Data analysis methods in weather and climate research

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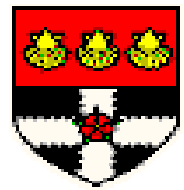


8. Multiple linear regression and non-linear regression

- Multiple regression
- Multivariate regression – the General Linear Model
- Non-linear responses – Generalized Linear Models
- Non-parametric regression

Probability problem

You meet a person who tells you that they have 2 children and that 1 (or more) of them is a girl.

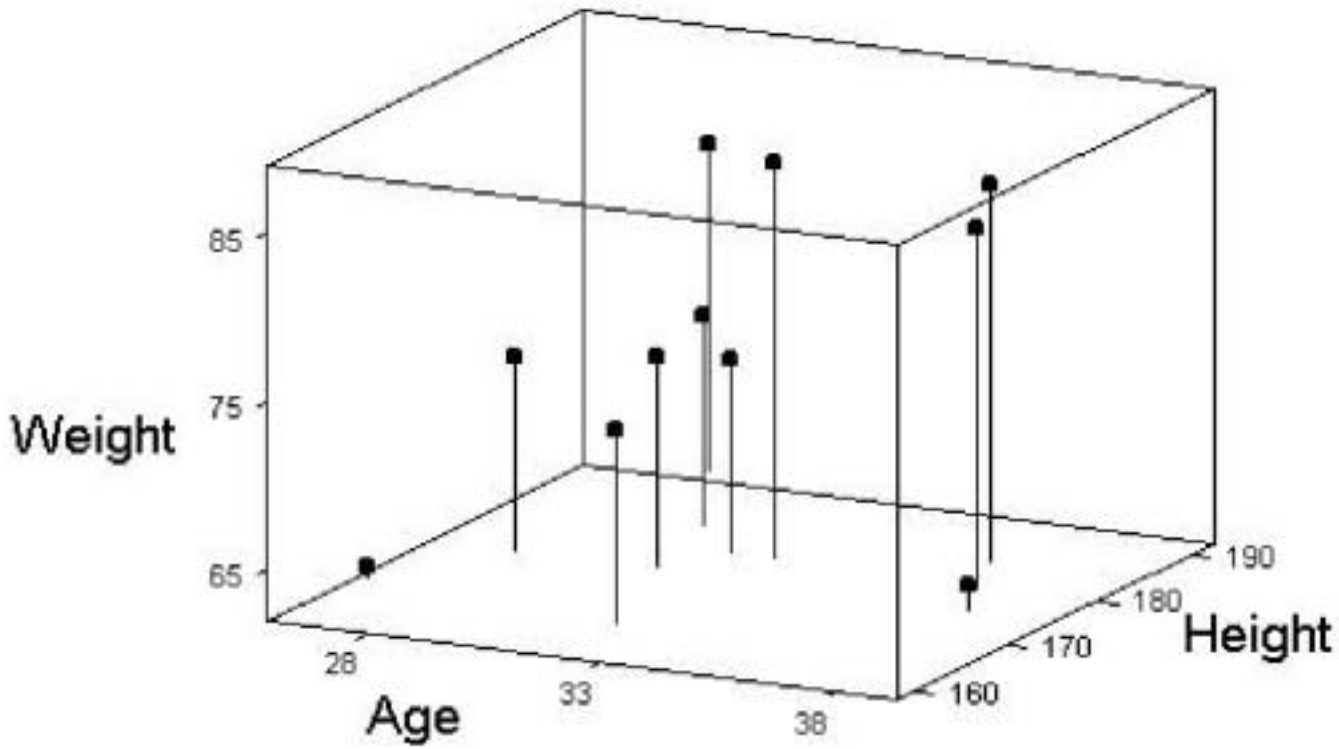
What is the probability that the other child is also a girl?

The person now tells you that the girl has a very rare name with probability p close to zero.

Would you revise your probability estimate? If so, what would be your new estimate?

