MTMG37 Example Solution to Class Exercise 4

1. Some example plots are shown in Figure 1.

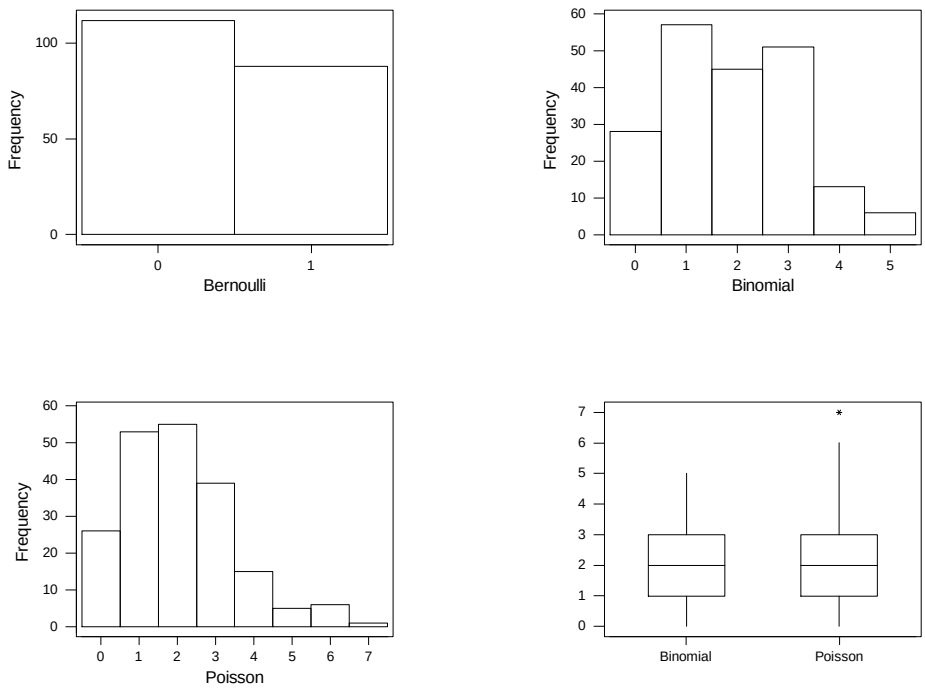

Figure 1. Histograms of samples of size 200 from three distributions: Bernoulli(0.4), Binomial(10, 0.2) and Poisson(2). Boxplots of the Binomial and Poisson data.

2. The Binomial distribution describes the number of successes in a fixed number of independent trials. Here, the number of 'trials' would be the number of days in a winter and a 'success' would be a stormy day. The Binomial distribution will be an unrealistic model, however, because days are unlikely to be approximately independent. The parameters of the model are n = 120 days and $\pi = 1/3$, with mean $n\pi = 40$ days and variance $n\pi(1 - \pi) = 80/3$. The probability of a winter having more than 40 stormy days is 0.4572. A plot of my 100 simulated winters is shown in Figure 2. The average number of stormy days in my sample is 40.5.

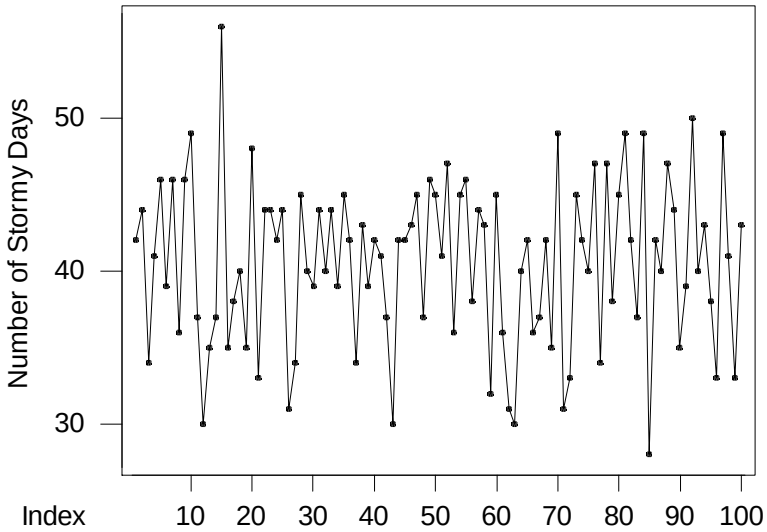

Figure 2. Time series of 100 simulated winters.

