Theory and Modelling of the Meridional Overturning Circulation

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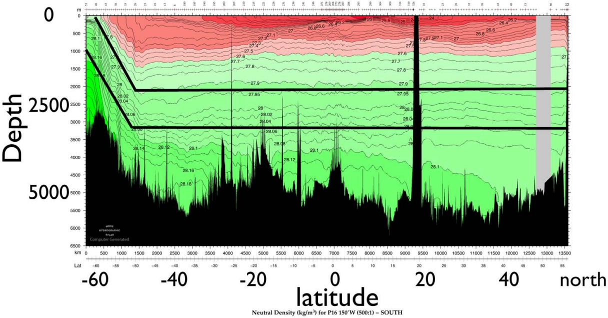
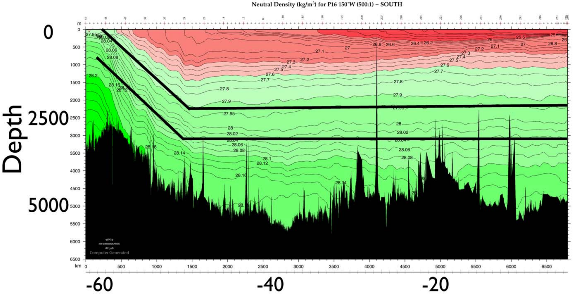
UCLA, May 2013

Ocean Stratification (density), Pacific, 150° W





(Unequal contours)

The problem, and why we care

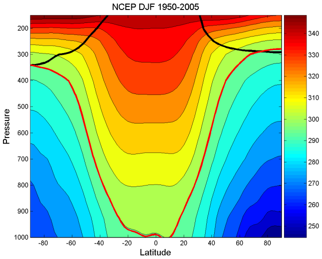
What?

- What processes determine the stratification and deep circulation of the ocean?
  - Mixing, advection, winds, surface buoyancy gradients...
- Previous theories. Primarily mixing driven (e.g., Stommel-Arons), but observational and numerical evidence (e.g., Toggweiler-Samuels) that wind over the Southern Ocean plays a key role.

Why?

- It is fundamental.
- Deep ocean (the meridional overturning circulation or MOC, aka the 'thermohaline circulation' or THC) carries a large fraction of the ocean's meridional heat transport.
  - Its variability would give rise to large variations in climate (inc. global warming and paleoclimate).
- Many results from GCMs, especially regarding past climates and climate variability, lack interpretation.

Atmospheric Stratification



The atmosphere is heated from below and cooled from above. So the forcing itself has a tendency towards being statically unstable.
The level of stratification might be maintained by:

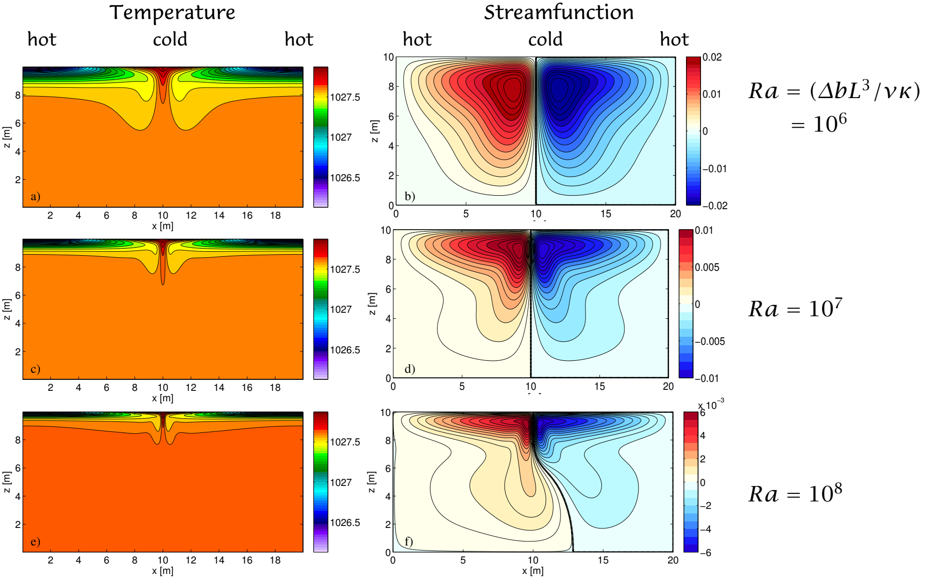
1. Vertical convection, moist or dry.
   - Moist convection is almost certainly the dominant process maintaining stratification in the tropics. (Having a given vertical structure simplifies remaining theory; leads to 'quasi-equilibrium theory' etc.)
2. Baroclinic instability. Moves energy upwards and sideways.

There is no shortage of ideas for maintaining stratification in the atmosphere!