8. Multiple regression

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$




```
> fit<-lsfit(cbind(x1,x2),y)
> ls.print(fit)
```

8. Regression summary

The regression equation is

Weight = -40.4 + 0.517 Age + 0.577 Height

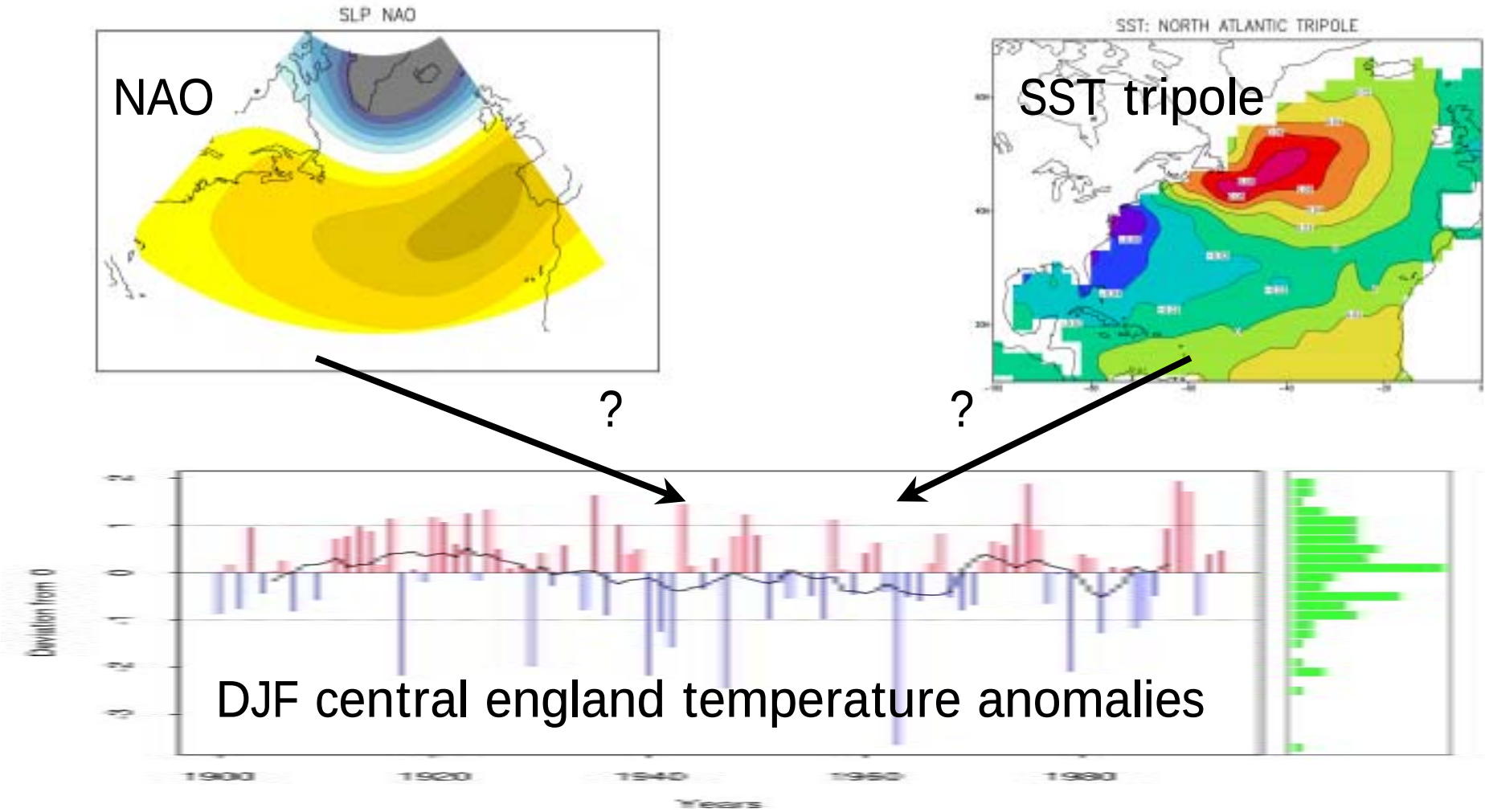
Predictor	Coef	St.Dev	t	p-value
Constant	-40.36	49.20	-0.82	0.436
Age	0.5167	0.5552	0.93	0.379
Height	0.5769	0.2671	2.16	0.063

S = 6.655 R-sq = 41.7% R-sq(adj) = 27.1%

Analysis of variance

Source	DF	SS	MS	F	p-value
Regression	2	253.66	126.8	2.86	0.115
Residual	8	354.34	44.29		
Total	10	608.00			