3. Some example plots are shown in Figure 3.

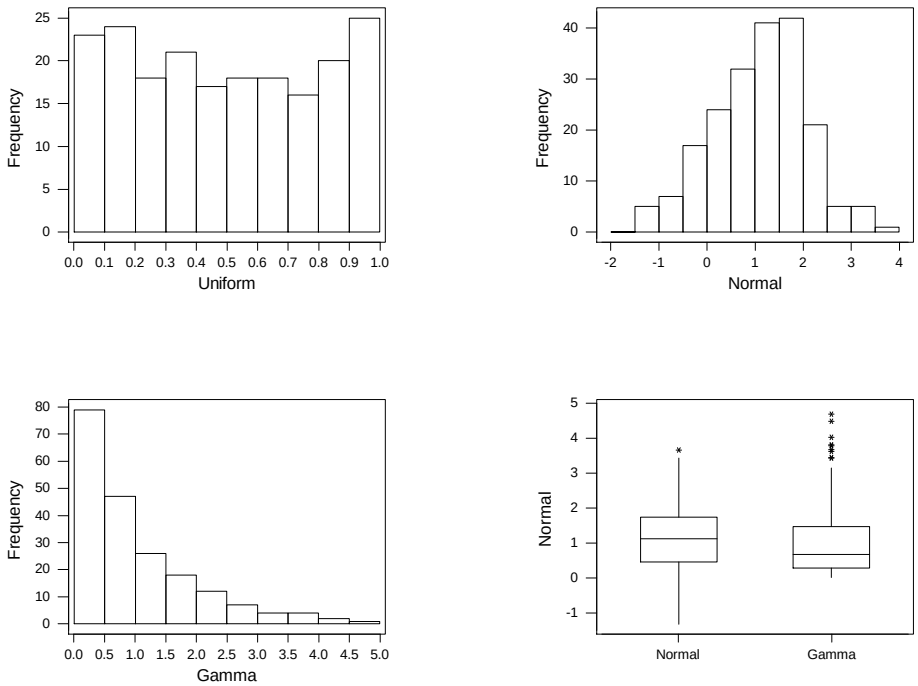

Figure 3. Histograms of samples of size 200 from three distributions: Uniform(0, 1), Normal(1, 1) and Gamma(1, 1). Boxplots of the Normal and Gamma data.

4. Histograms of the temperature data are shown in Figure 4. Normal densities with the same mean and variance as the data are superimposed on each plot. The daily mean temperatures appear reasonably Normal although there is an apparent discrepancy around 13°C. Daily minimum temperatures also appear reasonably Normal. Daily maximum temperatures, on the other hand, seem to have a longer upper tail and a shorter lower tail than the Normal distribution.

5. The probability that a Normal random variable with mean 10.964 and standard deviation 5.843 exceeds 15 is 0.245. This is lower than the sample proportion (0.29) because the Normal distribution is not a good fit to the data. Both values are likely to be poor estimates for 2004 because the temperature distribution in 2003 is unusual for Reading: a better estimate would be based on a longer temperature record.

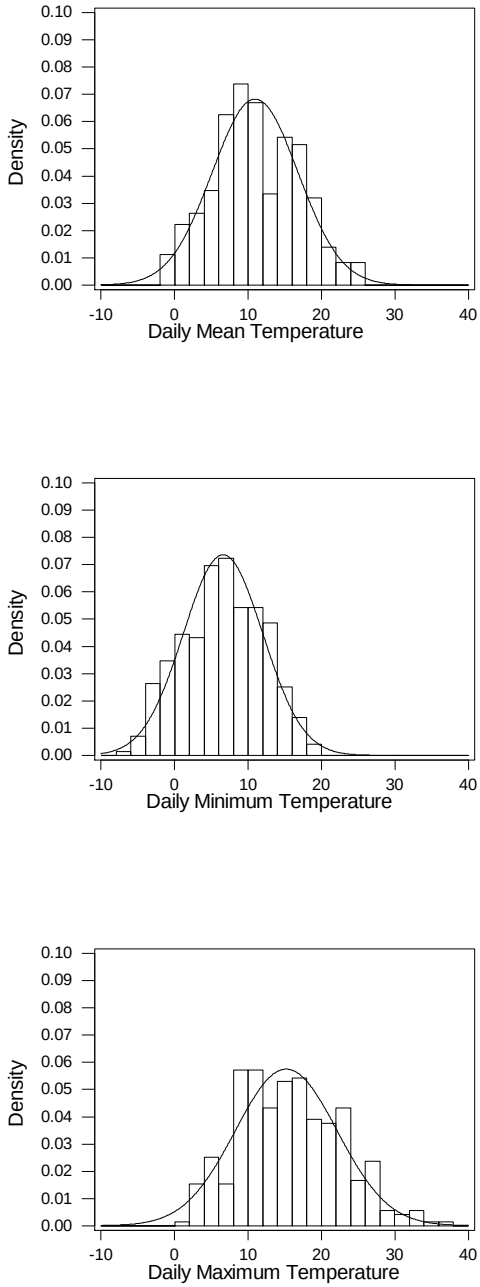

Figure 4. Daily mean, minimum and maximum temperatures (°C) with Normal densities superimposed.