

MTMG37 Example Solution to Class Exercise 8

This investigation aims to assess the evidence for time trends in wintertime Central England Temperature (CET). The data are mean winter CETs for the years 1866 to 1997 with winter defined to cover months December, January and February, the latter months identifying the year associated with a particular winter. Mean winter sea-level pressure (SLP) at four stations in Iceland, the Azores, Gibraltar and Lisbon for the same time period will also be used in the analysis.

The mean winter CETs are plotted against year in Figure 1. There appear to be some slow variations over time. The presence of an occasional very low temperature is reflected in the histogram of Figure 2, which is unimodal but has a longer cold tail. The mean temperature is 4.1°C and the standard deviation is 1.3°C.

A simple linear regression of temperature on year is fitted by ordinary least squares. The model is

$$T_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$

where T_i is the temperature in year x_i and the ε_i are independent Normal random variables with zero mean and constant variance. The parameter estimates are $\hat{\beta}_0 = 1.1$ and $\hat{\beta}_1 = 0.0015$ with standard errors 5.7 and 0.003. The slope is not different from zero at any reasonable level of significance (p-value = 0.6) and the coefficient of determination is only 0.2%, suggesting that there is no discernable time trend in the wintertime CETs. Residual plots such as Figure 3 do not suggest any non-linear time trend that could be simply modelled.

A simple linear regression of temperature on mean SLP at Iceland is now estimated. The model is

$$T_i = \beta_0 + \beta_1 p_{I_i} + \varepsilon_i,$$

where p_{I_i} is the Icelandic SLP in year x_i . The parameter estimates are $\hat{\beta}_0 = 170$ and $\hat{\beta}_1 = -0.166$ with standard errors 13 and 0.013. The slope is highly significantly different from zero (p-value < 0.001) and the coefficient of determination is 55%. Icelandic SLP is a powerful explanatory variable for wintertime CET, explaining more than half of the variation in the temperatures. The CET increases by about 1°C for every decrease of 6hPa in Icelandic SLP. Residual plots indicate two, slight departures from the model assumptions: the distribution of the residuals has a long lower tail, and the residuals have a slight positive trend over time (Figure 4).

A linear time trend is added to the regression model:

$$T_i = \beta_0 + \beta_1 p_{I_i} + \beta_2 x_i + \varepsilon_i.$$

The parameter estimates are now $\hat{\beta}_0 = 165$, $\hat{\beta}_1 = -0.169$ and $\hat{\beta}_2 = 0.004$ with standard errors 13, 0.013 and 0.002. Although the time trend is slight, an increase in CET of about 0.4°C per century, it is now significantly different from zero at the 5% level (p-value = 0.043). The explanatory power of the regression model is only marginally improved, however: the coefficient of determination is 56%.

Perhaps the most illuminating model includes the Gibraltar SLP:

$$T_i = \beta_0 + \beta_1 p_{Ii} + \beta_2 p_{Gi} + \beta_3 x_i + \varepsilon_i,$$

where p_{Gi} is the Gibraltar SLP in year x_i . The parameter estimates are $\hat{\beta}_0 = -104$, $\hat{\beta}_1 = -0.11$, $\hat{\beta}_2 = 0.21$ and $\hat{\beta}_3 = 0.004$ with standard errors 57, 0.02, 0.04 and 0.002. Both of the SLP terms are highly significantly different from zero (p-value < 0.001) and the time trend, of about 0.4°C per century remains significantly different from zero at the 5% level (p-value = 0.041). The opposite signs of the coefficients for the Iceland and Gibraltar SLP terms indicate that the (weighted) pressure difference between these two locations contributed most to the explanatory power of the model, which has coefficient of determination equal to 63%.

This analysis has shown that there is evidence, at the 5% level of significance, for an increasing time trend in CET of about 0.4°C per century. This trend is discernible only after accounting for the influence of atmospheric circulation patterns on CET, in particular the pressure differential between Iceland and Gibraltar.