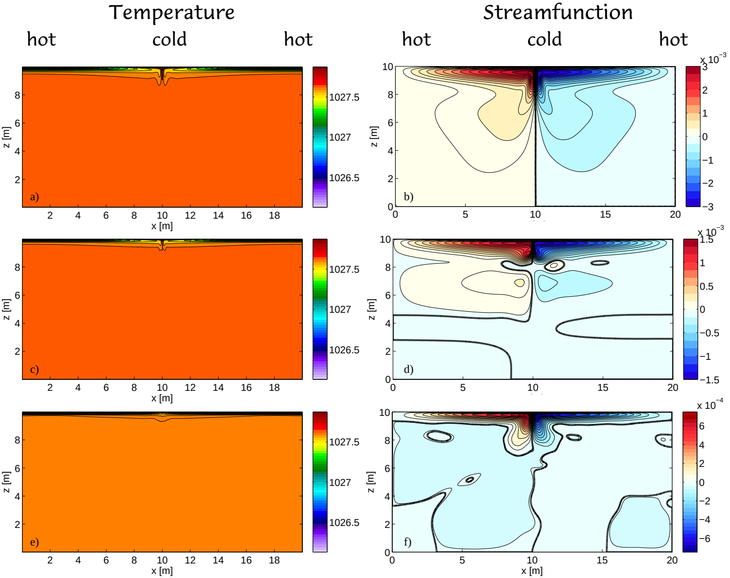
The Ocean

But the ocean is heated *and* cooled from above. In the absence of winds and mixing there will be no deep stratification (Sandström).

Horizontal Convection Fluid heated and cooled from above (Ilicak and Vallis)



Horizontal Convection with still higher Rayleigh number.



$Ra = 10^9$

$Ra = 10^{10}$

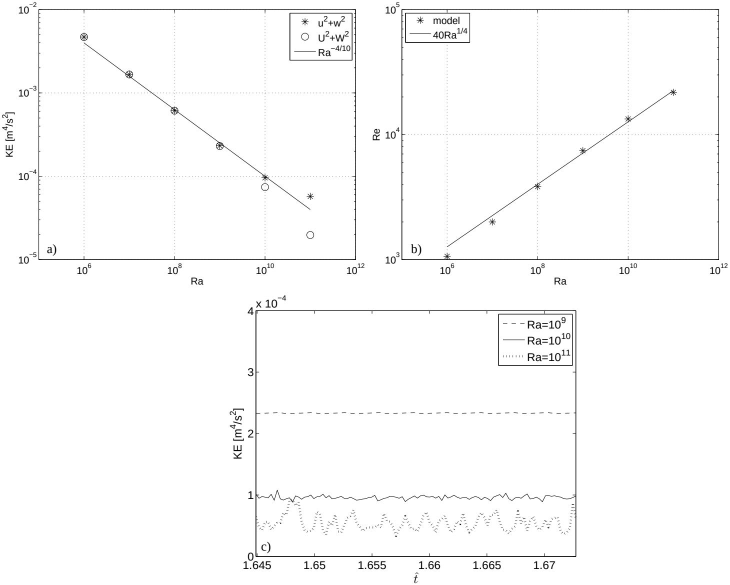
$Ra = 10^{11}$

For real ocean:

$Ra \approx 10^{30}$.

KE and Reynolds number vs Rayleigh number

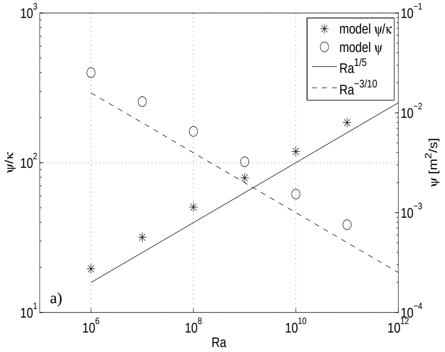
As Diffusivity decreases

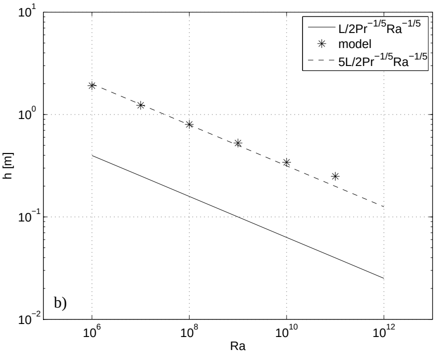


Scalings
(T. Rossby,1965)

$$\Psi = Ra^{1/5}\sigma^{-4/5}\nu = (\kappa^3 L^3 \Delta b)^{1/5}$$

$$H = L\sigma^{-1/5}Ra^{-1/5} = \left(\frac{\kappa^2 L^2}{\Delta b}\right)^{1/5}$$



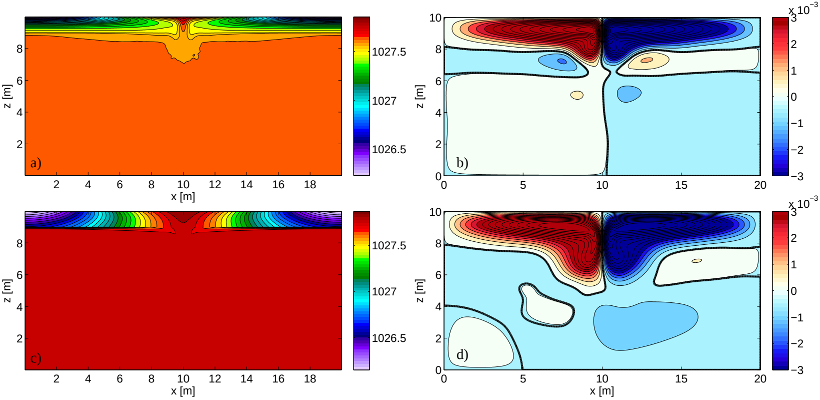


For realistic oceanic parameters $H \approx 2\,\text{m}$ and $V \approx 10^{-3}\,\text{m}\,\text{s}^{-1}$!

