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SOME STUDIES IN TERRESTRIAL RADIATION

BY

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(Throughout this paper the unit of radiation is the gram calorie per cm^2 per min.)

In a recent paper R. Mugge[1] has calculated the terrestrial radiation which leaves the atmosphere in different latitudes and comes to the surprising result that the outgoing radiation from each square centimetre of the upper atmosphere in the polar regions is more than twice as great as the corresponding radiation in equatorial regions, and is even greater than the average incoming solar radiation per square centimetre at the equator. This result is reached in a simple and straightforward way and at first sight is convincing. Mügge's work is based on the assumption that the air in the stratosphere is in radiative equilibrium so that every cubic centimetre radiates as much energy as it absorbs. If this is true then the radiation through the air can be calculated from its temperature. As the temperature of the stratosphere is higher at the poles than at the equator, the radiation through the stratosphere must be greater at the poles than at the equator; further, there is a considerable amount of solar radiation at the equator which is absent at the poles. When this solar radiation is subtracted from the calculated radiation at the equator, the result leaves a large excess of outgoing terrestrial radiation at the poles over the outgoing radiation at the equator.

Mügge's results are collected in the following table:

TABLE I.—TEMPERATURE OF THE STRATOSPHERE AND SOLAR AND TERRESTRIAL RADIATION AS DETERMINED BY R. MÜGGE.

Latitude.	Temperature of the Stratosphere.	Incoming[2] Solar Radiation.	Outgoing[3] Radiation.
°	°a	cal/cm^2/min.	cal/cm^2/min.
0	190	·363	·203
9	196	·359	·229
18	202	·345	·258
27	207	·324	·292
36	212	·294	·325
45	217	·256	·356
54	221	·213	·383
63	224	·165	·408
72	226	·112	·423
81	227	·057	·438
90	228	·000	·449

[1] R. Mügge, *Zeitschrift fur Geophysik*, 2, 1926, p. 63.

[2] After deducting 43 per cent for loss by reflection.

[3] Mügge does not give a table of the total outgoing radiation, he only gives the difference between the outgoing and the incoming radiation. To the latter I have added the value of the incoming radiation, using the formula contained in the paper. In carrying out the calculation I have substituted the value $8{\cdot}26 \times 10^{-11}$ for Stefan's constant in place of $7{\cdot}68 \times 10^{-11}$ used by Mügge.

It will be seen from this table that whereas the outgoing radiation at the equator is only ·203 it is ·449 at the poles. This result is based on assumptions and methods of calculation which have been extensively used by writers on atmospheric radiation, and they are commonly accepted. Seeing no reason to reject them I was much exercised to find some explanation of how a cold polar atmosphere with little water vapour can emit more energy than the hot equatorial atmosphere heavily charged with water vapour.

The consideration of this problem has led me to make an attempt to calculate the radiation from the earth's atmosphere, employing the known temperature and humidity data which observations of the upper atmosphere have provided. No branch of atmospheric physics is more difficult than that dealing with radiation. This is not because we do not know the laws of radiation, but because of the difficulty of applying them to gases. There have been many papers written on this problem, the best known being those by Gold and by Emden, but they have all been of a general nature, and, so far as I know, none have discussed the difference between the radiation from the atmosphere in different latitudes. There is no hope of getting an exact solution; but by making suitable simplifying assumptions, it ought to be possible to get a qualitative result which would give useful information as to the variation of radiation with latitude.

I commenced the work with the following assumptions:

(*a*) *The only constituent of the atmosphere which absorbs and emits long wave radiation is water vapour.* With this assumption we neglect the air itself; this is legitimate, because so far as experiments can show neither oxygen nor nitrogen have absorption bands[4] at wave lengths longer than 1μ, and all the radiation emitted by bodies at terrestrial temperatures has a wave length greater than 2μ.

Carbon dioxide is provisionally neglected; but it is not difficult to show that it cannot modify the problem to any appreciable extent. Ozone also is neglected, because we shall be dealing only with the lower atmosphere where there is very little, if any, ozone present.

(*b*) *Water vapour absorbs terrestrial radiation like a "grey body."* This is an assumption made by practically every worker in the subject. We know it is not correct; but without it it is practically impossible to make any computations.

(*c*) *The water vapour is uniform in horizontal layer.* This assumption makes it possible for us to consider all the radiation with which we have to deal as parallel and travelling only in the vertical direction. The problem then reduces to one of radiation from, and through, parallel layers for which the calculations are very simple.

(*d*) *The temperature of the atmosphere decreases uniformly with height at the rate of 6°C. per kilometre.* One of the outstanding results of the investigation of the upper atmosphere is that the mean lapse rate within the troposphere is practically the same in all parts of the world. I therefore propose to consider an atmosphere built up of a number of layers, each one kilometre thick, and each 6° colder than the layer below. Although each of these layers is a kilometre thick their actual height above sea-level cannot be specified. For instance, the layer with a mean temperature of 241° is at 6·2 kilometres at Pavlovsk, 6·9 at Hamburg, 8·3 in the United States (Lat. 40°N.) and 9·8 at Batavia. Instead of labelling the layers by their height above sea-level we shall number them

[4] Burmeister, *Berlin, Verh. D. physik. Ges.*, **15**, 1913, p. 612.

consecutively from a convenient temperature. It was found convenient to make the layer having a range of temperature 220° to 226°, mean 223°, number 1, lower layers being numbered 2, 3, 4, etc., and higher layers − 1, − 2, − 3, etc.

(*e*) *The water vapour content can be computed by the formula given by Hergesell.* In the *Beiträge zur Physik der Freien Atmosphäre*, Vol. VIII., page 86, 1918, Hergesell gives the following formula to determine the mean relative humidity at various heights:

$$\log r = 1{\cdot}8333 + 1{\cdot}603\frac{t}{T},$$

in which r = relative humidity, t = temperature in degrees centigrade, and T = temperature in degrees absolute. It is admittedly only an empirical formula; but it fits the observations at Lindenberg and Batavia sufficiently well to justify its use. It is convenient to us because it involves only the temperature and not the height above sea level. With this assumption the relative humidity of each of our layers can be determined, and then by means of meteorological tables the quantity of water vapour in each layer can be computed from its temperature and relative humidity. The most convenient way of expressing the quantity of water vapour is to give the "depth of precipitable water." Thus the amount of water vapour in each kilometre layer will be given as the depth of water in mm. which would be deposited if all the water vapour in the layer were condensed into water.

Fig. 1 represents the section of the atmosphere considered, and in it the adopted values of the temperature and humidity are shown. This will be referred to as the standard atmosphere.

Having fixed the temperature and water content of each layer, we only need to know the absorption coefficient of water vapour to be in a position to calculate the radiation. Unfortunately, we do not know this coefficient. Abbot and Fowle [5] estimate that only one-tenth of the heat radiation from the surface passes right through the atmosphere. This, however, does not give us the true absorption coefficient, for it only gives the proportion of the energy which escapes absorption in the atmosphere, and this is mainly energy contained in a limited region of the long wave spectrum for which the absorption is small, while some wave lengths are totally absorbed by a very little water vapour. Still, as we have no other determination, we will commence with this value and see where it leads to.

We define the transmission coefficient of water vapour, y, as the proportion of radiation transmitted through a column containing one mm. of precipitable water (in vapour form). The proportion of radiation absorbed in the same column is then z, and $z = 1 - y$. z is called the absorption coefficient of water vapour.

If Y is the proportion of radiation transmitted through a column containing W mm. of precipitable water, and Z the proportion absorbed, then we have the general relationships:

$$\log Y = W \log y,$$
$$Z = 1 - Y.$$

These relationships hold quite independently of the length of the column containing the water vapour, for a short column containing W mm. of precipitable water absorbs and transmits the same amount of the

[5] Abbot and Fowle, *Washington, Ann. Astroph. Obs.*, **11**, 1908, p. 172.

incident radiation as a long column in which the same amount of water vapour is contained.

Assuming that Abbot and Fowle's value of $\frac{1}{10}$ surface radiation transmitted applies to middle latitudes, we can determine the total vapour in the atmosphere by summing the water contained in the layers given in

Number of Layer.	Mean Temperature.	Relative Humidity.	Precipitable Water.	
	°a	%	mm.	
				Tropopause in Batavia.
-5	193	23	·000	
-4	199	25	·000	
-3	205	27	·001	
-2	211	29	·003	
-1	217	32	·006	
				Tropopause in lat. 50°.
1	223	35	·01	
2	229	37	·03	
3	235	41	·06	
4	241	44	·12	
5	247	48	·24	
				Sea-level at Pole.
6	253	52	·46	
7	259	56	·86	
				Sea-level in lat. 70°.
8	265	61	1·56	
				Sea-level in lat. 60°.
9	271	66	2·74	
10	277	72	4·55	
				Sea-level in lat. 50°.
11	283	78	7·28	
				Sea-level in lat. 40°.
12	289	85	11·40	
13	295	92	17·70	
				Sea-level at Equator.
14				
15				

FIG. 1.

Fig. 1 down to layer 11. This is, approximately, 18 mm. of precipitable water.

$$\text{Hence } Y = 0{\cdot}1 \text{ for } 18 \text{ mm.,}$$

$$\text{therefore } \log y = \frac{1}{18} \log 0{\cdot}1,$$

$$y = {\cdot}88.$$

We may now apply this value of y to the atmosphere specified in Fig. 1 and calculate the radiation. The result is shown in Table II.

