Data analysis methods in weather and climate research

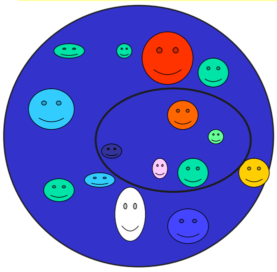
Dr. David B. Stephenson
Department of Meteorology
University of Reading
www.met.rdg.ac.uk/cag

3. Basic probability concepts

- Why we need probability? (uncertainty)
- “Events” and “event space”
- Definitions of “probability” and odds
- Joint and conditional probability
- Bayes’ theorem

3. Why do we need probability?

Probability is THE concept needed to make the link between a sample of data and the whole population.



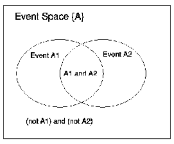
- **Descriptive statistics** – exploration and summary of a sample of data x that came from a population.
- **Inferential statistics** – use of sample data to infer properties of the whole

3. “Events” and “event space”

An **event** A is a possible outcome of an uncertain process e.g. A=Heads, A=Tails, A=Wet day, A=(T>20C), etc.

Events can be **simple** (e.g. A=Heads)
or **compound** (e.g. A=Heads & (T>20C))

Event space is the set of all possible events {a1,a2,…}



3. The Concept of Probability

Impossible
p=0

Certain
p=1

Laplace (1812) Théorie analytique des probabilités

- Axiomatic definition
- Interpretations:
  - Frequentist interpretation – probability as relative frequency of an event repeated many times
  - Non-frequentist subjective interpretation

3. Axiomatic definition

Kolmogorov (1933) axioms for probability:

- All probabilities P{A} >= 0
- The probabilities of all events sum to one
- P{A or B}=P{A}+P{B} if A and B are mutually exclusive events

Note: the axioms are valid also for all conditional probabilities P(.|C) where C is prior information.

3. Relative frequency interpretation

Suppose an event occurs m times in n repeated trials, then the *relative frequency* m/n provides an increasingly accurate estimate of the probability in the limit as n goes to infinity.

$$\lim \frac{m}{n} \xrightarrow{n\to\infty} p$$

This "**Law of large numbers**" is the basis of frequentist estimation of probabilities.

Note: individual weather and climate events are unique and can't be repeated! (Unlike traditional laboratory experiments).

3. Probability problem

You meet a person who tells you that they have two children and that one (or more) of them is a girl.

Q1: What is the probability that the other child is also a girl?

The person now tells you that the girl has a very rare name with probability p close to zero.

Q2: Would you revise your probability estimate? If so, what would be your new estimate?

3. Unbelievable frequentist estimation

If I toss a coin 10 times and get 2 heads then what is your estimate of the probability of heads?

2/10=0.2

Do you believe this probability?

How much would you be prepared to pay me every time a head happened, if I paid you £1 for every tail?

<£1 £1 £2 £4 >£8

Why not £4? £1*(8/10)-£4*(2/10)=0

You ignored your prior belief that my coins are normal unbiased ones with probability of heads =0.5.

3. Subjective interpretation

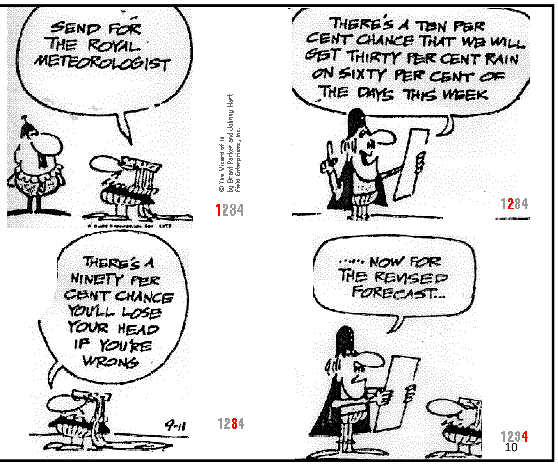
Probability = a measure of belief that is not solely based on past data (e.g. could also incorporate scientific beliefs)

One way to estimate *subjective belief* is to ask (elicit):

How much money (B) you would be prepared to gamble on a fair bet divided by the amount you could win (W):

$$(1-p)(-B) + p(W-B) = 0$$
$$\Rightarrow \frac{p}{1-p} = \frac{B}{W-B}$$
$$\Rightarrow p = \frac{B}{W}$$

odds = P{event}/P{not event}=p/(1-p)
An example of *expert elicitation* – for more explanation see
http://www.shef.ac.uk/pas/research/clusters/bayesian/elicitation.html



3. Marginal, Joint, and Conditional probabilities

Marginal probability = P{A}

Joint probability = P{A and B}

Conditional probability = P{A and B}/P{B} = P{A|B}

i.e. probability of event A GIVEN that event B occurs

$$P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$$

3. Exclusive, exhaustive, and independent events

- ***Exclusive events*** are events that cannot occur simultaneously so P(A and B)=0. Hence P(A|B)=0 if P(B)>0.
- ***Exhaustive events*** are events that describe all possible outcomes so P(A or B)=1.
- ***Independent events*** are ones where P(A and B)=P(A)P(B) and so P(A|B)=P(A). The probability of A is unaffected by conditioning on B. Events where P(A|B) differs from P(A) are known as *dependent events* since the occurrence or non-occurrence of event B affects the chance of event A occurring.

Events can share one or more of these properties, for example, the events heads and tails of a single coin toss provides an example of exclusive and exhaustive events.

3. Prosecutor's fallacy

$$P(A|B) \neq P(B|A)$$

Example:
Probability of you having no heart disease if you eat wheat cereal = probability of you eating wheat cereal if you have no heart disease.

BUT NOT TRUE
because P(no disease|wheat)=P(wheat|no disease)P(no disease)/P(wheat)

"How health claims over Shredded Wheat went too far"
The Guardian, Wednesday November 15, 2000
http://www.guardian.co.uk/guardiansociety/story/0,3605,397312,00.html
… the magistrate commented that it was "clear beyond doubt that the statements about Shredded Wheat attached to each of the campaign steps invite an irresistible inference that eating Shredded Wheat will reduce the risk of coronary heart disease".

3. Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Useful for getting from <u>prior</u> P(A) to <u>posterior</u> P(A|B)
  e.g. conditioning hypothesis A based on new information B
- Posterior depends on likelihood (e.g. data) and prior P(A)
- Bayesian makes the prior explicit – frequentist ignores it!

A Bayesian approach treats all variables AND parameters as uncertain:
- relativity interpretation of uncertainty
  P(A|you), P(A|me), … rather than P(A) absolute probability

Reverend Thomas Bayes
1701-1761

An Essay towards Solving a Problem In the Doctrine of Chances. Philosophical Transactions of the Royal Society, 1763

Frequentist approach:
The future weather event W is a realisation of the weather forecast F. All other information is ignored.

Bayesian approach:
We update/revise our beliefs about the future weather event W based upon the new information available in the forecast F.

$$\mathbf{p(W\,|\,F) \propto p(W)\;\, p(F\,|\,W)}$$

posterior ← prior likelihood

Summary

- Probability is a fundamental concept for quantifying uncertainty
- Interpretation of probability requires care!
- Defined by axioms but then estimated by either frequentist or subjective approaches.
- Dependency is a key concept – allows one event to be conditioned on another one.
- Bayes' theorem can be used to estimate subjective probabilities