Enhanced diffusion in upper ocean — a mixed layer

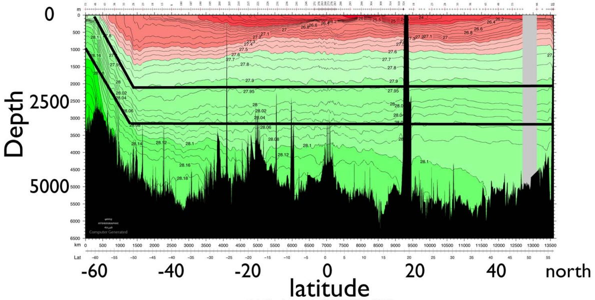
Beef up the Surface Fluxes

Surface fluxes are large, but still no deep stratification.

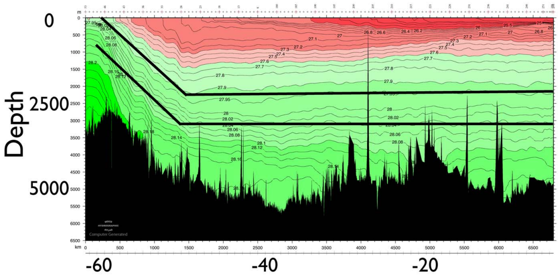


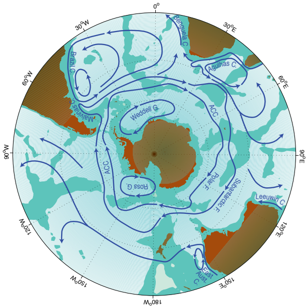
We can beef things up using an ’eddy diffusivity’ throughout the ocean’s depth but (putting in the numbers) there is still too little deep stratification.

Ocean Stratification, Pacific, WOCE, 150° W



Neutral Density (kg/m³) for P16 150°W (500:1) – SOUTH

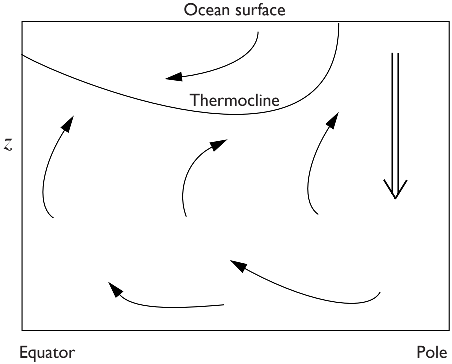




(Unequal contours)

The Mixing Driven Circulation

Robinson and Stommel (diffusive thermocline, 1959); Munk (abyssal recipes, 66).



Upwelling diffusive balance:

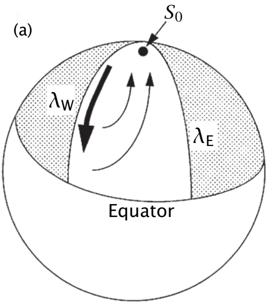
$$w \frac{\partial T}{\partial z} = \kappa \frac{\partial^2 T}{\partial z^2}.$$

- *But*, the diffusivity needed to produce the observed circulation is too high.
- Needed value: $\kappa \approx 10^{-4}\,\mathrm{m^2\,s^{-1}}$ Observed value in main thermocline: $\kappa \approx 10^{-5}\,\mathrm{m^2\,s^{-1}}$
- Values in abyss and in coastal regions may be much higher, but likely insufficient because abyssal stratification is small.

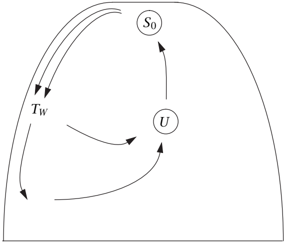
The so-called 'missing mixing' problem.