TABLE II.

1	2	3	4	5	6	7	8	9
Number of Layer.	Mean Temperature. °a.	Relative Humidity. %	Water Vapour, mm. of Precip. Water.	Transmission.	Absorption.	Radiation per Layer.	Transmission through Atmosphere.	Radiation transmitted through Atmosphere.
	T	r	W	Y	Z	E	$[Y]$	R
−4	199	25·4	·000	1·000	·000	·000	1·000	·000
−3	205	27·2	·001	1·000	·000	·000	1·000	·000
−2	211	29·5	·003	1·000	·000	·000	1·000	·000
−1	217	32·0	·006	1·000	·000	·000	1·000	·000
1	223	34·7	·014	·998	·002	·000	1·000	·000
2	229	37·5	·031	·995	·005	·001	·998	·001
3	235	40·8	·062	·991	·009	·003	·992	·003
4	241	44·3	·120	·984	·016	·004	·983	·004
5	247	48·0	·242	·968	·032	·010	·967	·010
6	253	52·0	·464	·942	·058	·020	·936	·019
7	259	56·4	·856	·895	·105	·039	·881	·034
8	265	61·2	1·56	·818	·182	·074	·789	·058
9	271	66·4	2·74	·701	·299	·133	·645	·086
10	277	72·0	4·55	·556	·444	·216	·452	·098
11	283	78·0	7·28	·391	·609	·322	·251	·081
12	289	84·6	11·4	·229	·771	·445	·100	·044
13	295	91·9	17·7	·100	·900	·563	·022	·013

The first four columns of Table II. contain the meteorological factors of our standard atmosphere.

We can at once calculate the proportion of radiation transmitted through each layer from the relationship

$$\log Y = W \log y = W \log \cdot 88$$

in which W is the value of the water vapour entered in column 4. Values of Y are entered in column 5.

The absorption, Z, of the layer is found at once from $Z = 1 - Y$. Values of Z are entered in column 6.

A layer of gas radiates in each direction, and the amount of radiation emitted in each direction is given by

$$E = Z\sigma T^4 \text{ gram calories per cm}^2 \text{ per min.}$$

in which σ is Stefan's constant[6]

$$= 8{\cdot}26 \times 10^{-11} \text{ gram calories per cm}^2 \text{ per min.}$$

and T is the absolute temperature of the layer.

Column 7 contains the values of E for each layer using the values of T and Z contained in columns 2 and 6 respectively.

In this paper we are chiefly concerned with the radiation which escapes out of the atmosphere. We shall therefore require to know how much of the radiation from each layer escapes, and we shall write R for this quantity. The radiation from each layer has to pass through all the

[6] As different values of Stefan's constant and the solar constant are used in the literature, I have adopted the values given in the 1919 edition of the Smithsonian Physical Tables as being the most authoritative available.

layers above it, therefore if R_n is the radiation which escapes through the atmosphere from layer n we have

$$R_n = E_n \times Y_{n-1} \times Y_{n-2} \times Y_{n-3} \dots \dots$$

$$R_n = E_n[Y]_n$$ in which $[Y]_n$ is written for the product of all the Ys above layer n.

Values of $[Y]_n$ are entered in column 8, and the resulting values of R are entered in column 9.

Table II. contains the radiation from each kilometre layer of the atmosphere, E (column 7), and the amount each provides, R (column 9), to the total outgoing radiation. The most important fact brought out by this table is that the first layer to provide any appreciable radiation is number 2, and that the next two layers, numbers 3 and 4, provide very little radiation. Now all these layers are well within the troposphere even in polar regions, therefore we may use Table II. without considering where the troposphere ends and the stratosphere commences, for the highest radiating layer is in the region of the atmosphere where the lapse is 6° C., to which conditions Table II. applies. To obtain the radiation from the atmosphere alone in any part of the world we have only to add up the radiation from the layers in Table II. which are above the surface at the place considered. Thus if the surface temperature is 274° layer 9 will lie on the surface, similarly all the layers will be above a place having a surface temperature of 298°a.

It now remains to consider how much the surface itself contributes to the total outgoing radiation.

If T_s is the temperature of the surface then the radiation emitted will be

$$E_s = \sigma T_s^4.$$

In order to escape from the atmosphere this radiation must traverse the whole of the overlying layers and suffer absorption in the process. If there are n layers above the surface, the absorption will be the same as that from the layer $n+1$ which the surface replaces; and Table II. contains the factor giving the proportion of radiation from layer $n+1$ which is transmitted through the atmosphere in column 8, namely $[Y]_{n+1}$. The amount of radiation transmitted by the atmosphere from the surface R_s is therefore

$$R_s = E_s[Y]_{n+1} = \sigma T_s^4 [Y]_{n+1}$$

in which n is the number of the layer resting on the surface.

The total radiation which leaves the atmosphere F is therefore

$$F = \Sigma R_n + \sigma T_s^4 [Y]_{n+1}$$

in which ΣR_n means the sum of the R's for each of the layers which emit radiation down to and including the layer resting on the surface.

The values of R and $[Y]$ are entered in Table II., and the value of σT_s^4 can be easily calculated.

We are now in a position to calculate the outgoing radiation from the different latitudes.

An example will help to show how the computation is carried through: we will therefore calculate the outgoing radiation from latitude 50°.

The mean annual temperature in latitude 50 is 5·6° C. *i.e.* 279°. The mean temperature of layer 10 is 277°, therefore the temperature at its under surface is $277 + 3 = 280°$. We may therefore represent the atmosphere in latitude 50 by the layers down to number 10 resting on a surface at 280°. We therefore have $T_s = 280°$, $n = 10$, $[Y]_{11} = \cdot 251$. Hence:

Total outgoing radiation from atmosphere $= \Sigma R_{10} = \cdot 313$
Total outgoing radiation from earth's surface $= \sigma 280^4 \times \cdot 251 = \cdot 127$.

Therefore:

Total outgoing radiation in latitude 50 = ·440 cal/cm²/min.

The outgoing radiation from other latitudes can be calculated in a similar way. Table III. contains the results for latitudes 90°, 70°, 60°, 50°, 40° and 0°.

TABLE III.—OUTGOING RADIATION FROM THE STANDARD ATMOSPHERE FOR VARIOUS LATITUDES, IF $y = \cdot 88$.

1	2	3	4	5	6	7	8
Latitude.	Surface temperature. T_s	Number of layer on surface. n	Radiation from surface. σT_s^4	Proportion transmitted. $[Y]_{n+1}$	Outgoing radiation from surface. R_s	Outgoing radiation from atmosphere. ΣR_n	Total outgoing radiation. F
Pole	250	5	·322	·936	·302	·018	·320
70	262	7	·380	·789	·306	·071	·377
60	268	8	·425	·645	·274	·129	·403
50	280	10	·508	·251	·127	·313	·440
40	286	11	·553	·100	·055	·394	·449
Equator	298	13	·652	·002	·001	·451	·452

It will be noticed from column 6 that much more radiation from the surface escapes from the atmosphere at the pole than at the equator; this is because the atmosphere has so little vapour at the pole that it can absorb very little radiation, and ·936 of the radiation emitted from the surface is transmitted. On the other hand, although the surface at the equator emits twice as much radiation as the surface at the pole only ·002 of it is transmitted, so that practically none escapes from the atmosphere.

As absorption and emission go hand in hand, the polar atmosphere emits little radiation while the atmosphere at the equator emits large quantities. The consequence is that there is a kind of compensation, and the total outgoing radiation is not very different at the equator and the pole, being ·452 and ·320 respectively.

In *Arbeiten des Preuss. Aeronaut. Obser. bei Lindenberg*, Vol. 13, 1919, Prof. Hergesell calculated the radiation leaving the atmosphere over Lindenberg in latitude 52°N. and Batavia in latitude 6°S. He adopted Abbot and Fowle's value for the absorption coefficient of water vapour, as we have done above; but used the actual temperature obtained from the very full series of sounding balloon records which are available for each place. His calculations gave the outgoing radiation as ·447 over Lindenberg and ·465 over Batavia.[7] These values are practically the same as we have calculated above: ·440 in latitude 50° and ·452 in equatorial regions. This agreement is very reassuring, as it shows that the simple method

[7] Hergesell's values are 24·7 and 25·7 cal/cm²/hour at Lindenberg and Batavia respectively; but he used an old value of Stefan's constant ($7 \cdot 59 \times 10^{-11}$ cal/cm²/min.). To convert them to the units used in this paper, in which the unit of time is the minute and Stefan's constant is taken as $8 \cdot 26 \times 10^{-11}$, it is necessary to multiply them by $\frac{8 \cdot 26}{7 \cdot 59 \times 60} = \cdot 0181$.

adopted in this paper gives practically the same results as the more elaborate method employed by Hergesell.

