MTMG37 Example Solution to Class Exercise 9

1. The Darwin SLP is plotted in Figure 1. There is a strong annual cycle that accounts for almost all of the variation in the data, which is spread evenly around the value 10; there is a slight increase over time.

2. For periodic data it is usual to take moving averages with lengths equal to multiples of the period. A moving average with length 60 months is shown in Figure 1. This highlights the main, low-frequency variation; shorter lengths have too much noise. The main pattern appears to be an overall increase with a dip in the early 1970s.

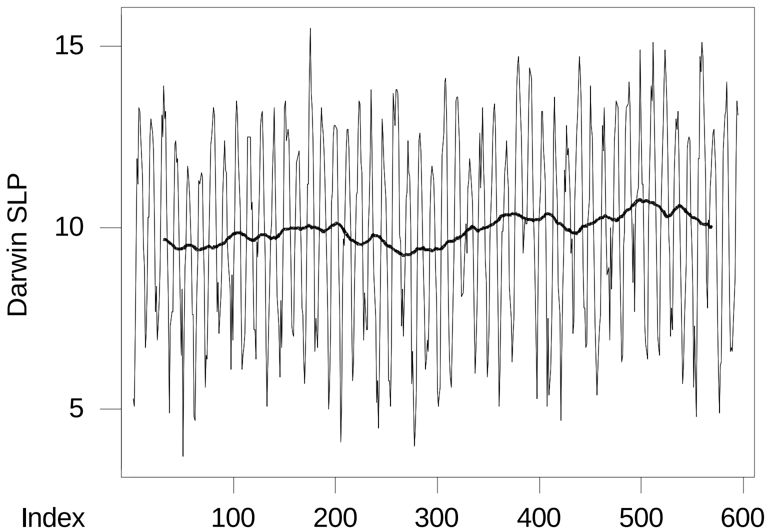

Figure 1. Monthly mean SLP (−1000hPa) at Darwin from January 1951 to July 2000. The centred moving average of length 60 months is superimposed.

3. The autocorrelation function is given in the lecture notes. The dependence at lag 12 months is illustrated in Figure 2.

4. The plot of the data after applying the backward difference filter of lag 12 is given in the lecture notes.

5. The series obtained after applying both lag 12 and lag 1 differences is shown in Figure 3. If this were white noise then the autocorrelation function would be zero for all lags. The actual autocorrelation function is shown in Figure 4 and has large, negative correlations at lags 1 and 12. The differenced series is therefore not white noise, but has some remaining dependence structure.

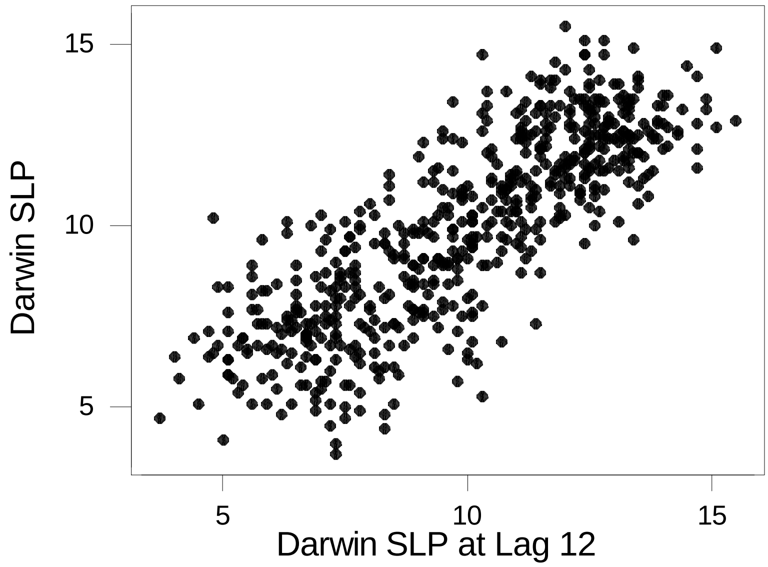

Figure 2. Darwin SLP plotted against itself at lag 12 months.