One potential shortcoming of this analysis is the assumption that the residuals are independent: they might be expected to exhibit serial correlation from one year to the next. If such behaviour is not accounted for in the model then the results, such as the statistical significance of a time trend, can be misleading. Models that do account for correlated residuals are beyond the scope of this module, however, but fortunately the correlation is weak for the data examined in this exercise. Another potential shortcoming is the linear time trend. This is very common practice in climate science, but processes rarely exhibit linear trends: they are usually more complicated, but appropriate statistical models are also beyond the scope of this module.

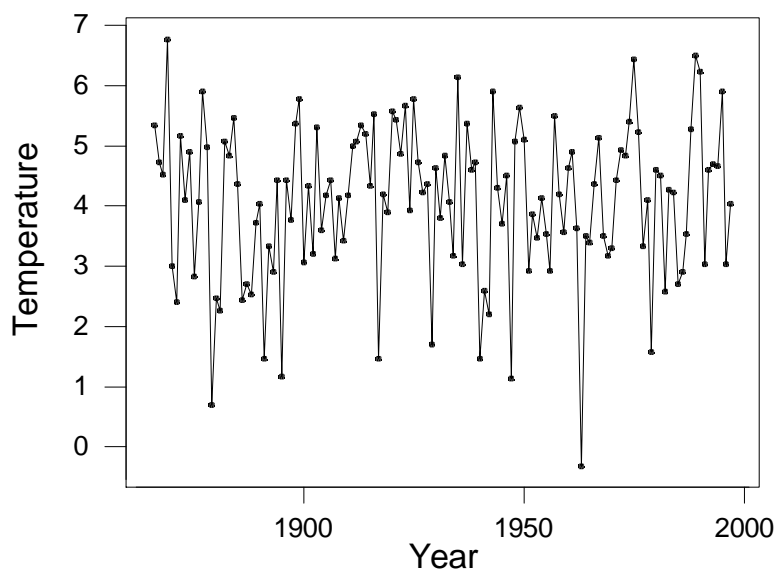


Figure 1. Mean winter CET ($^{\circ}\text{C}$) against year for 1866 to 1997.

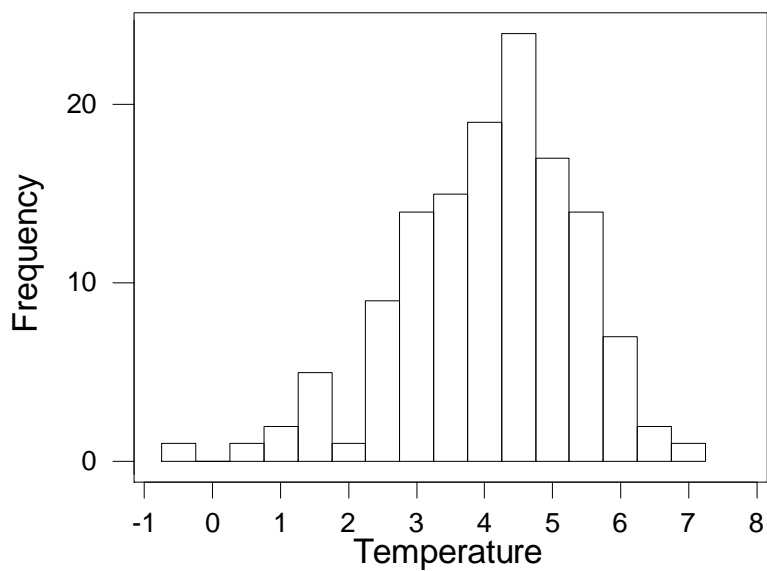


Figure 2. Histogram of mean winter CET ($^{\circ}\text{C}$) from 1866 to 1997.