In Fig. 2 curve *A*, the values of the outgoing radiation from Table III. have been plotted against the sine of the latitudes. This has been done in order to give due weight to the area of the zones, for the area of a zone in latitude θ is proportional to $\sin\theta$. By this method the total outgoing radiation from the earth as a whole is proportional to the area below the curve plotted in Fig. 2. The first thing we notice is that over $\frac{7}{10}$ of the earth's surface the outgoing radiation is practically uniform at ·450 cal/cm²/min. At higher latitudes than 40° the intensity of the outgoing radiation *decreases* and at the pole it is ·320, that is about $\frac{3}{4}$ of its value at

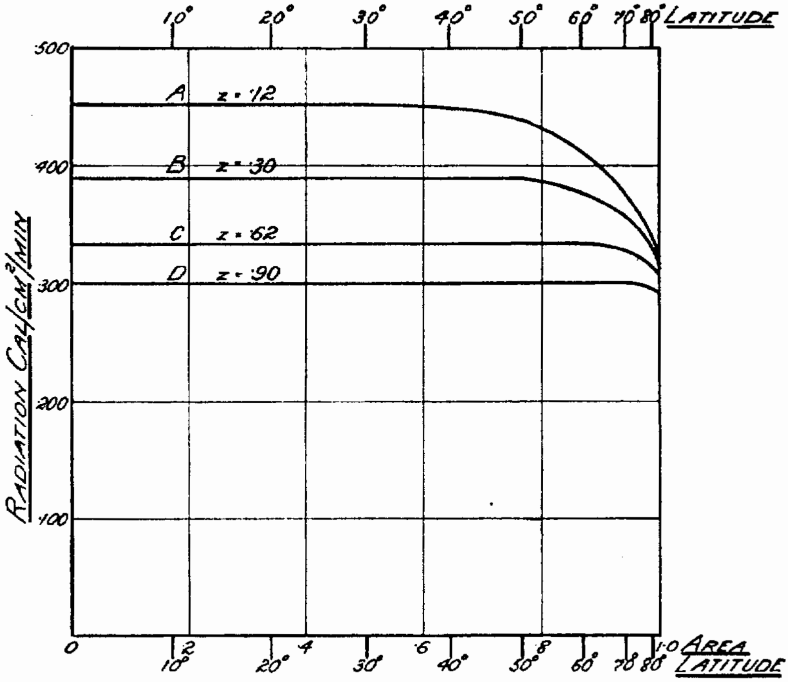


FIG. 2.

the equator. This is an entirely different result from that which we should expect from Mügge's work, for he found the outgoing radiation at the pole to be more than twice as much as at the equator.

We must, however, turn our attention to another aspect of the result of this preliminary investigation.

From Fig. 2 it will be seen that the average of the outgoing radiation from the earth as a whole is about ·430, the decrease in high latitudes bringing the mean value a little lower than the actual value between the equator and latitude 40°.

Now this average outgoing radiation must be equal to the incoming radiation averaged over the whole earth and over a complete year. The value of the latter can be calculated from known data.

The total incoming radiation depends on the "solar constant," the value of which is given in the Smithsonian Physical Tables, 1919, as 1·953 cal/cm²/min. This is the energy received on a square centimetre exposed at right angles to the solar rays. The total energy intercepted by the earth is therefore $\pi R^2 \times 1{\cdot}953$. This amount has, however, to be spread over the whole surface of the earth, therefore

$$\left.\begin{array}{l}\text{Average solar radiation}\\ \text{received by the earth's}\\ \text{surface}\end{array}\right\} = \frac{\pi R^2 \times 1{\cdot}953}{4\pi R^2} = {\cdot}488 \text{ cal/cm}^2\text{/min.}$$

Of this incoming radiation a large proportion is reflected back by the clouds and dust in the atmosphere without taking part in the heat exchanges of the atmosphere. Aldrich has determined this proportion (the albedo) to be 0·43; therefore the total effective incoming radiation is $\cdot 488 \times \cdot 57 = \cdot 278$ cal/cm²/min.

This value is however much smaller than the ·430 which we have found for the average outgoing radiation. Assuming that the work of Abbot and Fowle and of Aldrich is correct, the outgoing radiation which we have calculated is materially too large. This may be due to our adoption of a wrong absorption coefficient for water vapour, for we mentioned when adopting it that it was doubtful and could only be used in a preliminary investigation. We must therefore take up the question of a more suitable value for the absorption coefficient. We shall see later that the outgoing radiation decreases with an increase in the absorption coefficient, therefore we must look in the direction of a larger absorption coefficient. There is, however, a limit to the absorption of water vapour, and a coefficient of ·90 is already much too large. This coefficient would mean that water vapour equivalent to 1 mm. of precipitable water would absorb 90 per cent of the radiation passing through it, and we know that this is more than unlikely for a number of reasons. The coefficient, therefore, must vary between $z = \cdot 12$ which we have already found too small, and $z = \cdot 90$ which we believe to be too large.

An investigation similar to that set out in detail above for $z = \cdot 12$ has been carried out for a number of values of z, and the results for $z = \cdot 90$, ·62, ·30, and ·12 are set out in Table IV., in which the outgoing radiation from each layer is tabulated as well as the values of $[Y]$.

TABLE IV.

			$z = \cdot 90$		$z = \cdot 62$		$z = \cdot 30$		$z = \cdot 12$	
Layer.	T	W	$[Y]$	R	$[Y]$	R	$[Y]$	R	$[Y]$	R
− 2	211	·003	·998	·001						
− 1	217	·006	·990	·003	1·000	·001				
1	223	·014	·977	·007	·995	·002	1·000	·001	1·000	·000
2	229	·031	·946	·015	·984	·004	·995	·002	·998	·001
3	235	·062	·880	·030	·965	·009	·984	·006	·992	·003
4	241	·120	·762	·051	·929	·017	·961	·012	·983	·004
5	247	·242	·579	·078	·858	·030	·920	·025	·967	·010
6	253	·464	·332	·074	·768	·051	·843	·044	·936	·019
7	259	·856	·114	·037	·618	·071	·714	·070	·881	·034
8	265	1·56	·016	·006	·428	·080	·525	·091	·789	·058
9	271	2·74	·004	·000	·231	·065	·301	·084	·645	·086
10	277	4·55	·000		·085	·032	·113	·044	·452	·098
11	283	7·28			·018	·009	·022	·011	·251	·081
12	289	11·4			·002	·001	·002	·000	·098	·044
13	295	17·7			·000	·000	·000		·022	·013
Total outgoing . .				·302		·335		·390		·451

The values of R'—the radiation from each layer which escapes from the atmosphere—given in Table IV. have been plotted in Fig. 3 (a) and the result is an extremely instructive diagram. In the first place we notice that each curve has the same general shape. For each absorption

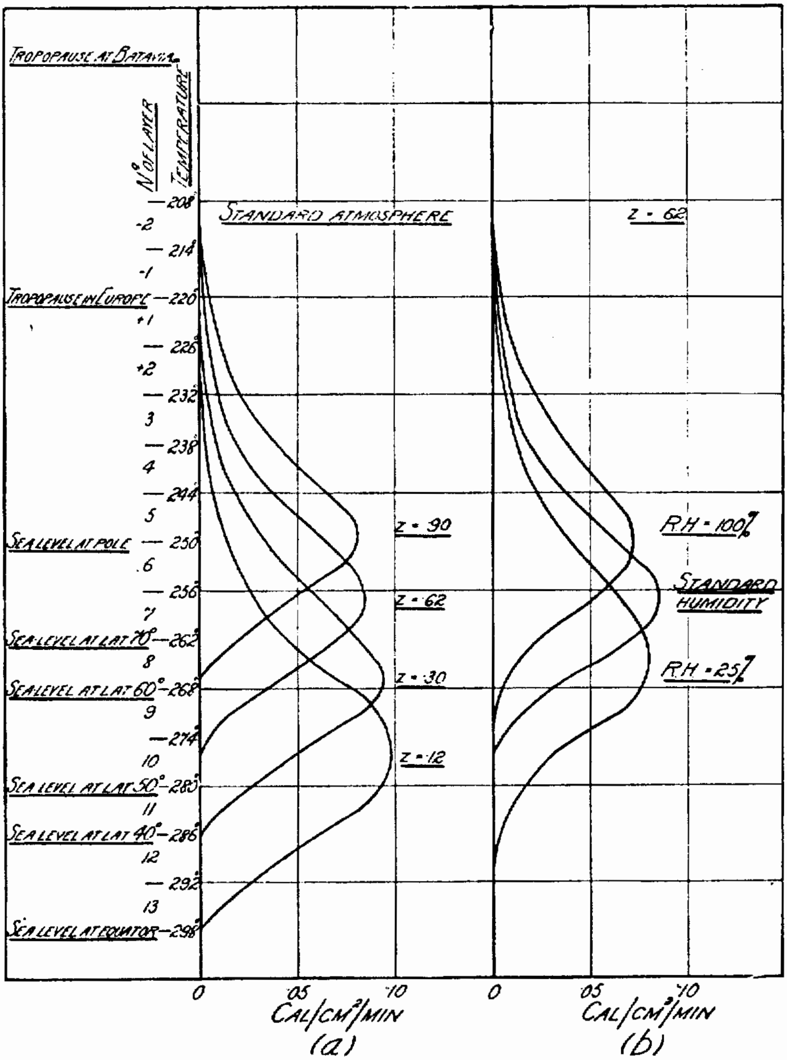


FIG. 3.

coefficient there is a layer in the atmosphere from which a maximum quantity of radiation escapes from the atmosphere.

The layers above and below contribute less to the outgoing radiation. We also notice that the greater the absorption the higher in the atmosphere is the layer of maximum outgoing radiation. The area contained between the curve and the zero ordinate is a measure of the total outgoing radiation from the atmosphere, and we notice that these areas increase with decreasing absorption coefficients. The values of the total outgoing radiation are entered in the bottom line of Table IV. and are plotted on Fig. 4.

