagram as being rotated about the axis joining the centres of A and B. Then the relation PHI corresponds to a connected volume of three-dimensional space. On the other hand JC still corresponds

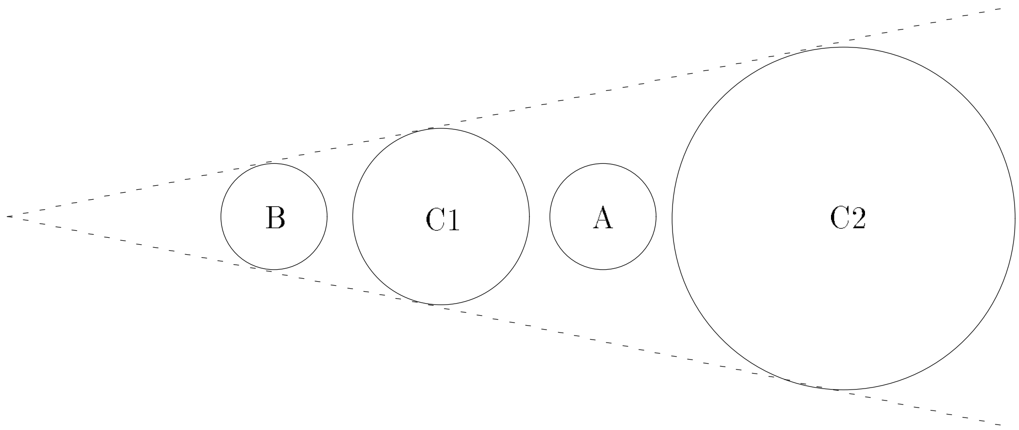

Figure 5: *Composition of HI with EH.*

to two disconnected surfaces; this is in turn a consequence of the fact that the figure is of necessity restricted to the case of fixed objects and movable observer. If the objects are allowed to move, then any JC configuration can be smoothly transformed into any other, indicating that this relation too constitutes a true conceptual neighbourhood. Thus the fact that the composition of simple relations such as HI with EH is not a conceptual neighbourhood must be accounted for in some other way.

Composition tables

To illustrate the principles involved in computing the full composition table for the 14 line-of-sight relations, we shall consider a *reduced set* of 7 relations, obtained by grouping the 14 in a natural way, as follows:

$$\begin{array}{rcl} c & = & \text{(C,JC)} \\ o & = & \text{PH} \\ f & = & \text{(F,JF)} \\ h & = & \text{(H,JH,EH)} \\ oi & = & \text{PHI} \\ fi & = & \text{(FI,JFI)} \\ hi & = & \text{(HI,JHI,EHI)} \end{array}$$

These relations can be derived from the direct product of two independent simpler sets. The idea is that we can treat the relative radial positions along the line of sight separately from the question of overlap or non-overlap. For example, h is like fi in that in both cases every point of B lies along the same line of sight as some point of A; the two relations differ, however, in that with h, B is further from the observer than A is, whereas with fi it is nearer. Thus h and f can be regarded as the product of the relations 'nearer' (N) and 'further' (NI), respectively, with the relation 'every line of sight intercepting B also intercepts A'.

For each body X, let $\mathcal{S}(X)$ denote the set consisting of those lines of sight, from the observer, which intercept X. Then the relations we shall require can be characterised set-theoretically as follows:

D	$\mathcal{S}(A) \cap \mathcal{S}(B) = \emptyset$	('disjoint')
O	$\mathcal{S}(A) \cap \mathcal{S}(B) \notin \{\mathcal{S}(A), \mathcal{S}(B), \emptyset\}$	('overlapping')
P	$\mathcal{S}(A) \subset \mathcal{S}(B)$	('part')
PI	$\mathcal{S}(B) \subset \mathcal{S}(A)$	('has as part')
E	$\mathcal{S}(A) = \mathcal{S}(B)$	('equals')

These are, in effect, spatial relations on the images of A and B in the visual field, considered as a spherical surface centred on the observer.

We can now define our reduced set of 7 relations as follows:

	D	O	P	PI	E
N	c	o	f	h	h
NI	c	oi	hi	fi	hi

Each relation can be expressed as an ordered pair in which the first component is taken from the set {N, NI} and the second is taken from the set {D,O,P,PI,E}, e.g., $f = [\text{N}, \text{P}]$. In the case of c, h, and hi, one of the components is complex, involving more than one element of the set it is taken from; this causes no particular problems, the notation we use being $c = [(\text{N}, \text{NI}), \text{D}]$, $h = [\text{N}, (\text{PI}, \text{E})]$.

The composition table for the set {D,O,P,PI,E} is comparatively straighforward, as follows:

	D	O	P	PI	E
D	any	D,O,P	D,O,P	D	D
O	D,O,PI	any	O,P	D,O,PI	O
P	D	D,O,P	P	any	P
PI	B,O,PI	O,PI	O,P,PI,E	PI	PI
E	D	O	P	PI	E

We also need the trivial composition table for {N, NI}:

	N	NI
N	N	N, NI
NI	N, NI	NI

Note that we do not include a symbol to represent the case where the two bodies are the same distance from the observer, which can only occur when the relation in question is c. The omission does not cause any problems in this case, though it would, perhaps, be rash to assert that omissions of this kind will always be similarly innocuous.

