Exercise 5. Parameter estimation

1. Generate 100 samples of size 100 from a Poisson distribution with mean 4. Compute the 100 sample means. What distribution does the Central Limit Theorem say will approximate the distribution of these means? Assess this claim with a suitable plot.

2. Suppose that the daily mean temperature at the Plato Cave weather station is precisely a sequence of independent Normal random variables with expectation 10°C and standard deviation 5°C. Generate a sequence of 100 daily mean temperatures. Now pretend that you do not know the true mean of the Normal distribution, and calculate a point estimate for it. By hand, also compute 90%, 95% and 99% confidence intervals. Do any of your intervals contain the true mean? What is the correct interpretation of the intervals?

   Point estimate =
   90% confidence interval =
   95% confidence interval =
   99% confidence interval =

3. Simulate 99 more samples of 100 temperatures and store the 100 sample means. Calculate the standard deviation of your sample means and compare it to the standard error that you would expect theoretically. Compute a 90% confidence interval from each sample, storing the lower and upper limits in separate columns. What proportion of the intervals contain the true value, 10°C? What proportion would you expect?

   Sample standard deviation =
   Theoretical standard error =
   Proportion of intervals =
   Expected proportion =

4. Now store the 100 sample medians. Compare the sampling distributions of the sample means and medians. What do you conclude about properties of the two estimators? Which estimator do you prefer?

5. Carefully work through the demonstrations ‘Sampling Distribution Simulation’ and ‘Confidence Intervals’ in the ‘Rice Virtual Lab in Statistics’.