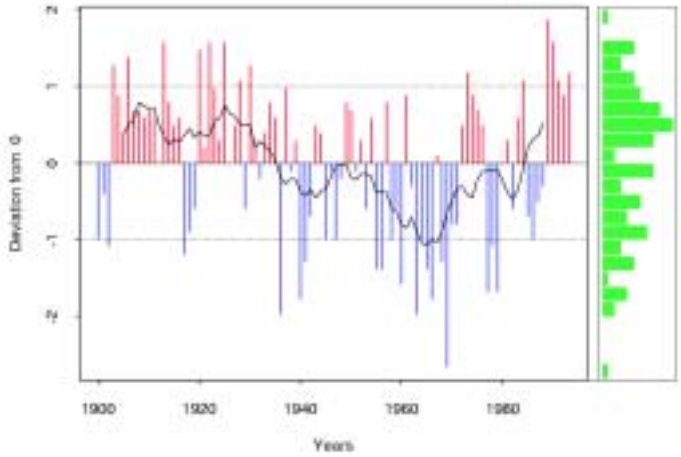
8. Climate example: role of the N. Atl ocean

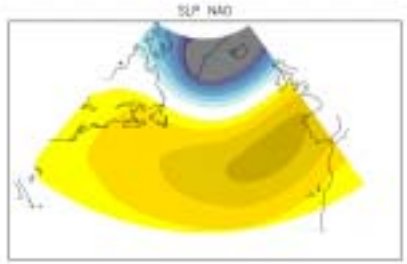


Stephenson, D.B. and M. Junge, 2003: Int. J. Climatology, 23, 245-261
www.met.rdg.ac.uk.cag/publications

MUTUAL CORRELATIONS

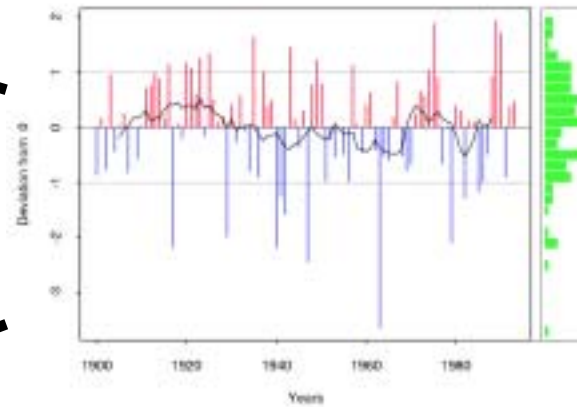
NAO





r=0.67

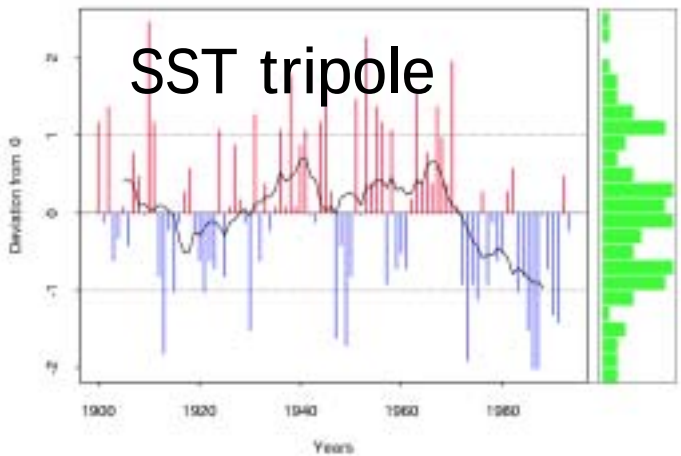
Central England Temperature

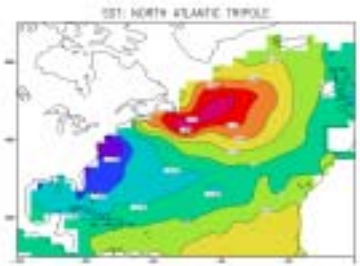


r=-0.38

r=-0.30

SST tripole





8. The linear modelling approach

To unravel the indirect from the direct effects we need to go beyond decriptive methods (correlation analysis) and introduce a model:

$$CET = \beta_1 NAO + \beta_2 SST + \varepsilon$$

Using data from 1900-1994, we obtain estimates of:

