

MTMG37 Example Solution to Class Exercise 6

1. The boxplots in Figure 1 show that the rainfall amounts from both unseeded and seeded clouds are positively skewed and that there are some large, outlying observations. The median rainfall amounts for the unseeded and seeded clouds are 44 and 222 acre-feet, indicating that the seeded clouds tend to yield more rain. The boxplots show that the rainfall from the seeded clouds also has a greater spread than the unseeded clouds.

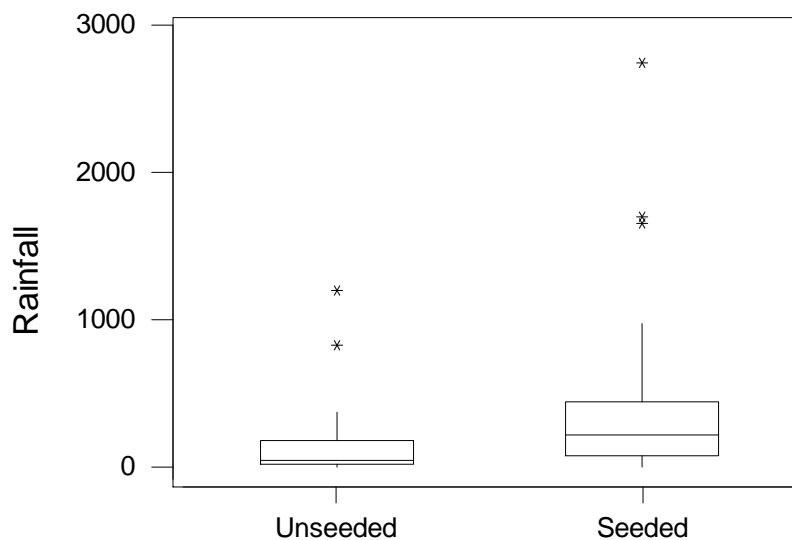


Figure 1. Rainfall amounts (acre-feet) for 26 unseeded and 26 seeded clouds.

2. A possible null hypothesis is $H_0 : \mu_1 = \mu_2$, where μ_1 is the mean rainfall from unseeded clouds and μ_2 is the mean rainfall from the seeded clouds. The general alternative hypothesis is $H_1 : \mu_1 \neq \mu_2$. (If the experimenters wanted to know if seeding a cloud *increased* rainfall then we could use $H_0 : \mu_1 \leq \mu_2$ and $H_1 : \mu_1 > \mu_2$, but we shall not consider this.) The two-sample t-test can be used to test such hypotheses. Since the twenty-six clouds in the two samples are not paired, the unpaired version of the test is appropriate. A reasonable significance level is anything between 1 and 10%, that is we conduct the test with the knowledge that we shall incorrectly reject the null hypothesis between 1 and 10% of the time on average.
3. The unpaired two-sample t-test assumes that each sample comprises independent Normal random variables with constant mean and variance, and that the two samples are independent with equal variances. The exploratory analysis in Question 1 shows that these assumptions are unreasonable for the data. Taking natural logarithms produces the data plotted in Figure 2, which

are well approximated by Normal distributions. Furthermore, the large, outlying observations noted in Figure 1 are no longer evident and the standard deviations of the two samples, 1.64 and 1.60 for the unseeded and seeded clouds, are similar. The assumptions are acceptable for the transformed data, so applying the test to the logged data will give an accurate result.



Figure 2. Log rainfall amounts for 26 unseeded and 26 seeded clouds.

- The means of the transformed rainfall from the unseeded and seeded clouds are 3.99 and 5.13, with standard deviations 1.64 and 1.60. The difference in the means is 1.14 and the pooled standard deviation is 1.62. The t-statistic is $t = 1.14\sqrt{13}/1.62 = 2.54$, which is compared to the T-distribution with 50 degrees of freedom. For significance level 5% the critical values are ± 2.01 , which are shown below on the sketch of the density. The statistical tables also show that the two-sided p-value is between 0.01 and 0.02 since $2.40 < t < 2.68$. R yields the p-value 0.014. The null hypothesis is therefore rejected at the 5% level, but not at the 1% level for example. I conclude that there is quite strong evidence to suggest that seeding clouds affects the mean amount of rainfall produced by a cloud. The data indicate that the effect is to increase the amount of rainfall.

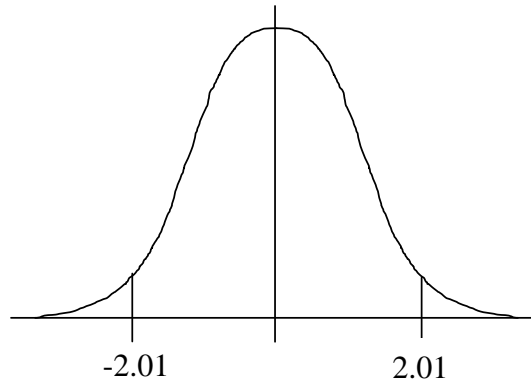


Figure 3. Density of the T_{50} distribution with critical values.

5. Without transforming the data the p-value is 0.051, and the null hypothesis would not be rejected at the 5% level.
6. Let X be the number of heads obtained in $n = 250$ spins of the coin, and let π be the probability that, when spun, the coin lands showing heads. We have no information to suggest that the spins are dependent, nor that the chance of heads changes during the experiment, so it is reasonable to assume that X has the Binomial distribution $\text{Bin}(n, \pi)$. We wish to test the null hypothesis $H_0 : \pi = \pi_0 = 1/2$ that the coin is unbiased against the general alternative $H_1 : \pi \neq 1/2$ that the coin is biased.

The experiment yielded $x = 140$ heads, so a point estimate for π is $\hat{\pi} = x/n = 140/250 = 0.56$. Using the Normal approximation $N(n\pi, n\pi(1-\pi))$ to the Binomial distribution, a 95% confidence interval for π is $(\hat{\pi} - 1.96\sqrt{\hat{\pi}(1-\hat{\pi})/n}, \hat{\pi} + 1.96\sqrt{\hat{\pi}(1-\hat{\pi})/n}) \approx (0.498, 0.622)$. This interval contains π_0 , so the null hypothesis is not rejected at the 5% level of significance.

A hypothesis test based on the Normal approximation compares the z-statistic, $z = (\hat{\pi} - \pi_0) / \sqrt{\pi_0(1-\pi_0)/n} = 1.90$, under the null hypothesis to the standard Normal distribution. The probability of obtaining a z-statistic at least as large as this is $2\Pr(Z > z) = 0.058$. We conclude that there is only weak evidence that the coin used in the experiment is biased.

A more accurate test that does not rely on the Normal approximation uses the Binomial distribution directly. The probability of obtaining a result as unlikely as 140 heads in 250 spins under the null hypothesis is $\Pr(X \geq 140) + \Pr(X \leq 110) = 0.066$.

The claim that the Belgian euro is struck asymmetrically assumes that the result for this single coin holds for all Belgian euros. If this were the purpose of the investigation then a better experiment would be to spin n different coins instead of the same coin n times.