Abyssal Circulation in the Horizontal

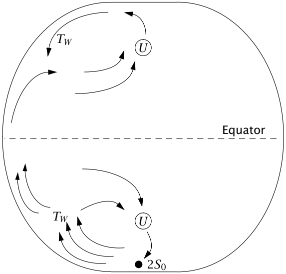
Stommel-Arons



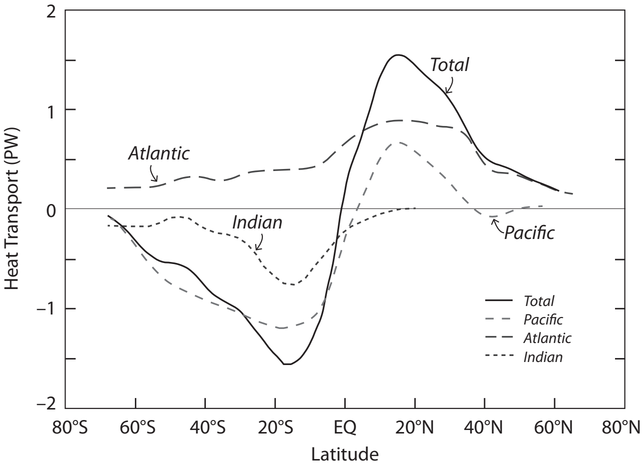
Hemispheric Circulation



Global Circulation



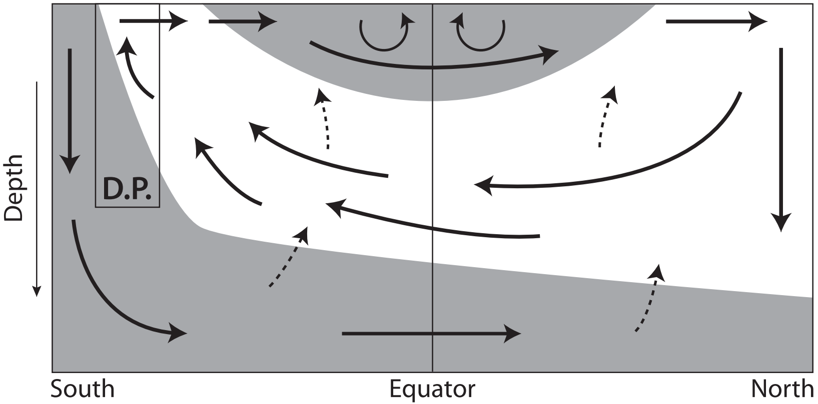
Heat Transport



Note *equatorward* heat transport in South Atlantic!

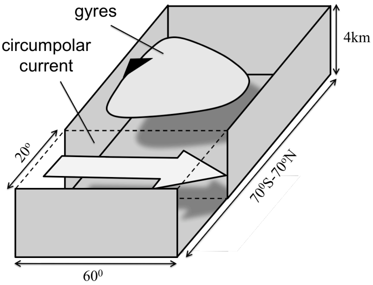
Interhemispheric Circulation

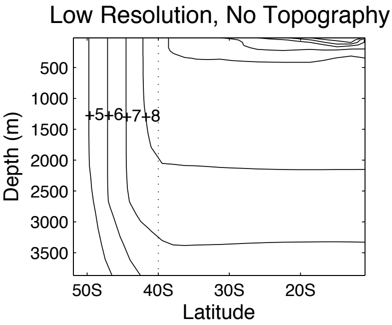
A More Modern Schematic

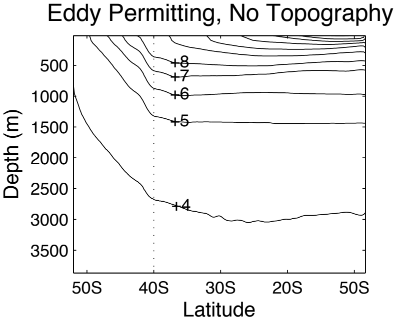


Schematic from observations and GCM simulations.

Wind and Eddies in the ACC







Use a 3D primitive equation model in simplified geometry to build intuition and phenomenology.

- Stratification in the ACC maintained by a balance between wind (steepens) and mesoscale eddies (slumping).
- Stratification then extends through the rest of the ocean.

(Henning and Vallis, 2005)

A Theory for Deep Stratification

Meta-theoretical musings...

Theory: a conceptual model that makes testable predictions.

Why have a theory?

- Provides a conceptual framework for a phenomenon.
  - Interpret observations and numerical model results.
- Predict things that comprehensive numerical models (GCMs) cannot.
- Build better GCMs in the future.
- Intellectual satisfaction.

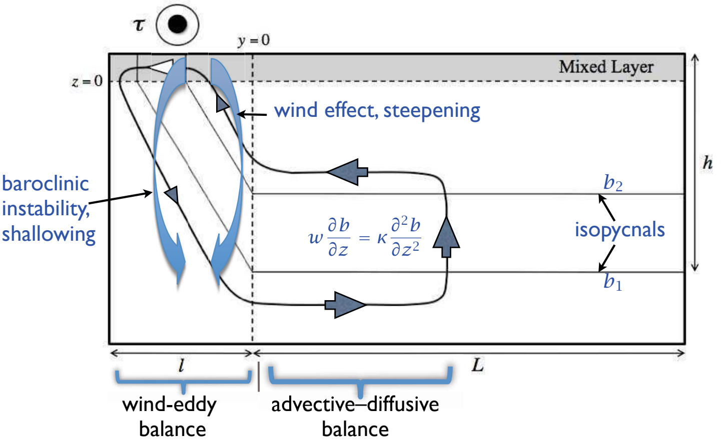
What can we expect from a theory?

- Predictions of how MOC depends on key parameters: winds, buoyancy forcing, diapycnal diffusivity.
  - So testable either via observations or using a complete numerical model.
- Human-interpretable analytic expressions or scalings.

Our theory:

- Zonally averaged.
- For one basin only (e.g., Atlantic, Pacific, Indian Ocean alone)
- Semi-analytic
  - Explicit scaling expressions; analytic in some limits, short numerical code in general.

A Theory for Deep Stratification — The conceptual model



- In channel, the wind induced circulation causes generates a clockwise 'Deacon Cell' that rotates tries to make the isopycnals vertical.
- The baroclinic eddies try to make the isopycnals horizontal.
- The circulation in the channel must connect smoothly with that in the basin.

Equations of Motion (Transformed Eulerian Mean)

Quasi-geostrophic equations of motion

$$\frac{\partial \overline{u}}{\partial t} - f_0 \overline{v} = -\frac{\partial}{\partial y}\overline{u'v'} + F,$$
$$\frac{\partial \overline{b}}{\partial t} + N^2 \overline{w} = -\frac{\partial}{\partial y}\overline{v'b'} + Q.$$

where b is buoyancy ('temperature') and $N^2 = \partial_z b$.