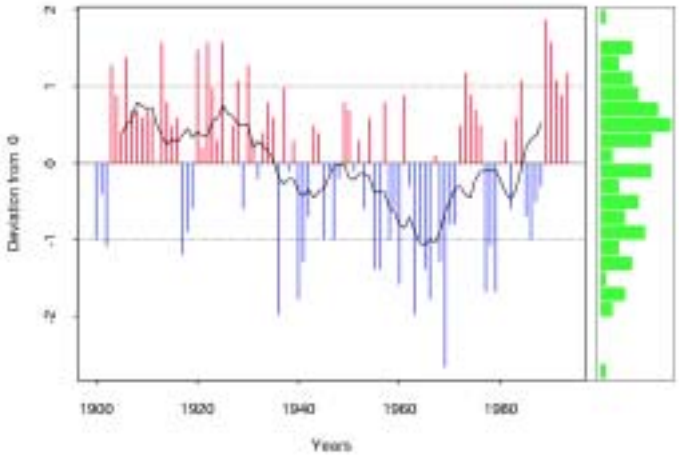
$$\hat{\beta}_1 = +0.64 \pm 0.08$$

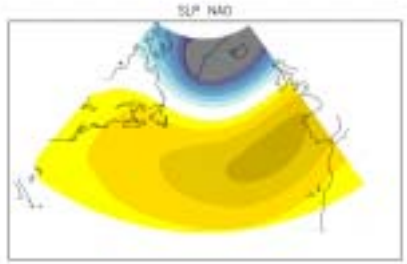
$$\hat{\beta}_2 = -0.06 \pm 0.08$$

The fit explains 45% of the total CET variance and is statistically significant at $p<0.001$

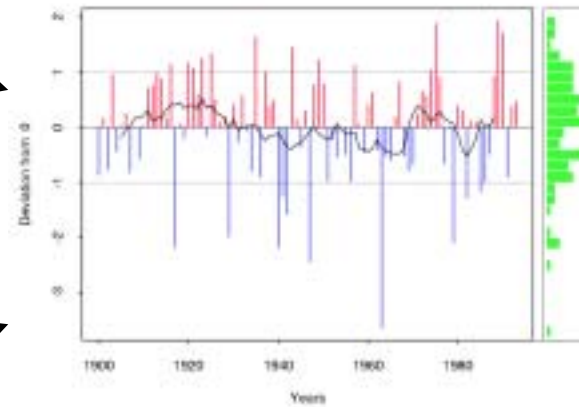
Direct and indirect effects

NAO





Central England Temperature

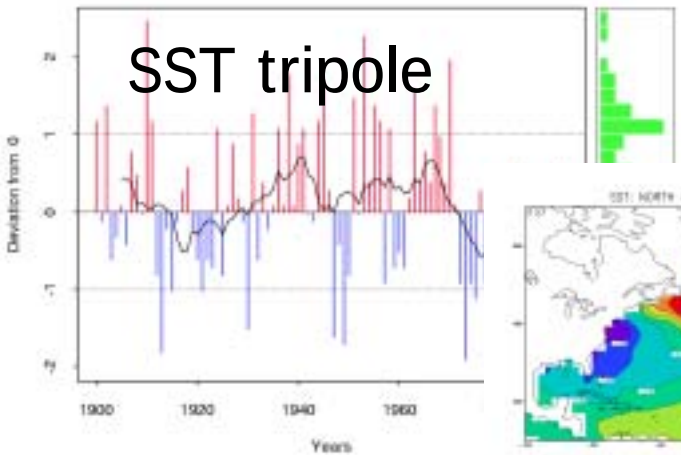


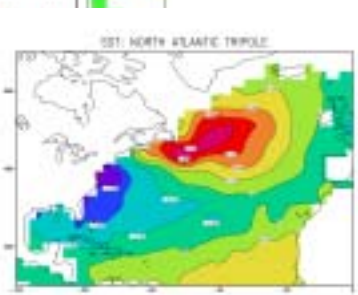
0.64

r=-0.38

-0.06

SST tripole





r(CET,NAO)=0.67=0.64+(-0.38)x(-0.06

r(CET,SST)=-0.30=-0.06+(-0.38)x(0.64

So most of correlation between SST an

CET is coming indirectly via the NAO's influence on both variables.