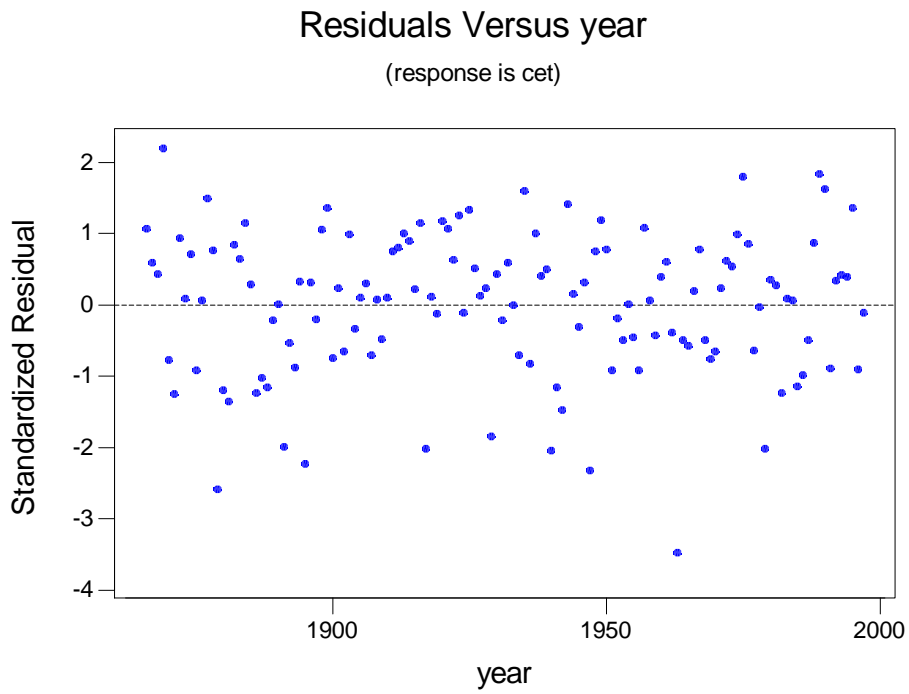


Figure 3. Residuals from the linear regression of CET (°C) on year.

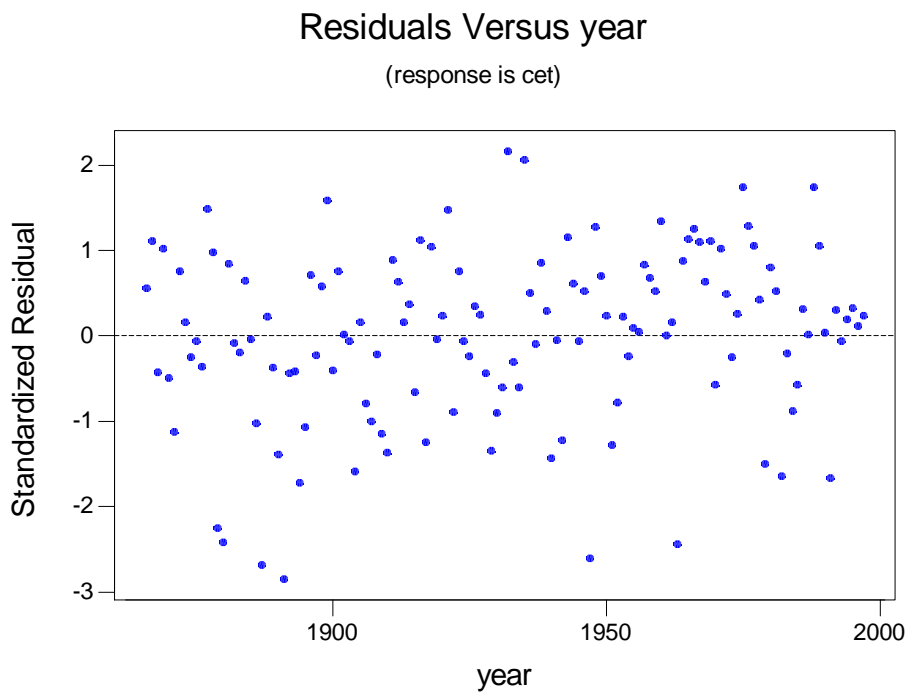


Figure 4. Residuals from the linear regression of CET (°C) on Iceland SLP.

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