Data analysis methods in weather and climate research

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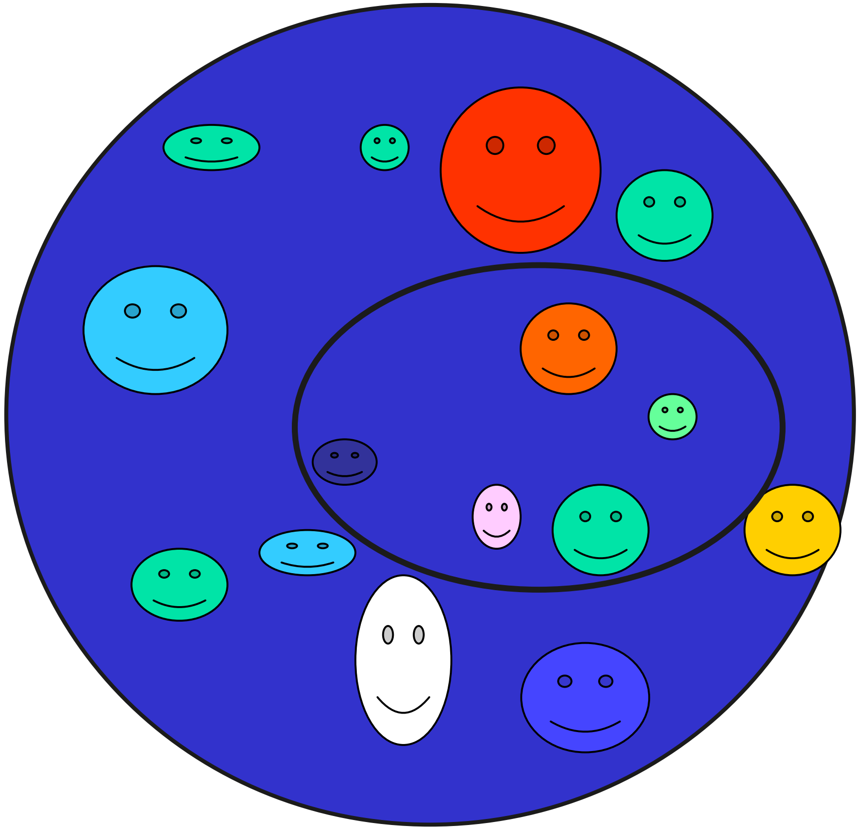
3. Basic probability concepts

- Why we need probability? (uncertainty)
- “Events” and “event space”
- Definitions of “probability” and odds
- Joint and conditional probability
- Bayes’ theorem

3. Why do we need probability?

Probability is THE concept needed to make the link between a sample of data and the whole population.



- **Descriptive statistics** – exploration and summary of a sample of data x that came from a population.
- **Inferential statistics** – use o sample data to infer propertie of the whole

3. “Events” and “event space”

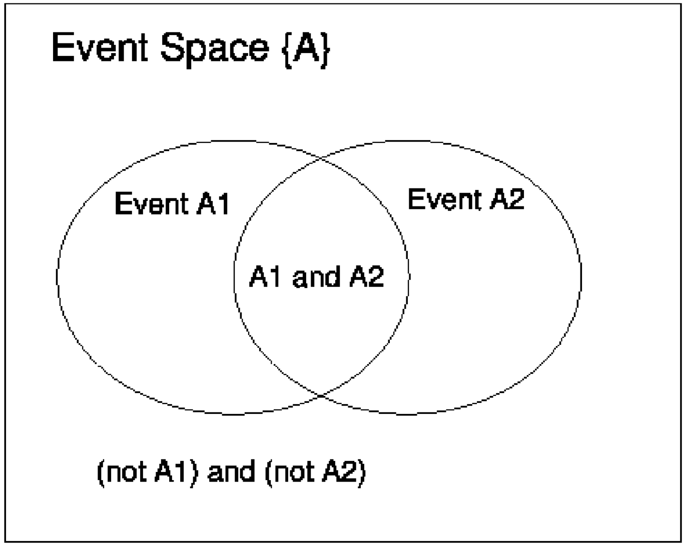
An **event** A is a possible outcome of an uncertain process

e.g. A=Heads, A=Tails, A=Wet day, A=(T>20C), etc.