8. Multivariate regression

The General Linear Model:

$$Y = X\beta + \varepsilon$$

$$Y = (n \times p) \text{ matrix of p response variables}$$

$$X = (n \times q) \text{ matrix of q explanatory variables}$$

$$\beta = (q \times p) \text{ matrix of model parameters}$$

$$\varepsilon = (n \times p) \text{ matrix of normally distributed errors}$$

Multiple regression extended to more than 1 response

```
> # R command
> lm(y~x)
```

8. Regression as a probability model

- Purely descriptive correlation r=0.67
- Least-squares regression

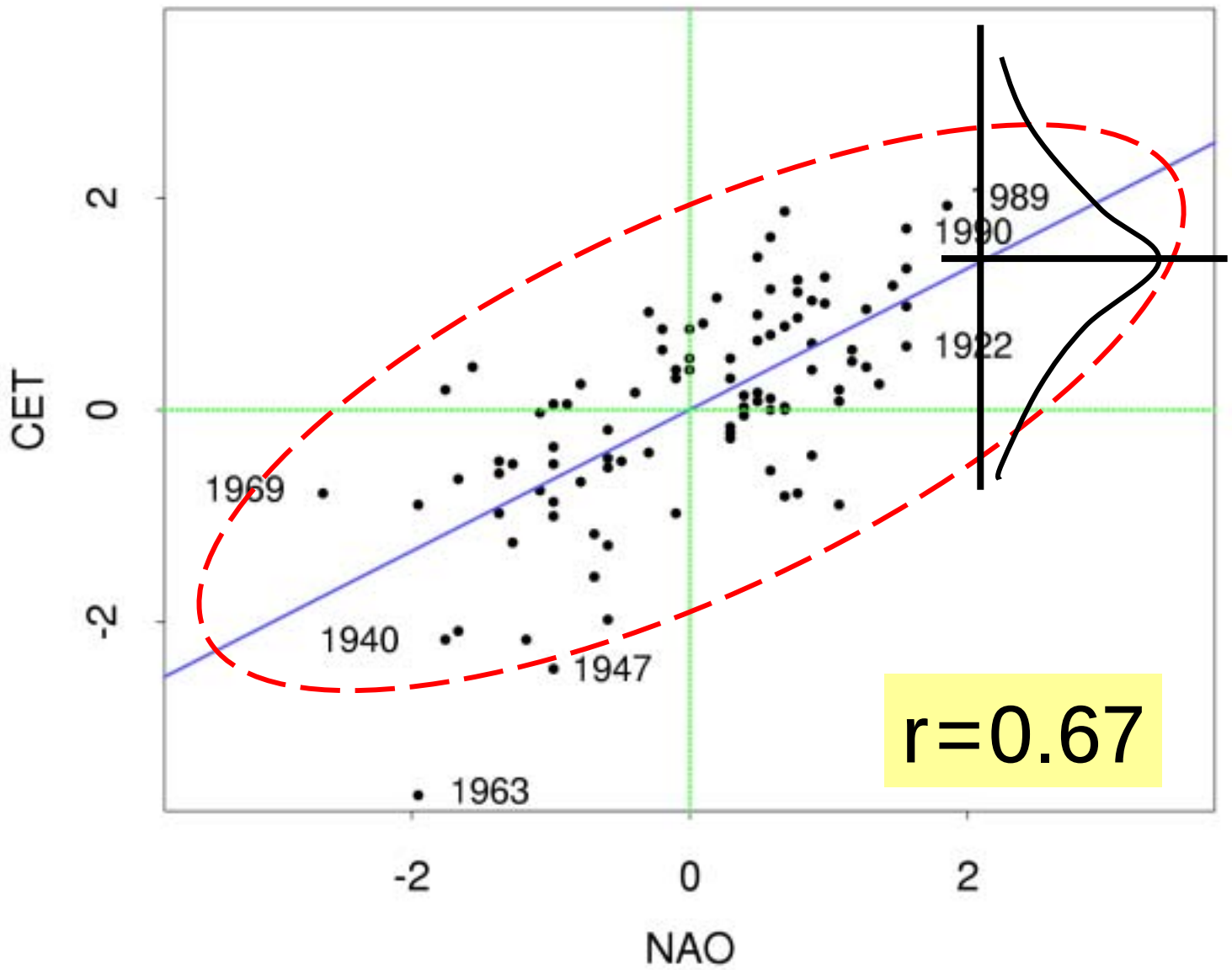
$$y_i = \alpha + \beta x_i + \varepsilon_i$$

$$\text{Minimise} \sum_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$$

- Probability model

$$Y \mid X \sim N(\alpha + \beta X, \sigma_\varepsilon^2)$$

8. Example: Regression of CET on NAO



8. Non-linear/non-normal responses

The Generalised Linear Model (GLM):

$$Y \mid X \sim F(\theta)$$

$$g(\theta) = X\beta$$

```
> # R command
> glm(y~x)
```

Some examples:

