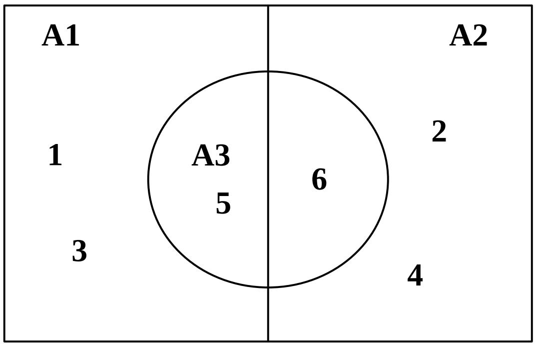
MTMG37 Example Solution to Class Exercise 3

1. The sample space for X is {1, 2, 3, 4, 5, 6}; the Venn diagram is shown below.



   a. Pr(A1) = 1/2, Pr(A2) = 1/2, Pr(A3) = 1/3
   b. A1 and A2 are clearly mutually exclusive (a single roll cannot be even and odd) but it may not be obvious which events are independent. Pr(A1 and A2) = Pr({}) = 0, Pr(A1 and A3) = Pr({5}) = 1/6, Pr(A2 and A3) = Pr({6}) = 1/6. Since Pr(A1 and A2) = 0, A1 and A2 are indeed mutually exclusive. Since Pr(A1 and A3) = Pr(A1)Pr(A3) and Pr(A2 and A3) = Pr(A2)Pr(A3), both A1 and A2 are independent of A3.
   c. Pr(A1|A3) = Pr(A1 and A3)/Pr(A3) = 1/2 and Pr(A2|A3) = Pr(A2 and A3)/Pr(A3) = 1/2. So Pr(A1|A3) = Pr(A1) and Pr(A2|A3) = Pr(A2), as expected for independent events.
      Pr(A3|A1) = Pr(A1|A3)Pr(A3)/Pr(A1) = 1/3 and
      Pr(A3|A2) = Pr(A2|A3)Pr(A3)/Pr(A1) = 1/3.

2. Let R be the event 'precipitation on Christmas day', F the event 'boundary layer below freezing' and W the event 'white Christmas'. We have Pr(R) = 1/2 and Pr(F) = 1/3. Assuming independence of R and F, Pr(W) = Pr(R and F) = Pr(R)Pr(F) = 1/6. If Pr(W) = 1/10 then Pr(R|F) = Pr(R and F)/Pr(F) = Pr(W)/Pr(F) = 3/10 < Pr(R). This appears to be inconsistent with independence of R and F, since in that case Pr(R|F) = Pr(R).

3. The results you get may differ because your random numbers will be different. For my five sample sizes I got proportions 0.4, 0.32, 0.358, 0.3306 and 0.33045. The Law of Large Numbers says that, if the data really are independent random numbers then the proportion should converge to Pr(A3) = 1/3 as the sample size increases. This is what we see here. Note that the convergence need not be monotonic: there is always some sampling variation.

4. The expected number of occurrences of each number is 100/6 = 16.7. In my sample I have the following counts.

	1	2	3	4	5	6
Observed	12	25	19	12	15	17

Again, there is some sampling variation but the observed counts are reasonably close to the expected values. Another way to test the random number generator is to count the number of times consecutive numbers are equal. The expected number is 99/6 = 16.5 since the probability that consecutive numbers are equal is 1/6. The observed number in my sample is 17, which can be computed using the 'Lag' function in Calc > Calculator.

5. $E(X) = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6) = 7/2$.
   $Var(X) = E(X^2) – E(X)^2 = 1(1/6) + 4(1/6) + 9(1/6) + 16(1/6) + 25(1/6) + 36(1/6) – (7/2)^2 = 91/6 – 49/4 = 35/12$.
   $E(X+Y) = E(X) + E(Y) = 2E(X) = 7$ since X and Y have the same expectation.
   $E(XY) = E(X)E(Y) = 49/4$ since X and Y are independent.
   $Cov(X, Y) = Cor(X, Y) = 0$ since X and Y are independent.
   $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y) = 2Var(X) = 35/6$.
   $Var(X – Y) = Var(X) + Var(Y) – 2Cov(X, Y) = 2Var(X) = 35/6$.

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