Events can be **simple** (e.g. A=Heads)

or **compound** (e.g. A=Heads & (T>20C))

Event space is the set of all possible events {a1,a2,…}



3. The Concept of Probability

Impossible
p=0

Certain
p=1

Laplace (1812) Théorie analytique des probabilités

- Axiomatic definition
- Interpretations:
  - Frequentist interpretation – probability as relative frequency of an event repeated many times
  - Non-frequentist subjective interpretation

3. Axiomatic definition

Kolmogorov (1933) axioms for probability:

- All probabilities P{A} >= 0
- The probabilities of all events sum to one
- P{A or B}=P{A}+P{B} if A and B are mutually exclusive events

Note: the axioms are valid also for all conditional probabilities P(.|C) where C is prior information.

3. Relative frequency interpretation

Suppose an event occurs m times in n repeated trials, then the *relative frequency* m/n provides an increasingly accurate estimate of the probability in the limit as n goes to infinity.

$$\lim \frac{m}{n} \xrightarrow{n\to\infty} p$$

This "**Law of large numbers**" is the basis of frequentist estimation of probabilities.

Note: individual weather and climate events are unique and can't be repeated! (Unlike traditional laboratory experiments).

3. Unbelievable frequentist estimation

If I toss a coin 10 times and get 2 heads then what is your estimate of the probability of heads?

Do you believe this probability?

How much would you be prepared to pay me every time a head happened, if I paid you $10 for every tail?

Why not $40? $10*(8/10)-$40*(2/10)=0

You ignored your prior belief that my coins are normal unbiased ones with probability of heads =0.5.

3. Subjective interpretation

Probability = a measure of belief that is not solely based on past data (e.g. could also incorporate scientific beliefs)

One way to estimate *subjective belief* is to ask (elicit):

How much money (B) you would be prepared to gamble on a fair bet divided by the amount you could win (W):