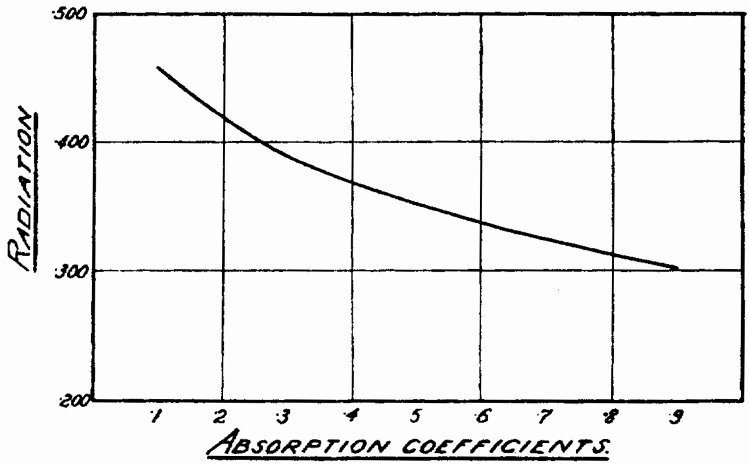


FIG. 4.

Another important point is that all the layers which contribute appreciable quantities of outgoing radiation have a temperature higher than 220°. As this is higher than the temperature at the tropopause in practically all regions of the atmosphere, all the effective layers are well within the troposphere.

By a process exactly similar to that described above the outgoing radiation has been calculated for a number of latitudes for each of the coefficients, using the values given in Table IV. The results are collected in the following table, and plotted on Fig. 2:

TABLE V.

Latitude.	Surface Temperature a.	Total outgoing radiation.			
		A [illegible] = ·12	B [illegible] = ·30	C [illegible] = ·62	D [illegible] = ·90
Pole	250	·320	·317	·308	·292
70°	262	·377	·360	·329	·302
60°	268	·403	·379	·335	·302
50°	280	·440	·390	·335	·302
40°	286	·449	·390	·335	·302
Equator	298	·452	·390	·335	·302
Average . . .		·430	·385	·332	·302

It will be noticed from Fig. 2 that with all coefficients the radiation remains uniform from the equator to high latitudes and then decreases towards the pole. With $z = \cdot 12$ the decrease commences at latitude 40°; with $z = \cdot 30$ at latitude 50°; with $z = \cdot 62$ at latitude 60°, and with $z = \cdot 90$ at latitude 70°.

The decrease towards the pole is however small, especially with the higher coefficients. Even with $z = \cdot 30$ eight-tenths of the earth's surface would have a uniform outgoing radiation.

In the last line of Table V. is given the average outgoing radiation, the area of the zones being taken into account. This last line gives the surprising result that even with an absorption coefficient of $z = \cdot 90$, which is greater than is permissible, the average outgoing radiation is ·302, which is still appreciably greater than the effective incoming radiation according to Abbot and Fowle's solar constant and Aldrich's value for the albedo of the earth. It would appear, therefore, that with the actual temperature distribution and assumed relative humidity outgoing radiation would be greater than the incoming radiation whatever may be the absorption coefficient of water vapour. This is of course an impossible result, and we must seek the error. There can be no question about the temperatures we have used, for the uniform lapse rate of 6° per kilometre is a well established fact within the limits which would affect our result. The relative humidity is more doubtful, and we will now see what alteration in our result we could obtain by making a different assumption about the relative humidity. In the above discussion we have used Hergesell's values of the relative humidity, and there is little doubt that they do represent the conditions as well as could be expected. Hergesell has shown that they reproduce the conditions at Lindenberg and Batavia very well indeed so far as the measurements of humidity are reliable—say up to where the temperature is −30° C., *i.e.* up to and including our layer 4. The higher layers contain so little vapour in any case that a large error in the relative humidity would produce little effect on the absolute value of the water vapour present. Still it is interesting to examine the effect of changing the humidity. There are natural limits: the relative humidity cannot be greater than 100 per cent, this then fixes an upper limit, also a reasonable lower limit to take would be a relative humidity of 25 per cent throughout the atmosphere. Calculations have been made with these limits for several of the absorption coefficients, but it will suffice to reproduce here the results with $z = \cdot 62$.

Table VI. contains the values of R for each layer with the water vapour in each layer shown in the column headed W. The values are plotted in Fig. 3(*b*), from which it is clear that the effect of increasing the humidity is the same as increasing the absorption coefficient. The greater the humidity the higher the effective layers are in the atmosphere and the *less* the outgoing radiation. This latter result is at first sight surprising. As the radiation is due to the water vapour alone, it would seem a natural inference that an increase in the contained water—the temperature remaining unchanged—would be accompanied by an increase in the outgoing radiation. This, however, is not the case, and the reason is that the increased water vapour causes the higher layers to absorb the radiation from below at a greater rate than they increase their own emission.

Our object in investigating the effect of changing the relative humidity was to see whether the discrepancy between the calculated outgoing radiation and the incoming radiation could be due to the adopted value for the relative humidity. We see from Table VI. that the total outgoing radiation with the three assumptions regarding the relative humidity is

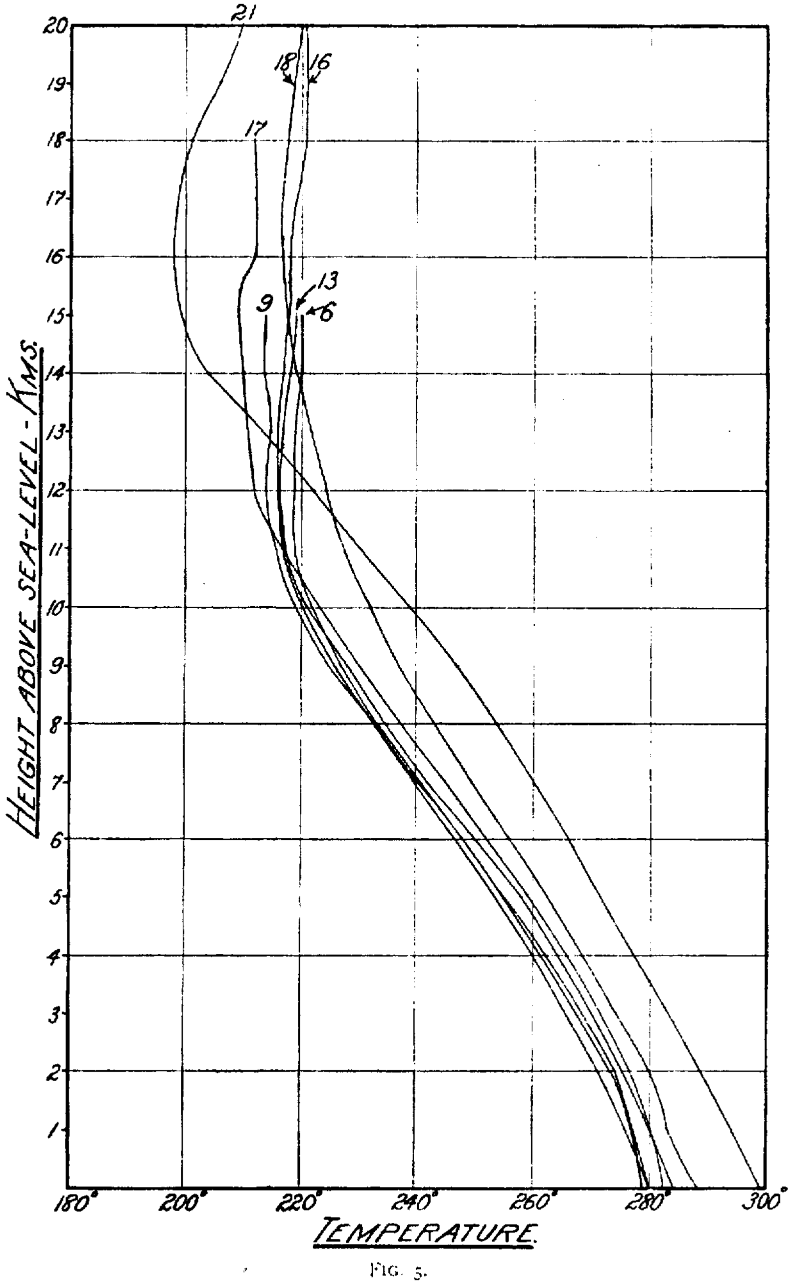
·300, ·335, and ·372. The change from the humidity distribution which we have adopted to either of the two limiting values of 100 per cent and 25 per cent uniform humidity produces only a change of 11 per cent in the outgoing radiation. As our adopted value of the humidity can

TABLE VI.

Layer.		Uniform Humidity. *R.H.* = 100 per cent.		Variable Humidity. Hergesell's Formula.		Uniform Humidity. *R.H.* = 25 per cent.	
	T	*W*	*R*	*W*	*R*	*W*	*R*
−2	211	·010	·002	·003	·000	·002	·000
−1	217	·020	·003	·006	·001	·005	·001
+1	223	·039	·007	·014	·003	·010	·002
2	229	·082	·016	·031	·007	·020	·004
3	235	·153	·030	·062	·013	·038	·009
4	241	·270	·048	·120	·027	·068	·017
5	247	·505	·069	·242	·051	·126	·030
6	253	·892	·069	·464	·077	·223	·051
7	259	1·52	·042	·856	·084	·380	·071
8	265	2·54	·013	1·56	·055	·635	·080
9	271	4·13	·001	2·74	·016	1·03	·065
10	277	6·33	·000	4·55	·001	1·58	·032
11	283	9·33		7·28	·000	2·33	·009
12	289	13·50		11·40		3·38	·001
13	295	19·22		17·70		4·80	·000
Total radiation from atmosphere			·300		·335		·372
Percentage			89%		100%		111%

only be slightly away from the true value—in any case only a fraction of the change in passing to either of the limiting values—the error due to having adopted a wrong distribution of water vapour can be only small, at the most 2 or 3 per cent in either direction. Thus we see that we cannot remove the discrepancy in this way.