Linear regression: $F = N(\mu, \sigma^2)$ $\quad g(\mu) = \mu$

Gamma regression: $F = G(\alpha, \beta)$ $\quad g(\mu = \alpha / \beta) = 1/$

Logistic regression: $F = Be(\pi)$ $\quad g(\pi) = \log(\pi / 1 - \pi)$

Poisson regression: $F = Po(\mu)$ $\quad g(\mu) = \log \mu$

8. Non-parametric regression

Know that Y depends on X but have no idea what the functional relationship is except that it is smooth.

$$Y = f(X) + \varepsilon$$

Need to use *smoothing methods* to estimate the unknown function f(X). Main approaches are:

- local robust polynomial fits **`>lowess(x,y)`**
- smoothing splines **`>smooth.spline(x,y)`**
- kernel smoothing **`>kernel(.)`**

8. Kernel estimation

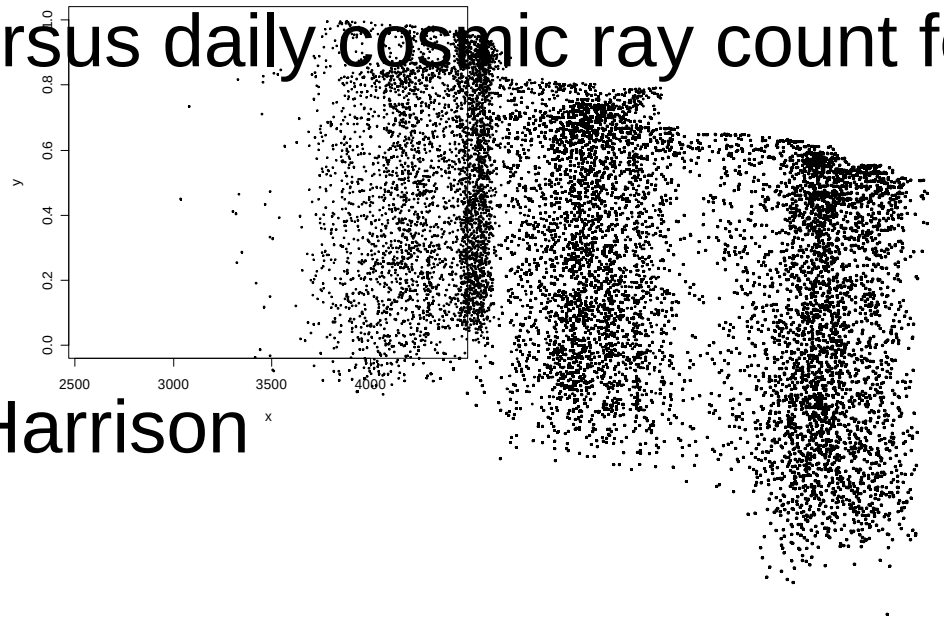
Smooth local composites:

$$\hat{f}(x) = \frac{\sum_{i=1}^{n} w(x - x_i) y_i}{\sum_{i=1}^{n} w(x - x_i)}$$

$$w(.) = \text{kernel weights}$$

8. Demo: scatter plot from hell …

Daily observations of diffuse solar radiation fraction (cloudiness) versus daily cosmic ray count for the past 50 years.



Source: Giles Harrison

Scientific question:

does cosmic radiation have an effect on cloudiness?

Summary

- Regression models the conditional probability p(Y|X)
- Many such models can be used depending upon the type of response(s), type of explanatory variable(s), and knowledge of distributional form.
- Linear regression can be extended to include multiple explanatory variables (multiple regression), multiple responses (multivariate regression), and non-linear/normal responses (Generalised Linear Models).
- Non-linear unknown relationships can be estimated using non-parametric regression approaches.