Thermal wind relation and mass continuity:

$$f_0 \frac{\partial \overline{u}}{\partial z} = -\frac{\partial \overline{b}}{\partial y}, \qquad \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0.$$

Define a *residual flow* such that

$$\overline{v}^* = \overline{v} - \frac{\partial}{\partial z}\left(\frac{1}{N^2}\overline{v'b'}\right), \qquad \overline{w}^* = \overline{w} + \frac{\partial}{\partial y}\left(\frac{1}{N^2}\overline{v'b'}\right).$$

whence

$$\boxed{\begin{aligned} \frac{\partial \overline{u}}{\partial t} - f_0 \overline{v}^* &= \overline{v'q'} + F \\ \frac{\partial \overline{b}}{\partial t} + N^2 \overline{w}^* &= Q \end{aligned}}$$

Only the PV flux, $\overline{v'q'}$, need be parameterized.

Eddy parameterizations

Diffusive: Prandtl and G. I. Taylor.   Use PV: Bretherton, Green, others.

Because $\mathrm{D}q/\mathrm{D}t = 0$, just use eddy diffusion:

$$\overline{v'q'} = -K_e \frac{\partial \overline{q}}{\partial y}.$$

where, approximately, for the large-scale ocean

$$\overline{q} \approx f_0 \frac{\partial}{\partial z}\left(\frac{\overline{b}}{\overline{b}_z}\right), \qquad \frac{\partial \overline{q}}{\partial y} \approx f_0 \frac{\partial}{\partial z}\left(\frac{\overline{b}_y}{\overline{b}_z}\right) = -f_0 \frac{\partial S}{\partial z}$$

where $S = -\overline{b}_y/\overline{b}_z$ is the slope of the isopycnals.

This is more-or-less the same as the 'Gent-McWilliams' parameterization.

Momentum equation:

$$-f_0 \overline{v}^* = \overline{v'q'} + \frac{\partial \tau}{\partial z} \qquad \Longrightarrow \qquad \boxed{f_0 \overline{\Psi}^* = -K_e \frac{\overline{b}_y}{\overline{b}_z} + \tau}.$$

Simple, robust and (like lots of turbulence modelling) a little *ad hoc*.

Equations

In the channel

Momentum equation: wind, Coriolis and eddies in TEM form

$$-fv^* = \frac{\partial \tau}{\partial z} + f\frac{\partial}{\partial z}\left(\frac{\overline{v'b'}}{\partial_z \overline{b}}\right)$$

becomes

$$\psi = -\frac{\tau}{f} + K_e S_b$$

where $S_b = -\partial_y b/\partial_z b$ is the slope of isopycnals and K_e is a baroclinic eddy diffusivity.

Buoyancy equation:

$$\boldsymbol{v}^* \cdot \nabla \overline{b} = \kappa_v \frac{\partial^2 \overline{b}}{\partial z^2} \qquad \rightarrow \qquad \frac{\partial \psi}{\partial y}\frac{\partial \overline{b}}{\partial z} - \frac{\partial \psi}{\partial z}\frac{\partial \overline{b}}{\partial y} = \kappa_v \frac{\partial^2 \overline{b}}{\partial z^2}$$

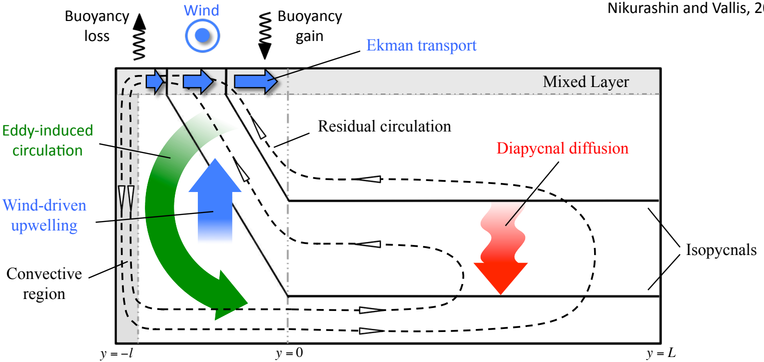
Boundary condition at edge of channel: (integrate $\partial \psi/\partial y \times \partial \overline{b}/\partial z = \kappa \partial^2 \overline{b}/\partial z^2$ wrt y).

$$\psi|_{y=0} = -\kappa_v L \frac{\partial_{zz}\overline{b}}{\partial_z \overline{b}}.$$

Schematic of the Theory

Equations of motion in TEM form (Andrews & McIntyre, Edmon, Hoskins, McIntyre)

Nikurashin and Vallis, 2011



In the channel

$$-\psi_z b_y + \psi_y b_z = 0$$

and

$$\psi = -\frac{\tau}{f} - K\frac{b_y}{b_z}$$

(e.g. Marshall and Radko, 2003)

In the basin

$$\psi_y b_z = \kappa b_{zz}$$

or

$$\psi\big|_{y=0} = -\kappa L \frac{b_{zz}}{b_z}$$

(e.g. Munk and Wunsch, 1998)

τ - wind stress K - isopycnal eddy diffusivity κ - diapycnal diffusivity $(v,w) = (-\psi_z, \psi_y)$ - velocity

Procedure

We proceed in three ways:

I. Scaling: gives basic parameter dependencies on wind, diffusivity, Coriolis parameter etc., in limits of weak and strong diffusion.

II. Analytic solutions: Obtainable, in limit of weak diffusion only, by regular asymptotics and method of characteristics.

III. Numerical solution of equations of the theory.

Compare predictions of the theory with full solutions of primitive equations using a comprehensive ocean GCM.