We will return to this discrepancy later, but before doing so it will be as well to discuss Mügge's results in the light of what we have already achieved. We have already remarked that our results do not support the increase of outgoing radiation in passing from the equator to the poles; on the contrary we have found that any difference would be in the opposite direction, the atmosphere over the pole emitting less radiation than is emitted in equatorial regions. We are, however, now in a position to fix perfectly definite limits to the possible variation between the radiation from polar and equatorial regions. The effect of the atmosphere in all the cases we have investigated in this paper has been to reduce the outgoing radiation to a value less than if the surface of the earth itself radiated directly into space. It would not be difficult to give a formal proof that this must always be the case when the temperature of the absorbing gas decreases from the surface upwards, but that is unnecessary here. From this it follows that the total outgoing radiation at the poles cannot be greater than the radiation emitted by the surface of the earth at the pole.



Height above Sea-Level - Kms.
Temperature.
21
18
16
17
13
9
6
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
180°
200°
220°
240°
260°
280°
300°

Fig. 5.

In column 4 of Table III. is given the radiation from the surface, and we see that at the pole this is ·322 cal./cm.²/min. This is, therefore, the maximum amount of radiation which can leave the atmosphere at the pole. Mugge's value of ·449 for the outgoing radiation at the pole is obviously much too great. Even if the atmosphere were entirely absent, the mean annual temperature at the pole would have to be 272°a, *i.e.* − 1°C., to emit the large amount of radiation calculated by Mügge.

A minimum value for the outgoing radiation at the equator can also be calculated. We have seen that the outgoing radiation decreases with an increase in the absorption coefficient and the relative humidity. We have also seen that the absorption coefficient cannot be greater than $z = \cdot 90$, while the relative humidity cannot be greater than 100 per cent. If, therefore, we carry through a calculation on the lines described above, using these limiting values of the absorption coefficient and relative humidity, we obtain the minimum outgoing radiation at the equator. I have carried out this calculation and find that, under these conditions, ·261 cal./cm.²/min. of radiation would be emitted by the equatorial atmosphere. It is, therefore, clear that the ·203 found by Mügge is impossibly low.

There must be some error in the reasoning which has led to the impossible results obtained by Mügge, and we will endeavour to find it. As already stated, Mügge adopts the temperature of the stratosphere given in Table I. These temperatures are quite in agreement with the generally adopted values. He then follows Emden in determining the radiation which must pass through the stratosphere to maintain these temperatures on the assumption that the stratosphere is in thermal equilibrium under the radiation alone.

As no fundamental objection can be made against the method of calculating the radiation from the temperatures, the reason for the discrepancy must lie either in the temperatures themselves or in the assumption that the air which has these temperatures is in radiative equilibrium.

The first step was obviously to examine the records of upper air temperatures. In doing this I was greatly assisted by Sir Napier Shaw, who has recently collected all the most reliable upper air data for use in his forthcoming book, *The Manual of Meteorology*, Part II. (now in the press). For his book Sir Napier has prepared tables giving the mean values of the temperature at each kilometre above sea-level for summer and winter separately. As we require the mean annual temperature it was necessary to take the means of the summer and winter values. With Sir Napier's permission I give here the results for all the stations for which observations reaching the stratosphere are available, in both the summer and the winter.

Mügge's values for the stratosphere temperatures range from 190° at the equator to 228° at the poles. On looking at the temperatures at the top of the columns in Table VII., *i.e.* the temperatures of the highest layers, I was surprised to find that they were all within the range 210° (Batavia) and 222° (Pavlovsk). While these extremes are in the right direction the range is less than a third of that given by Mügge.

I then plotted the values (see Fig. 5) and was struck by the way the curves for the higher ascents all close together towards a temperature in the neighbourhood of 220°. It is clear that the curve for Batavia is approaching this temperature of 220°; and the few additional observations for Batavia at heights above 20 kilometres, although too few to give mean

TABLE VII.—MEAN ANNUAL UPPER AIR TEMPERATURES. FROM "MANUAL OF METEOROLOGY," VOL. II., BY SIR NAPIER SHAW.

	2 Kiruna.	3 Pavlovsk.	5 Kutchino.	6 Hamburg.	7 Lindenberg.	8 S. England.	9 Uccle.	10 Paris.	11 Nijni-Oltchédaeff.	12 Strassburg.	13 Vienna.	14 Munich.	16 Pavia.	17 Canada.	18 United States.	19 St. Louis.	21 Batavia.
Lat. Long.	68° N. 20° E.	60° N. 30° E.	56° N. 38° E.	54° N. 10° E.	52° N. 14° E.	52° N. 1° W.	51° N. 4° E.	49° N. 2° E.	49° N. 28° E.	49° N. 8° E.	48° N. 16° E.	48° N. 12° E.	45° N. 9° E.	43° N. 81° W.	40° N. 100° W.	39° N. 90° W.	6° S. 107° E.
Height.																	
20													221		220		210
19													220		219		206
18													221	212	218		201
17													219	212	217		199
16												218	218	212	217		198
15				220			214	220	221	218	219	219	218	209	218	216	199
14	219			220	220	219	214	220	222	217	219	219	217	210	219	214	204
13	220			219	219	219	215	221	223	216	217	218	216	211	222	214	213
12	219	222	218	219	218	219	214	221	222	215	216	218	216	212	224	216	222
11	219	222	218	219	219	219	216	220	221	217	217	219	217	217	227	220	230
10	221	223	219	222	221	223	219	223	223	221	220	223	221	223	232	226	239
9	228	225	225	227	226	229	225	230	226	228	226	229	228	230	237	232	247
8	234	229	232	233	233	235	233	237	233	235	232	236	235	237	243	239	254
7	241	235	240	240	240	242	241	244	241	242	240	243	242	245	249	248	260
6	248	242	247	247	248	249	248	252	248	250	248	250	250	252	256	256	266
5	254	249	253	254	255	255	255	258	255	256	255	257	258	259	262	262	271
4	260	255	259	260	261	262	262	264	261	263	261	263	264	265	268	268	277
3	265	261	266	265	267	268	268	270	266	269	267	269	270	271	274	273	283
2	269	266	270	271	272	272	274	275	271	274	273	274	275	276	280	277	288
1	273	271	275	276	277	277	277	279	276	278	277	279	280	280	283	282	294
Ground	269	275	278	280	281	282	279	281	290	281	280	—	284	282	288	286	299

values, do show a still further increase of temperature with height. In Fig 6 (*a*) I have plotted the lowest temperatures observed at each station against the latitude, and in Fig. 6 (*b*) the temperatures at the highest points reached. On each diagram the temperatures of the stratosphere used by Mügge have been indicated by the curved lines.

As the lowest temperatures generally occur near the bottom of the stratosphere, Fig. 6 (*a*) may be said to give the temperatures at the base of the stratosphere; and it cannot be said that they disagree with Mügge's values; while Fig. 6 (*b*) gives the temperature at some distance within the stratosphere. From these curves it appears legitimate to draw the conclusion that, while the temperature at the base of the stratosphere increases from the equator towards the pole, the temperature well within the stratosphere is independent of the latitude and tends towards a constant temperature of about 220°.

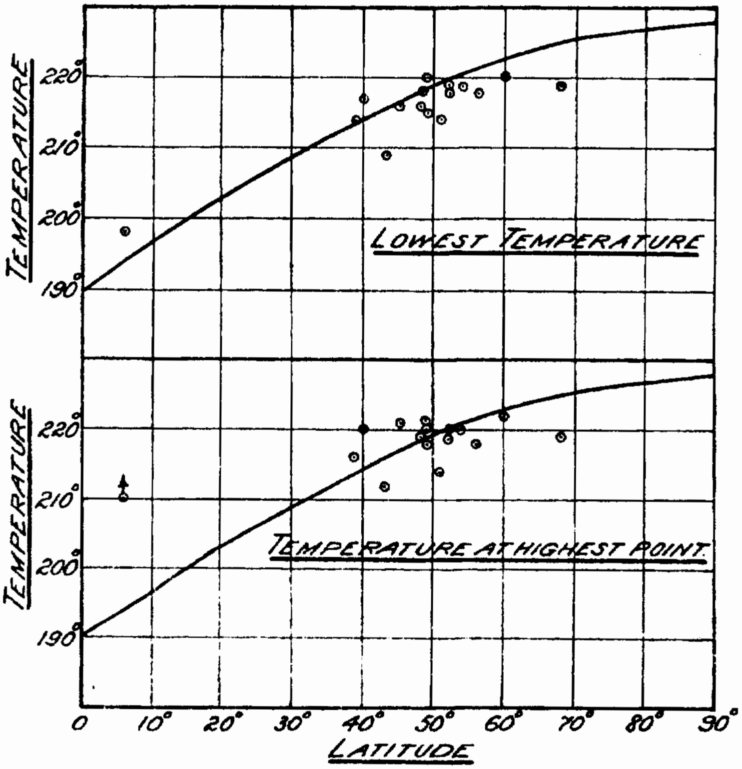


FIG. 6.