We illustrate the procedure for computing the full table (shown in Figure 6) by computing the entry for $oi \circ f$, as follows:

$$\begin{aligned} oi \circ f &= [\text{NI}, \text{O}] \circ [\text{N}, \text{P}] \\ &= [\text{NI} \circ \text{N}, \text{O} \circ \text{P}] \\ &= [(\text{N}, \text{NI}), (\text{O}, \text{P})] \\ &= ([\text{N}, \text{O}], [\text{N}, \text{P}], [\text{NI}, \text{O}], [\text{NI}, \text{P}]) \\ &= (o, f, oi, hi) \end{aligned}$$

	h	f	o	c	oi	fi	hi
h	h	h, f, o	h, o	h, o, c oi, fi	$h, o,$ oi, fi	h, fi	$h, f, o,$ oi, fi, hi
f	$h, f, o,$ c	f	f, o, c	c	$f, o, c,$ oi, hi	any	f, hi
o	h, o, c	f, o	h, f, o c	h, o, c oi, fi	any	h, o, c oi, fi	f, o oi, fi
c	c	f, o, c oi, hi	f, o, c oi, hi	any	f, o, c oi, hi	c	f, o, c oi, hi
oi	$h, o, c,$ oi, fi	$f, o,$ oi, hi	any	$h, o, c,$ oi, fi	$c, oi,$ fi, hi	$c, fi,$ oi	oi, hi
fi	h, fi	h, f, o oi, hi, fi	h, o oi, fi	$h, o, c,$ oi, fi	oi, fi	fi	$oi, fi,$ hi
hi	any	f, hi	f, o, c oi, hi	c	c, oi, hi	c, oi fi, hi	hi

Figure 6: *Composition table for the reduced set of 7 line-of-sight relations.*

Exactly the same technique can be used to generate the full 14×14 table. As in the case just considered, we can regard each line-of-sight relation as the product of a radial distance relation (either N or NI) with a relation on images in the visual field. In this case, however, the latter set of relations is more finely discriminating, corresponding exactly with the 8 relations identified by Randell *et al.*, namely

DC	Disconnected
EC	Externally connected
PO	Partially overlapping
E	Equals
TPP	Tangential proper part
NTPP	Non-tangential proper part
TPPI	Has as tangential proper part
NTPPI	Has as non-tangential proper part

The line-of-sight relations are now generated as follows:

	DC	EC	PO	E	TPP	NTPP	TPPI	NTPPI
N	C	JC	PH	EH	JF	F	JH	H
NI	C	JC	PHI	EHI	JHI	HI	JFI	FI

Then, for example, we have:

$$\begin{aligned} \mathrm{HI} \circ \mathrm{EH} &= [\mathrm{NI}, \mathrm{NTPP}] \circ [\mathrm{N}, \mathrm{E}] \\ &= [\mathrm{NI} \circ \mathrm{N}, \mathrm{NTPP} \circ \mathrm{E}] \\ &= [(\mathrm{N}, \mathrm{NI}), \mathrm{NTPP}] \\ &= ([\mathrm{N}, \mathrm{NTPP}], [\mathrm{NI}, \mathrm{NTPP}]) \\ &= (\mathrm{F}, \mathrm{HI}) \end{aligned}$$

agreeing with the result obtained earlier and illustrated in Figure 5.

Conclusions

We have taken some key ideas from spatial and temporal reasoning and applied them to a novel domain, that of line-of-sight relations amongst convex objects within the visual field of an observer. The main ideas are

- the partitioning of a continuous domain into a fixed finite set of qualitative relations;
- the second-order relation of neighbourhood defined on this set of relations;
- the composition table defined by the pairwise composition of these relations.

The case of the line-of-sight relations differs interestingly from the standard spatial (Randell, Cui & Cohn 1992) and temporal (Allen 1984) relations in that the result of composing two relations does not always form a conceptual neighbourhood (Freksa 1992*a*).

We further illustrated what may prove to be a productive technique for generating composition tables. The set of line-of-sight relations can be defined in terms of the direct product of two simpler sets of relations; the composition table for the product set can then be straightforwardly generated from the composition tables for its factors. This represents a departure from existing methods such as those of Randell *et al.* (Randell, Cohn & Cui 1992*a*) and Bennett (Bennett 1994); in terms of simplicity of operation, our approach compares favourably with those, although it is admittedly dependent on finding a suitable factoring of the state space.

Finally, in addition to being a useful testing-ground for techniques developed in connection with other domains, the line-of-sight calculus has potential applications of its own. We have not, in fact, developed it with this in view, but it would appear probable that it might find some applicability in the field of computer vision.

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