Nondimensionalization

Let $y = l\hat{x}$, $z = h\hat{z}$, $\psi = \hat{\psi}\,\tau_0/f_0$, etc.

Buoyancy evolution: $$\partial_y \hat{\psi} + \hat{s}_\rho \partial_z \hat{\psi} = -\epsilon \left(\frac{l}{L}\right) \frac{\partial_{zz}\hat{b}}{\partial_z \hat{b}},$$

Momentum balance: $$\hat{\psi} = -\frac{\hat{\tau}}{\hat{f}} + \Lambda \hat{s}_\rho,$$

Boundary condition: $$\hat{\psi}|_{y=0} = -\epsilon \frac{\partial_{zz}\hat{b}}{\partial_z \hat{b}},$$

where

$$\boxed{\Lambda = \frac{\text{Eddies}}{\text{Wind}} = \frac{K_e}{\tau_0/f_0}\frac{h}{l} \sim 1} \quad \text{and} \quad \boxed{\epsilon = \frac{\text{Mixing}}{\text{Wind}} = \frac{\kappa_v}{\tau_0/f_0}\frac{L}{h} \sim 0.1 - 1,}$$

The parameter h is a characteristic depth of the stratification, and will be a part of the solution.

Scaling

Weak diffusiveness

$$\epsilon \ll 1, \qquad \Lambda = 1, \qquad \text{and} \qquad \frac{l}{L} \ll 1$$

$$\boxed{h = \frac{\tau_0/f_0}{K_e} l, \qquad \Psi = \kappa_v \frac{K_e}{\tau_0/f_0} \frac{L}{l}}.$$

Depth of stratification is determined by wind and eddies only. Circulation is weak, and goes to zero with the diffusivity.

Strong diffusiveness

$$\epsilon \gg 1, \qquad \Lambda = \epsilon, \qquad \text{and} \qquad \frac{l}{L} \ll 1$$

$$\boxed{h = \sqrt{\frac{\kappa_v}{K_e} L l}, \qquad \Psi = \sqrt{\kappa_v K_e \frac{L}{l}}}$$

Depth of stratification is determined by diffusion and eddies. Circulation is stronger, goes as half power of diffusivity.

Analytic Solutions

If diffusion is weak then the surface conditions are propagated into the interior along characteristics (helpfully found by my Russian collaborator). Requires particular forcing at the surface. Choosing

$$b_0(y) = \Delta b \left(1 + \frac{y}{l}\right)^2,$$

we obtain

$$\psi^{(0)} = 0$$

and

$$\psi^{(1)}(y, z) = \kappa_v \frac{K_e}{\tau/f} \frac{L}{l} \left(1 + \frac{y}{l} - \frac{K_e}{\tau/f} \frac{z}{l}\right)^{-1}$$

Arguably, solutions are of no especial interest in themselves, except that we can compare them against numerical solutions to check the theory.

Numerical Solutions of Equations of the Theory

Provide a bridge between analytics and full GCM. Step forward the zonally-averaged TEM buoyancy equation:

$$\frac{\partial \overline{b}}{\partial t} + J(\psi, \overline{b}) = \kappa_v \frac{\partial^2 \overline{b}}{\partial z^2}$$

with

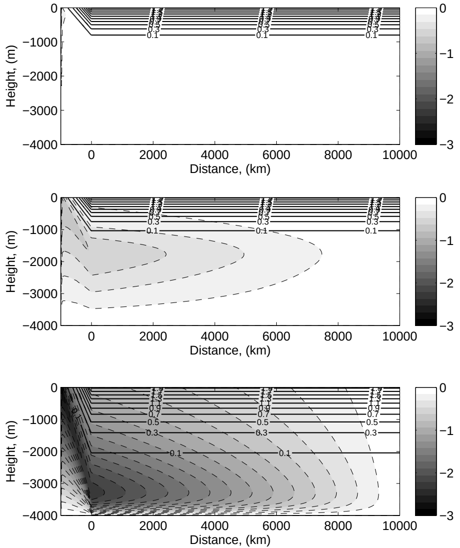
$$\psi = -\frac{\tau}{f} + K_e S_b.$$

and boundary conditions:

At top: $\quad \overline{b}|_{z=0} = b_0(y).$

At channel edge: $\quad \overline{\psi}|_{y=0} = -\kappa_v \dfrac{\partial^2 \overline{b}/\partial z^2}{\partial \overline{b}/\partial z}.$

Figures (from top):
Buoyancy distribution and circulation with low, medium and high diffusion.



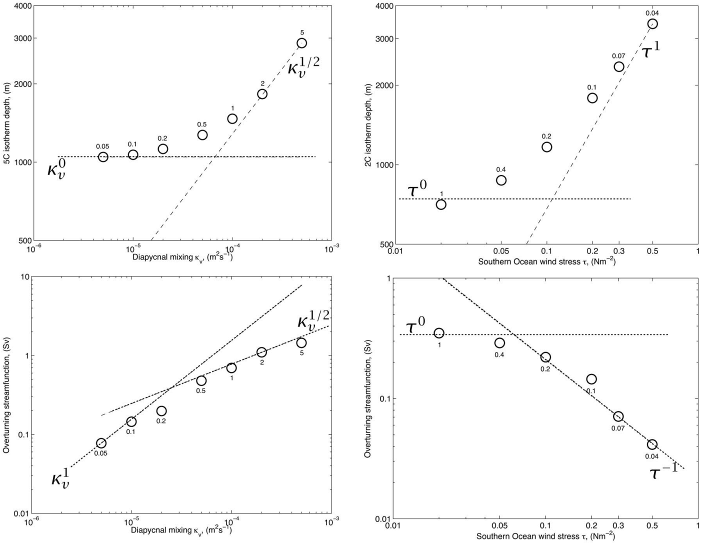
Test of Scaling with General Circulation Model