Now, which of these temperatures—the temperature at the base, or the temperature well within the stratosphere—is likely to be the temperature of radiative equilibrium? There is little doubt that it is the uniform temperature well within the stratosphere, for the temperature at the base of the stratosphere is necessarily that at the top of the troposphere which is determined by convection and turbulence within the troposphere and not by the outgoing radiation. It is only at some distance within the stratosphere where even minor and occasional convection currents cease to penetrate that radiation alone can determine the temperature.

The explanation of Mügge's results is now clear, he has calculated the radiation from temperatures which are not those of radiative equilibrium, and, therefore, from temperatures which do not give a measure of the radiation.

This investigation of the upper air temperatures has, however, led to a result which is of great importance in our own investigation, for if the temperature of the upper stratosphere, where radiative equilibrium has probably been established, is the same in all latitudes, we have another proof that the outgoing radiation is independent of latitude. Not only is this important because it confirms our previous result; but also because it gives us an independent measure of the intensity of the outgoing radiation; for we can use this temperature to determine the radiation in the way Mügge has done.[8]

Mügge took the incoming solar radiation into account; but, as the formula for doing this is very uncertain, and as we know that the absorption of the short wave solar radiation is very small, we can safely neglect the solar radiation and assume that the temperature of the upper stratosphere is determined by the long wave outgoing radiation alone. We may therefore use Humphreys' relationship

$$F = 2\sigma T^4,$$

in which F is the outgoing radiation and T is the uniform temperature of the upper stratosphere. Putting in the values $T = 220°$ and $\sigma = 8{\cdot}26 \times 10^{-11}$ we find

$$F = {\cdot}386 \text{ cal./cm.}^2\text{/min.}$$

We have now two values for the average outgoing radiation, (*a*) the value determined from consideration of the solar constant and the albedo, ·278, and (*b*) the value determined by the temperature of the stratosphere, ·386. We have already seen that the former is inconsistent with radiation from water vapour in the troposphere, for it necessitates an absorption coefficient which is quite impossible. On the other hand, this objection cannot be raised against the latter value, for we see from Table V. that an absorption coefficient of $z = {\cdot}30$ gives exactly the right amount of outgoing radiation. A coefficient of $z = {\cdot}30$, *i.e.* 1 mm. of precipitable water in the form of vapour absorbs 30 per cent of the incident long wave radiation, is quite consistent with the laboratory observations of Rubens and Aschkinass[9] on the absorption of water vapour, and is also consistent with a number of observations on atmospheric radiation made by Ångström and others.

It would therefore appear from this that the real outgoing radiation is likely to be nearer ·386 than ·278. The latter value was obtained by considering the solar constant and the earth's albedo. The adopted value of the solar constant is based on many measurements by a method which has been much discussed and generally approved. The solar constant is almost certainly correct to less than 1 per cent. The adopted value of the albedo, ·43, has, however, nothing like the same reliability. I do not

[8] As air does not absorb and emit long wave radiation, it would appear at first sight as though the temperature of the stratosphere would be independent of the terrestrial radiation. There is, however, in the absence of convection and conduction, sufficient water vapour throughout the stratosphere to maintain radiative equilibrium.

[9] *Ann. Physik u. Chemie*, **64**, 1898, p. 593.

wish to go into the whole question of the determination of the albedo of the earth here; but I am sure that anyone who has examined the question will agree that we have far too few data on which to establish a reliable value. In the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. II., p. 145, Abbot obtains the value ·337 for the albedo, and this was the accepted value for many years. In 1918 Aldrich, as the results of observations from a balloon, corrected this value to the one we have adopted, ·43, *i.e.* a change of nearly 30 per cent in the value of the albedo. In order that the outgoing radiation should have the value we have found from the temperature of the stratosphere the albedo would have to be ·21. It is difficult to believe that Abbot and Aldrich's method of determining the albedo is so far in error as to give twice the real amount. Too much stress, however, is not to be laid on our actual numerical value, and it is quite possible that our determination of the outgoing radiation is on the high side; but it is equally possible that the adopted value of the albedo is too large. I have no doubt that this discrepancy will disappear with future work. In the meantime I propose to adopt $z = ·30$ as the absorption coefficient of water vapour, and ·386 cal./cm.^{2}/sec. as the mean value of the outgoing radiation.

With these values it is possible to examine some interesting properties of the thermal state of the atmosphere, and to exhibit them in a very instructive graphical form. The reader may be interested to compare the results which follow with those contained in a valuable paper by Mr. W. H. Dines,[10] in which the method of calculation adopted in this paper was first used.

TABLE VIII

RADIATION IN LATITUDE 50°.

$z = ·30.$

1	2	3	4	5	6	7	8	9	10	11	12	13
Layer.	Height above sea-level. km.	Mean temperature °a.	Water content mm.	Absorption.	Black body radiation.	Radiation from layer.	Transmission through atmosphere.	Radiation transmitted through atmosphere.	Total upward radiation.	Total downward radiation.	Excess of upward radiation.	Loss of heat in layer.
	H	T	W	Z	σT^4	E	$[Y]$	K	F_1	F_2	F_3	
									↑	↓	↑	
1	9–10	223	·014	·005	·204	·001		·001	·391	·000	·391	·000
2	8–9	220	·031	·011	·227	·002	·995	·002	·392	·001	·391	·000
3	7–8	235	·062	·023	·252	·006	·984	·006	·394	·003	·391	·003
4	6–7	241	·120	·043	·276	·012	·961	·012	·397	·009	·388	·007
5	5–6	247	·242	·084	·307	·026	·920	·025	·402	·021	·381	·016
6	4–5	253	·464	·153	·339	·052	·843	·044	·410	·045	·365	·035
7	3–4	259	·856	·264	·372	·098	·714	·070	·420	·090	·330	·056
8	2–3	265	1·56	·427	·407	·174	·525	·091	·438	·164	·274	·080
9	1–2	271	2·74	·624	·445	·273	·301	·084	·462	·268	·194	·089
10	0–1	277	4·55	·803	·487	[illegible]	·113	·044	·491	·286	·205	·064
Surface	0	280			·508	·508			·508	·467	·041	·041

[10] *Quarterly Journal of the Royal Meteorological Society*, Vol. XLVI., 1920, p. 163.

The necessary calculations have been carried out to determine the long wave radiation which traverses all parts of the atmosphere in latitude 50° using the following data. Surface temperature, 280°; lapse rate, 6° per kilometre; humidity given by Hergesell's formula; absorption coefficient of water vapour, ·30 for 1 mm. of precipitable water. The results are given in Table VIII. and plotted in Fig. 7.

The method of obtaining the values in columns 1 to 9 of Table VIII. has already been described and need not be repeated here.

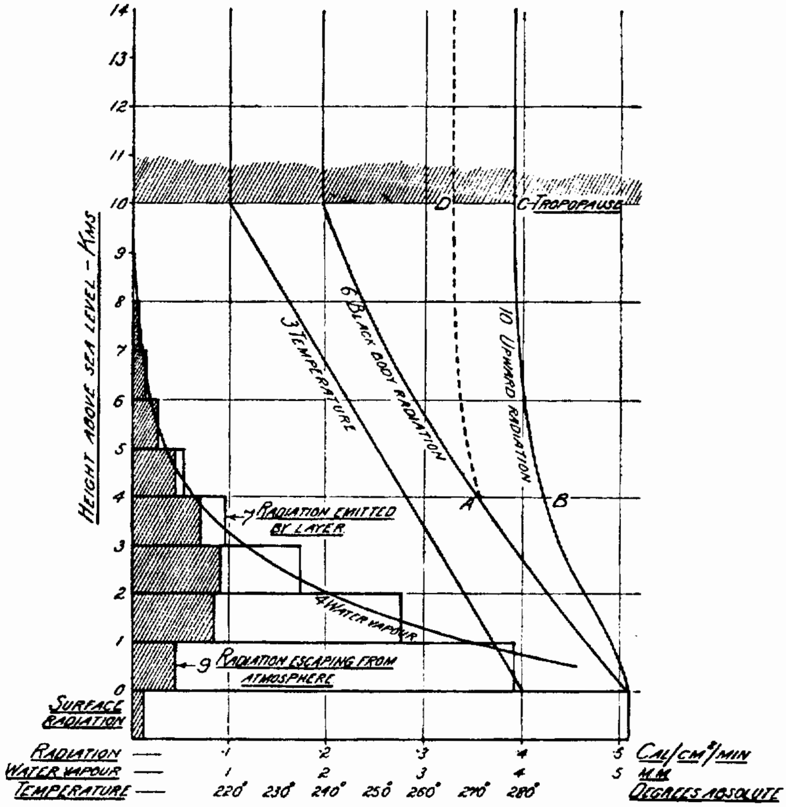


Fig. 7.

Column 10—Total Upward Radiation. This column gives the total long wave radiation which crosses the surfaces between consecutive layers in the upward direction. Starting at the ground the upward radiation is ·508 (column 7); of this ·803 (column 5) is absorbed in the first layer, leaving ·100 to proceed upwards. To this is added the radiation from the first layer ·391 (column 7) so that ·491 (column 10) crosses the surface between the first and second layers. The same process is repeated at the successive layers until the radiation becomes constant owing to the absence of further water vapour at the top of layer 1.