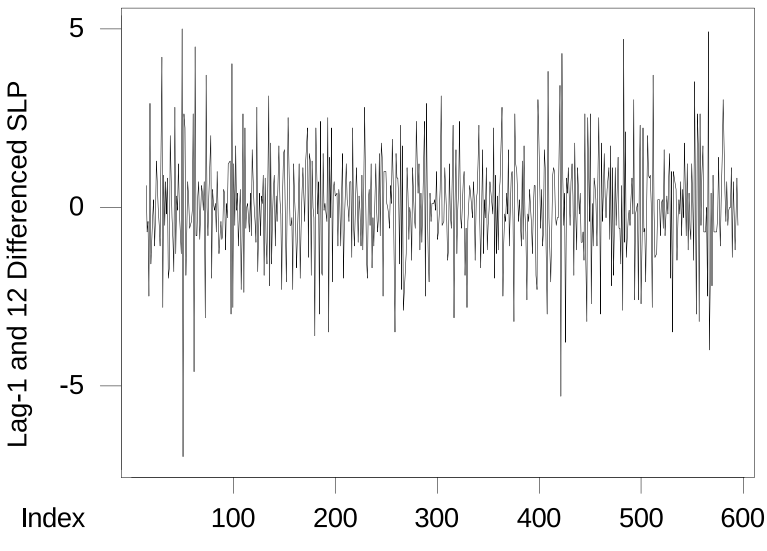

Figure 3. Darwin SLP after applying lag 1 and lag 12 backward difference filters.

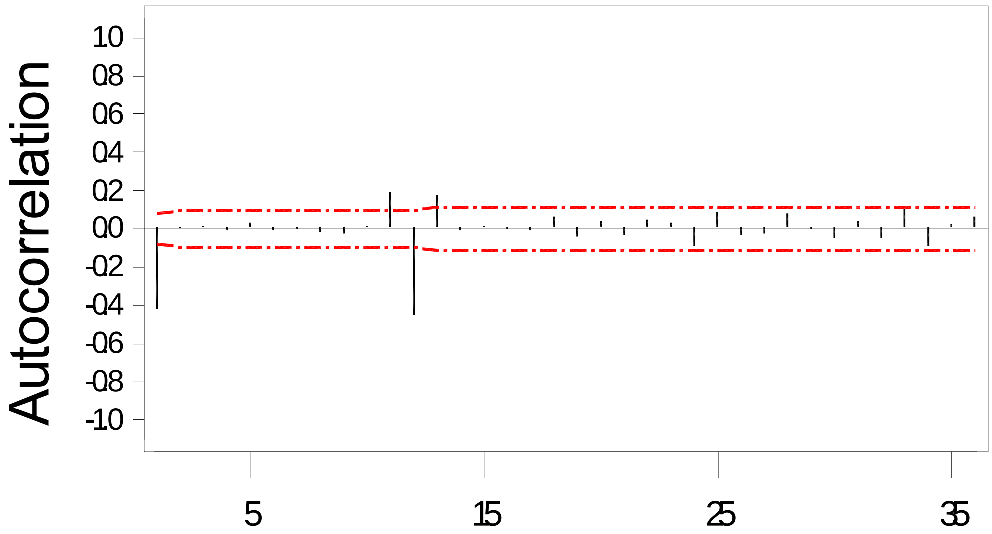

Figure 4. Autocorrelation for the series in Figure 3 with 95% confidence intervals.

6. The histogram and autocorrelation function from the seasonal ARIMA model are shown in Figure 5. The distribution of the residuals is approximately Normal (in fact the residuals have slightly heavier tails) and there is no significant autocorrelation at the 5% level, indicating that the residuals are well approximated by white noise. The fitted model has equation

$$Y_t = Z_t - 0.59Z_{t-1} - 0.96Z_{t-12} + 0.56Z_{t-13},$$

where Z_t is white noise and Y_t is the lag-1 and lag-12 differenced data, i.e.

$$X_t = X_{t-1} + (X_{t-12} - X_{t-13}) + Z_t - 0.59Z_{t-1} - 0.96Z_{t-12} + 0.56Z_{t-13}.$$

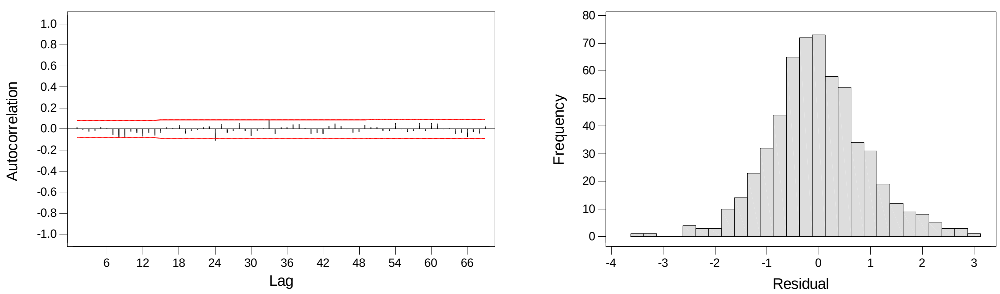

Figure 5. Autocorrelation function (with 95% confidence intervals) and histogram for the residuals from the seasonal ARIMA model.