$$h = \begin{cases} \dfrac{\tau/f_0}{K_e}\, l \\[2ex] \sqrt{\dfrac{\kappa_v}{K_e} L l} \end{cases}$$

$$\Psi = \begin{cases} \kappa_v \dfrac{K_e}{\tau/f_0} \dfrac{L}{l} \\[2ex] \sqrt{\kappa_v K_e \dfrac{L}{l}} \end{cases}$$

h

Ψ

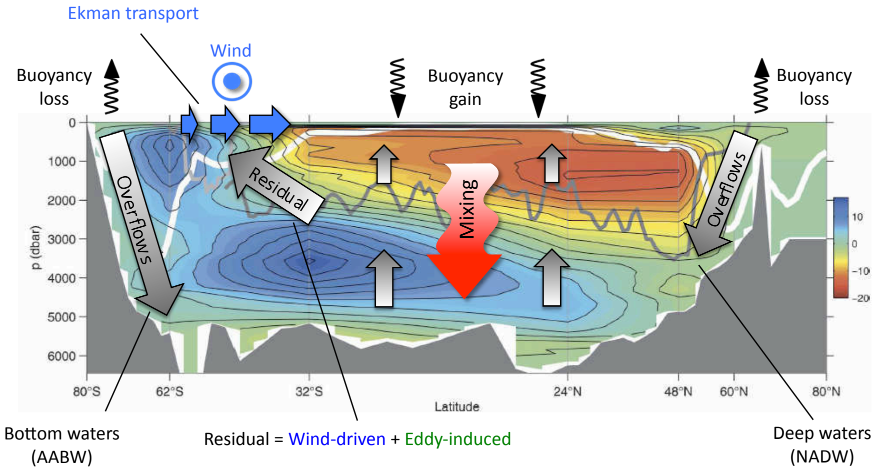


Diffusivity

Wind stress

Interhemispheric Circulation

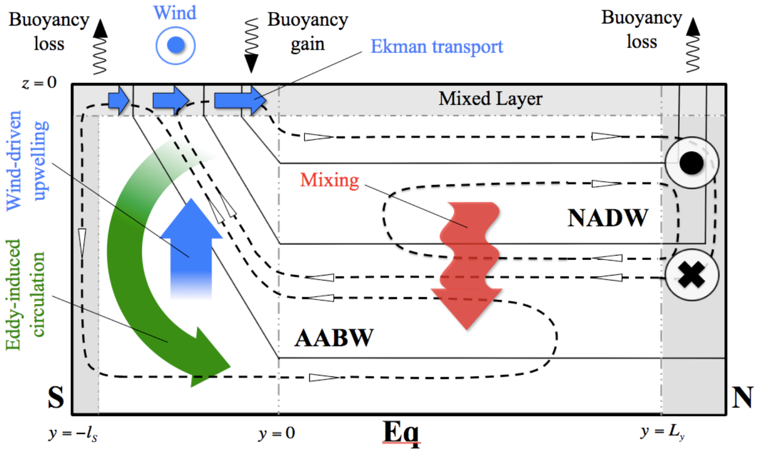
More complicated plot



To maintain a deep stratification and circulation we need:

- Gradients of buoyancy fluxes (to get 'water masses')
- Mixing and/or winds.

Schematic for Interhemispheric Flow



Circumpolar Current region: wind-driven upwelling balanced by eddy-induced circulation

Ocean interior: downward mixing of buoyancy balanced by upward vertical advection

North Atlantic high latitudes: convection due to buoyancy loss

The three regions are matched so the solutions are smooth.

The Theory is an Algorithm

- SH Circumpolar Channel
  - (i) Buoyancy: Nearly adiabatic buoyancy advection.
  - (ii) Momentum: Advection by residual flow. Balance between wind driving and eddy (GM) effects
  - (iii) Boundary conditions on buoyancy:
    (i) Surface fluxes, (ii) matching to basin region.
- The Ocean Basin
  - (i) Isopycnals are flat
  - (ii) Advective-diffusive balance: $w\partial_z b = \partial_z(\kappa_v \partial_z b)$.
  - (iii) Boundary conditions on buoyancy (surface restoring).
- NH convective region
  - (i) Buoyancy distribution from the basin interior is matched convectively to the surface buoyancy profile.
  - (ii) Meridional buoyancy gradient drives eastward flow in the upper ocean and the return westward flow in the deep ocean (thermal wind).
  - (iii) Zonal flows are connected to the meridional circulation at the western boundary.