Column 11—Total Downward Radiation. The same process is repeated in the opposite direction. No appreciable long wave radiation is received from above, therefore the stream of downward radiation starts with the contribution from layer 1. To this is added the radiation from layers 2, 3, and 4 before appreciable absorption takes place. Afterwards the absorption of the downcoming radiation and the emission of new radiation is taken into account. Finally the stream has an intensity of ·467 when it reaches the ground.

Column 12—Excess of Upward Radiation. The actual flow of long wave radiation is the difference between the upward and downward radiation, *i.e.* $F_1 - F_2$. Values of this quantity are given in column 12.

Column 13—Loss of Heat in each Layer. The resultant upward stream of radiation increases from the ground upwards (column 12). Each layer therefore receives from below less radiation than it passes on to the layer above. This would result in a loss of heat from the layer if a source of heat other than long wave radiation were not present. The magnitude of the loss is given in column 13 by subtracting the successive values of the resultant upward radiation entered in column 12. This loss is made good by the heat liberated in the condensation of water vapour and by heat transferred from one place to another by the general circulation of the atmosphere. It is not proposed to discuss that aspect of the subject further in this paper.

The data contained in Table VIII. are shown graphically in Fig. 7, the curves on the diagram being numbered according to the columns in the table. In this diagram the ordinates are heights above sea-level in latitude 50°, the abscissæ being different for the different curves as indicated by the scales at the foot of the diagram. It is not feasible to discuss this diagram in detail here; but the following points may however be mentioned. Each layer emits radiation, in both directions, represented by the length of the "steps," while the amount of the radiation which escapes from the upper atmosphere is represented by the part of each step which is shaded. The total shaded area represents the total amount of radiation which escapes from the upper atmosphere. It will be seen that all the layers contributing to the outgoing radiation are well below the stratosphere, and very little radiation is contributed by the upper two kilometres of the troposphere. The most effective layer is the one between 2 and 3 kilometres above sea-level, this layer alone providing 23 per cent of the total outgoing radiation, while the surface provides less than 2 per cent. The diagram as a whole is a very instructive summary of the long wave radiation in the atmosphere in latitude 50°.

In the discussion so far we have neglected the clouds, and as these obviously must seriously modify the radiation through a clear atmosphere their effect must be considered. It is not possible to calculate the actual effect of the clouds, and we will therefore confine our investigations to determining the magnitude of the largest possible effect and then deduce from this the probable actual effect. The simplest way of thinking of the effect of a cloud is to consider each particle as obstructing all the radiation which falls on it; but emitting the radiation appropriate to a black body at its own temperature. When a cloud is sufficiently thick all the radiation which falls on it will be absorbed, and the surface of the cloud will emit the full radiation of a black body at the temperature of the cloud surface. Now imagine a layer of cloud with its upper surface 4 kilometres above sea-level. This will cut off all the normal upward flowing radiation, which in the clear atmosphere would be represented by the point *B* on curve 10 of Fig. 7. The upper surface will, however, radiate as a black body with an intensity given by the point *A* on curve 6 of Fig. 7. The layers above the top of the cloud will contribute their usual radiation, and from the top of the cloud the upward stream of radiation will run parallel to the original curve 10; but starting now from *A* instead of from *B*. This is shown in Fig. 7 by the dotted curve *AD*. The effect of the cloud layer on the total outgoing radiation has therefore been to reduce the outgoing radiation from *C* to *D* on Fig. 7. But *CD* is practically equal to *AB*, therefore the reduction in the outgoing

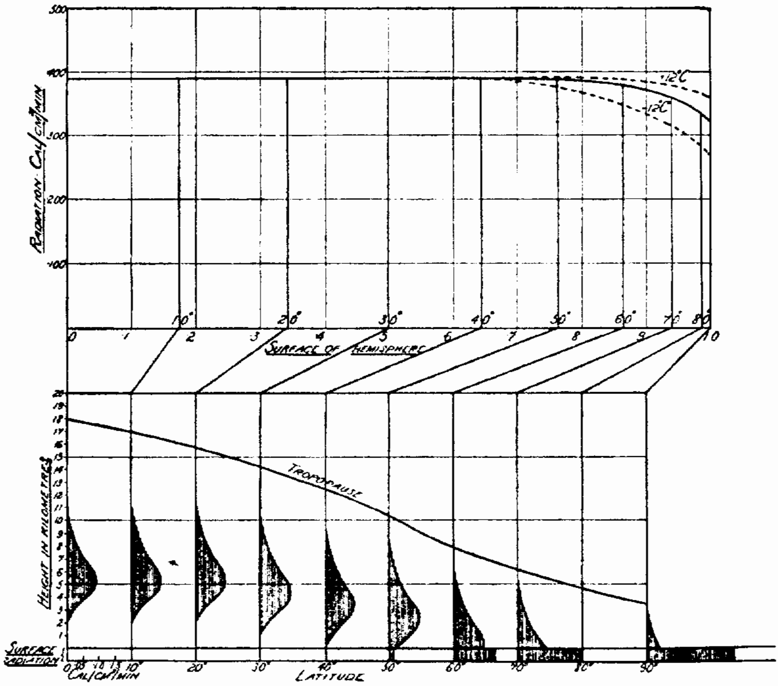


FIG. 8.

radiation is represented by the distance between the curves 10 and 6 at the height of the top of the cloud.

This, however, is a maximum reduction, and is only strictly true when the cloud is sufficiently dense. A thin layer of high cloud would no more absorb all the outgoing long wave radiation than it absorbs all the incoming sun light, therefore the higher clouds only reduce the outgoing radiation by a fraction of the distance between the two curves. We do not know the frequency of the different forms of clouds, still less the frequency of the height of the surfaces of the clouds; but we do know that the average "cloud amount" of the whole world is about five-tenths of the sky.[11] As a large proportion of the clouds going to form this total are thin high clouds, and most of the heavy low clouds do not extend more than two or three kilometres above the surface, we shall certainly over-estimate the effect of the clouds on the radiation if we assume that they

[11] C. E. P. Brooks gives 5·44 tenths: *Memoirs, R. Meteor. Soc.*, **1**, 1927, No. 10, p. 135.

are all replaced by a heavy layer of clouds at a height of 4 kilometres.[12] In other words, we will assume that the actual clouds are equivalent to a layer of clouds covering half the sky with its upper surface at 4 kilometres in latitude 50°, and at the height having the same temperature 238° (−35° C.) in other latitudes. The effect of this layer of clouds will be half that of a full layer at 4 kilometres, which we have shown to be equivalent to AB of Fig. 8. Now $AB = \cdot 06$ cal./cm.²/min. so that the loss is ·03 cal./cm.²/min., while the outgoing radiation in the absence of the cloud is ·391 cal./cm.²/min. Hence the reduction is 7 per cent, and as we have purposely chosen the maximum effect, we may say that the clouds are not likely to affect the calculated outgoing radiation by 5 per cent.

The numerical work in this paper lays no claim to be accurate to 5 per cent, so we are justified in neglecting the effect of the clouds. It is however desirable to see the direction in which our results would have been changed if the effect of the clouds had been taken into account.

It will be remembered that we calculated the amount of outgoing radiation with various absorption coefficients and then chose that absorption coefficient which gave the same value for the outgoing radiation as that which we had calculated from the temperature of the upper stratosphere. To allow for the clouds we should have taken into consideration that the clouds would reduce the outgoing radiation from a clear atmosphere by about 5 per cent, and therefore have calculated the absorption coefficient, which would give 5 per cent more radiation than that required in the absence of clouds. Had we done this we should have found an absorption coefficient $z = \cdot 23$ instead of the $z = \cdot 30$ which we have used. This could have produced no appreciable difference in the qualitative results we have obtained, and as we cannot be sure of our numerical results to 5 per cent, it seemed best to neglect the clouds as introducing an unnecessary complication. It will, however, be well to remember that our value of $z = \cdot 30$ is probably too large.

We now come to the most interesting part of our investigation, for we must consider how the earth's atmosphere will react towards a change in solar radiation. We shall be helped in this investigation if we have a clear mental picture of the present conditions in all latitudes. In order to obtain this I have prepared Fig. 8, which summarises what has been said above and applies it to the atmosphere as a whole. The diagram represents the source and magnitude of the outgoing radiation from different latitudes. In the lower half of the diagram the ordinates are height above sea-level and the main abscissæ degrees of latitude starting at the equator on the left and extending to the pole on the right. On the ordinates at each 10° of latitude a curve is plotted to show the outgoing radiation from each layer in the atmosphere, the abscissæ for these small curves being shown by the small scale between 0° and 10° on the main latitude scale.

The areas between these curves and the ordinates to which they are attached, shaded in the diagram, represent the total outgoing radiation at the latitude considered. Thus the first shaded area on the left of the diagram represents the outgoing radiation from the equator; the next to the right the radiation from latitude 10°, and so on. The

[12] The mean heights of the various forms of cloud at Potsdam are given in Hann's *Lehrbuch der Meteorologie* as follows:

Ci	8·2 km.	A-Cu	3·3 km.	Cu-Nb, summit	4·4 km.
Ci-St	7·9 ,,	St-Cu	1·7 ,,	Cu-Nb, base	2·4 ,,
Ci-cu	5·5 ,,	Cu, summit	1·8 ,,	Nb	1·6 ,,
A-St	3·3 ,,	Cu, base	1·3 ,,		

temperature at the surface is taken to be approximately the mean temperature of the latitudes determined by Meinardus, and the lapse rate is taken to be 6° per kilometre everywhere in the troposphere.