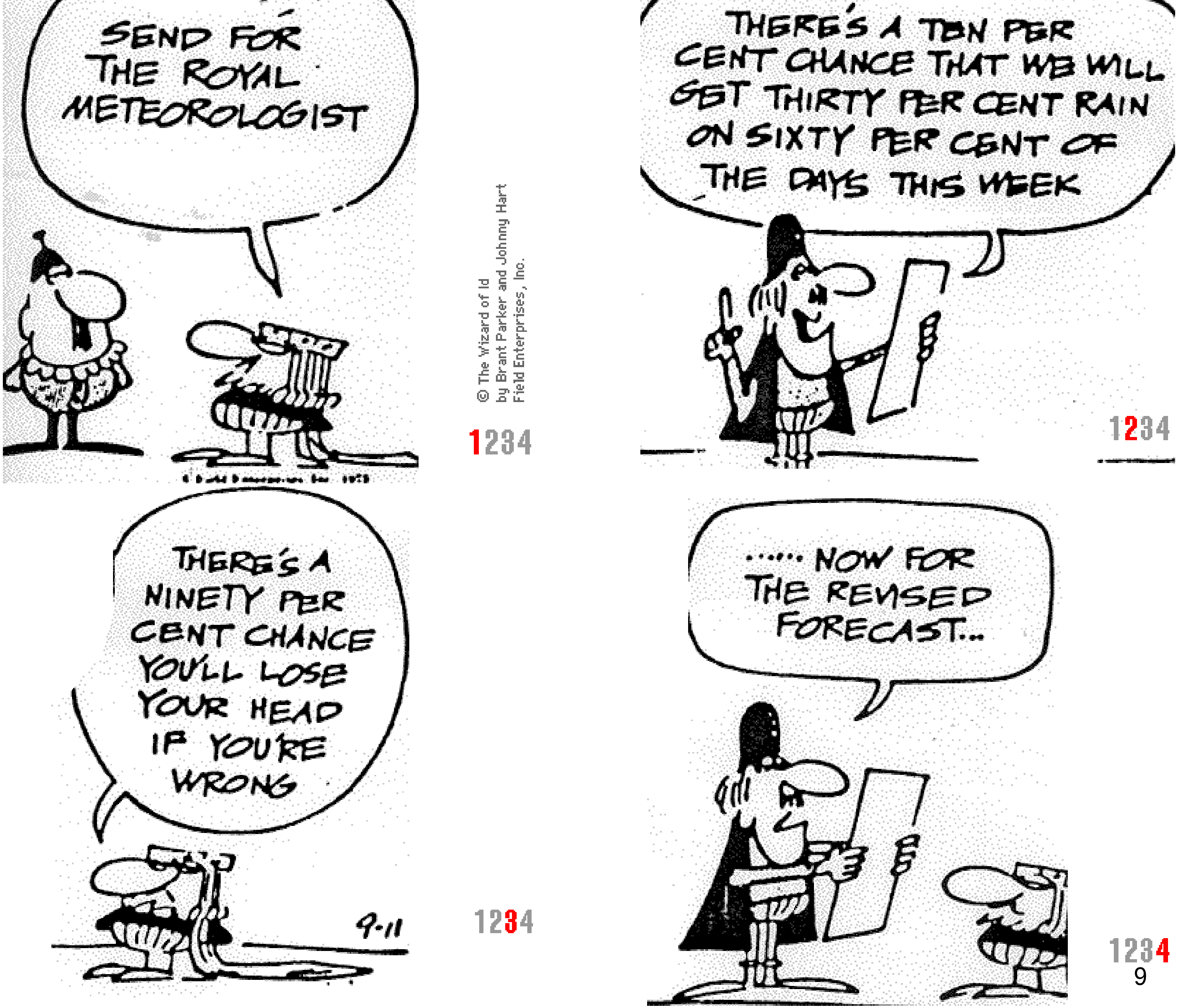
$$(1-p)(-B) + p(W-B) = 0$$

$$\Rightarrow \frac{p}{1-p} = \frac{B}{W-B}$$

$$\Rightarrow p = \frac{B}{W}$$

odds = P{event}/P{not event}=p/(1-p)

An example of *expert elicitation* – for more explanation see http://www.shef.ac.uk/pas/research/clusters/bayesian/elicitation.html


SEND FOR THE ROYAL METEOROLOGIST
THERE'S A TEN PER CENT CHANCE THAT WE WILL GET THIRTY PER CENT RAIN ON SIXTY PER CENT OF THE DAYS THIS WEEK
THERE'S A NINETY PER CENT CHANCE YOU'LL LOSE YOUR HEAD IF YOU'RE WRONG
9-11
...... NOW FOR THE REVISED FORECAST...

3. Joint and conditional probability

Joint probability = P{A and B}

Conditional probability = P{A and B}/P{B} = P{A|B}

i.e. probability of event A GIVEN that event B occurs

$$P(A\ and\ B) = P(A|B)P(B) = P(B|A)P(A)$$

Prosecutor's fallacy: $P(A|B) = P(B|A)$

e.g. probability of you drinking the local wine if you are a drunk

= probability of you being a drunk if you drink the local wine

BUT NOT TRUE because P(being a drunk)<P(drinking the local wine)!

3. Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Useful for getting from <u>prior</u> P(A) to <u>posterior</u> P(A|B)

  e.g. conditioning hypothesis A based on new information B
- Posterior depends on likelihood (e.g. data) and prior P(A)
- Bayesian makes the prior explicit – frequentist ignores it!

A Bayesian approach treats all variables AND parameters as uncertain:

- relativity interpretation of uncertainty

  P(A|you), P(A|me), … rather than P(A) absolute probability

Reverend Thomas Bayes
1701-1761

An Essay towards Solving a Problem In the Doctrine of Chances. Philosophical Transactions of the Royal Society, 1763

Frequentist approach:
The future weather event W is a realisation of the weather forecast F. All other information is ignored.

Bayesian approach:
We update/revise our beliefs about the future weather event W based upon the new information available in the forecast F.

$$\mathbf{p(W \mid F) \propto p(W)\ \ p(F \mid W)}$$

posterior ← prior likelihood

Summary