The tropopause is indicated by the line extending from a height of 18 kilometres at the equator to 4 kilometres at the pole. The exact position of this line is not known, so I have drawn it at the position in the atmosphere where the temperature would be that given by Mügge for the temperature of the stratosphere in the various latitudes.

At the equator the only layers in the atmosphere which contribute to the outgoing radiation are those between 2 and 12 kilometres above sea-level. The energy emitted by the two lower layers and by the surface is entirely absorbed by the overlying layers, while the layers above 12 kilometres are too cold to contain sufficient water vapour at those low temperatures to contribute anything appreciable to the radiation. From the equator to latitude 40° the effective layers are all above the surface, and the total outgoing radiation is the same at each latitude. At latitude 50° the lower radiating layer has disappeared and is replaced by the surface, a small proportion of the radiation from which is now able to escape from the atmosphere. The total radiation here is slightly less than from the lower latitudes. The details for this latitude are given in Table VIII. and are plotted on a larger scale in Fig. 7. At higher latitudes progressively less radiation goes out from the atmosphere, and progressively more from the surface; while the total radiation emitted decreases slightly from latitude to latitude.

Owing to the decreasing area of equal latitude zones from the equator to the poles the lower half of Fig. 8 does not give a proper picture of the outgoing radiation from the earth as a whole. To rectify this the upper half of Fig. 8 has been prepared. In this diagram the abscissæ are equal areas of the earth's surface, and the position of each 10° of latitude is indicated by connecting each of the nine ordinates in the lower diagram to the corresponding ordinates in the upper diagram, according to the sine of the latitude. The thick line curve gives the average outgoing radiation in all parts of the world, and as the abscissæ represent areas of the earth's surface the area of the diagram below the curve represents the total outgoing radiation from a single hemisphere.

The problem we have now to investigate is what effect will be produced if the effective solar radiation (that is the total solar radiation reduced by the albedo) changes? As the only way in which heat can leave the earth is by radiation, the total outgoing radiation must equal the incoming radiation, unless the temperature of the earth as a whole is going to continue to change. At present the earth is in thermal equilibrium, therefore we must assume that the outgoing radiation balances the incoming radiation. If now the solar radiation increases, say by 10 per cent, how will the outgoing radiation readjust itself to re-establish equilibrium?

One's natural instinct is to say that the temperature will rise until the increased temperature raises the outgoing radiation to the required amount. A rise in temperature, however, produces no increase in the outgoing radiation at the equator (assuming that there is no other change, as for instance in the lapse rate or the relative humidity). The outgoing radiation at the equator does not depend at all on the surface temperature, it simply depends on the water vapour in the layers having a temperature between 220° and 286°. At present these layers commence two kilometres above sea-level at the equator, and if the temperature at sea-level were increased they would simply be raised in the atmosphere without undergoing any change. Similar conditions exist from the equator to about latitude 45°, which comprises an area of three-quarters of the total surface of the earth. Thus an increase of temperature over three-quarters of the earth's surface would add nothing to the outgoing radiation. At latitudes higher than 45° an increase of sea-level temperature would lead to additional outgoing radiation; but to an insignificant amount. If the temperature of polar regions were increased by 30°C., so that the maximum outgoing radiation were emitted at all latitudes, the outgoing radiation from the earth as a whole would be increased only by 1·3 per cent. A similar result is obtained when we consider a decrease in solar radiation. A fall of the sea-level temperature would result in a slight lowering of the outgoing radiation in high latitudes; but the radiation would be unaltered from low latitudes until the temperature had fallen many degrees. To get a numerical value I have calculated the effect of raising and lowering the existing temperatures in all latitudes by 12°C., and the result is shown by the dotted curves in Fig. 8. The only change has been in higher latitudes than 40°. The changes in outgoing radiation are represented by the areas between the full curve and the two dotted curves, and by measuring these it will be seen that the change is ·6 per cent of the total radiation in the case of a rise of temperature and less than 2 per cent in the case of the fall of temperature. Now we know that the sun's radiation undergoes, even in our time, changes of several per cent; but there are no changes of sea-level temperature even approaching those which we have seen result in less than 2 per cent change in the outgoing radiation. It is therefore clear that the primary adjustment between incoming and outgoing radiation cannot be effected by changes of sea-level temperature.

As a simple rise and fall of temperature affects the outgoing radiation to such an insignificant extent, we must seek for some other mode of adjustment. We naturally turn to changes in water content; that is to changes in relative humidity. Even here the possible range of adjustment is very small. We saw above that, if the relative humidity everywhere were at its maximum of 100 per cent, the outgoing radiation would be reduced only by 11 per cent; while if the relative humidity everywhere were reduced to the uniformly low value of 25 per cent, the outgoing radiation would be increased also by 11 per cent. We ought, however, to notice that with increasing relative humidity would also go an increase in the amount of cloud, the effect of which on the outgoing radiation would be in the same direction. It is, however, difficult to imagine any mechanism by which an increase in solar radiation would produce the *decrease* in relative humidity and cloud formation which would be necessary to increase the terrestrial radiation. The change would appear to go more naturally the other way, an increase in solar radiation being accompanied by more ascending currents carrying more water into the upper atmosphere and producing more cloud. Thus changes in the relative humidity do not appear a very hopeful solution of the problem.

The only other variable factor affecting the outgoing radiation is the lapse rate. Throughout this work we have adopted a constant lapse rate of 6° C. per kilometre. Changing the lapse rate is equivalent to changing the thickness of the layer having a given range of temperature. This is equivalent to putting more water vapour in the track of the outgoing radiation when the lapse rate decreases, and less when the lapse rate increases. The effect on the outgoing radiation is to decrease it in the former case and to increase it in the latter case.

The lapse rate, however, cannot be increased indefinitely, the maximum being the adiabatic lapse rate for dry air: that is 10° C. per kilometre. Carrying through the calculations for this lapse rate it is found that the outgoing radiation is increased only by 7 per cent. The increased turbulence necessary to produce such an increased lapse rate would inevitably result in higher relative humidities in all layers of the atmosphere, and this would affect the outgoing radiation in the opposite direction. Thus the 7 per cent increase could never be completely effective even if the adiabatic lapse rate could be maintained, showing that there is little adjustment of the outgoing radiation possible through changes in the lapse rate.

We have now considered all the variations in the physical state of the atmosphere which could possibly affect the magnitude of the outgoing radiation, and found that none of them appears to give a solution of how the outgoing radiation is adjusted to the incoming radiation. This problem must therefore stand over for further study.

In conclusion, I should like to make one or two general remarks on the method employed in this paper. The results are strictly dependent on the assumptions made. In the case of two of the assumptions, those dealing with the lapse rate and the relative humidity, we have been able to estimate the effect of departure from the assumption and found the result little affected. There can be no question about the suitability of the assumption that the water vapour is in uniform horizontal layers, and the experimental evidence is very strong that water vapour is practically the only constituent of the atmosphere which absorbs and radiates long wave radiation. This leaves only the assumption that water vapour radiates as a grey body. It is not difficult, however, to show that, so long as we deal only with radiation from the vapour, it is legitimate to employ a mean absorption coefficient. We have only to carry out our calculations for individual wave lengths and then sum the results to get a complete statement of the outgoing radiation. Now, for each wave length we have the two expressions:

$$Y_\lambda = W \log y_\lambda \text{ and } Z_\lambda = 1 - Y_\lambda,$$

so that the form of the result for the individual wave lengths is the same as the one we have obtained. The form of the result would therefore not be altered; but there might be considerable difference in the numerical values. This is obviously the direction along which future research must proceed. It is strange that with all the recent research on the absorption of long wave radiation by gases no one has made accurate measurements of water vapour. The measurements by Rubens and Aschkinass in 1898 are the only ones we have which give the absorption as a function of wave length, and they were made with a single mass of water vapour within the beam of radiation, and are not sufficiently complete to use in a detailed investigation. They do, however, show, as mentioned above, that a mean absorption of 30 per cent for 1 mm. of precipitable water is of the right order of magnitude.

Thus, while admitting that the numerical values may need adjustment as more data become available, there can be no doubt that the general conclusion that the outgoing radiation is almost independent of the surface temperature, and is practically the same in all latitudes, will continue to be true.

SUMMARY.

It is assumed that water vapour is the only constituent of the atmosphere which absorbs and emits long wave radiation. From the results of upper air observations approximate values of the temperature and water content of the atmosphere at all heights and in all latitudes are adopted. Knowing the amount of water vapour and the temperature, it is possible to calculate the outgoing radiation if the absorption coefficient of water vapour is known. From consideration of the temperature of the upper stratosphere it is found that one millimetre of precipitable water in vapour form absorbs 30 per cent of the incident long wave radiation. With this value for the absorption coefficient the total outgoing radiation is calculated for each latitude from the equator to the pole, and it is found that over three-quarters of the earth's surface—from the equator to latitude 50°—the outgoing radiation is uniform and independent of the temperature of the surface. This result is due to the fact that it is only the layers of the atmosphere whose temperature lies between 220° and 286° which contribute to the outgoing radiation. As these layers are well within the troposphere between the equator and latitude 50°, the outgoing radiation is uniform from 50°S to 50°N. At higher latitudes the radiation falls off slightly, and at the pole itself it is 20 per cent below its value at the equator.

As the outgoing radiation is practically independent of the temperature of the surface, the problem arises as to how the temperature of the atmosphere readjusts itself to changes in solar radiation. This problem is considered in detail, but no solution is found.