The Theory is Equations

1 SH Circumpolar Channel

Buoyancy advection: $$J(\psi_1, b_1) = \kappa_v \frac{\partial^2 b_1}{\partial z^2}$$

Momentum: $$\psi_1 = -\frac{\tau(y)}{f} - K_{GM} s$$

Surface boundary: $$-\kappa \frac{\partial b_1}{\partial z} |_{z=0} = \lambda(b^* - b_1)$$

Match to interior: $$b_1(z)|_{y=y_c} = b_2(z)$$

2 The Ocean Basin

Advective diffusive: $$\frac{(\psi_3 - \psi_1)}{L} \frac{\partial b_2}{\partial z} = \kappa_v \frac{\partial^2 b_1}{\partial z^2}$$

Surface boundary: $$-\kappa \frac{\partial b_2}{\partial z} |_{z=0} = \lambda(b^* - b_2)$$

3 NH convective region

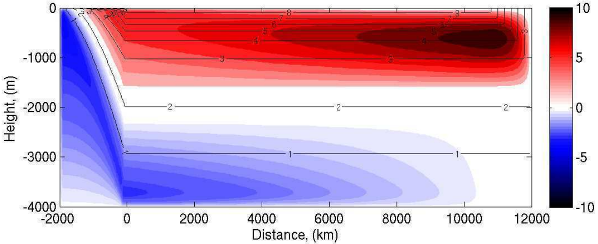
Convective matching: $$b_2(z) \Rightarrow b_3(y, z)$$

Thermal wind: $$f \frac{\partial u_3}{\partial z} = \frac{\partial b_3}{\partial y}$$

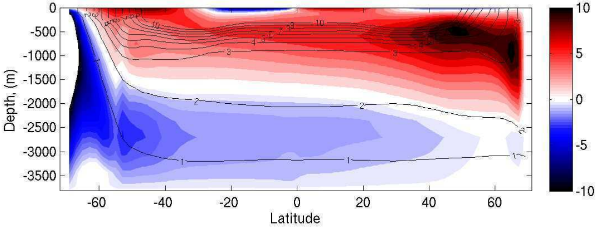
Mass Continuity: $$\psi_3(z) = \iint u_3 \, dy \, dz$$

Results

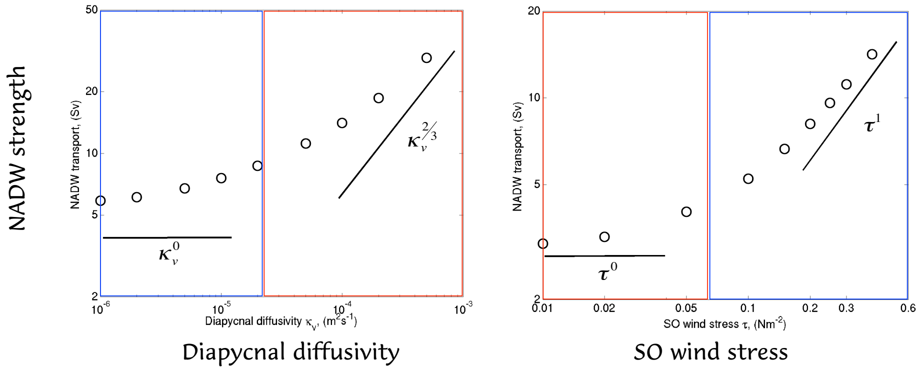
Theory:
Temperature (lines) and overturning circulation (Sv, colour).



GCM simulation:
(MOM)



Scaling

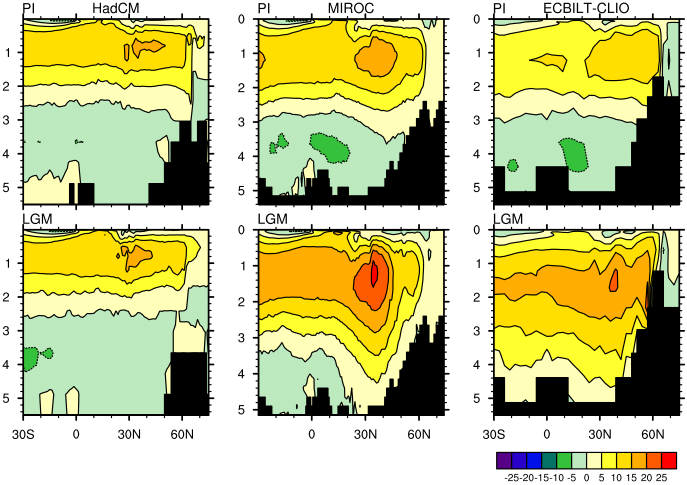

NADW strength
NADW transport, (Sv)
50
20
10
5
2
10^{-6}
10^{-5}
10^{-4}
10^{-3}
Diapycnal diffusivity κ_v, (m^2s^{-1})
κ_v^0
$\kappa_v^{2/3}$
Diapycnal diffusivity
NADW transport, (Sv)
20
10
5
2
0.01
0.02
0.05
0.1
0.2
0.3
0.6
SO wind stress τ, (Nm^{-2})
τ^0
τ^1
SO wind stress

Conclusions

- Developed a theoretical model for deep stratification and overturning circulation.
    - Differs in fundamental ways from classical ideas of Stommel-Arons and Munk.
    - Deep circulation remains in the limit of zero diapycnal diffusivity.
    - Tries to quantify and make concrete various GCM simulations and conceptual models (Toggweiler-Samuel, others)
    - Extends existing theories (Radko-Marshall, others) by linking the ACC to the basins.
- Stratification depends on mesoscale eddies and wind in ACC.
- Overturning circulation depends (inter alia) on diapycnal mixing and Northern Hemisphere temperature.
- Possible implications for carbon cycle and paleoclimate.

Paleoclimate Implications

GCM results



Present Day

LGM

Otto-Bliesner et al 2007

- Relative sizes of top and bottom cells (NADW and AABW) was different in Last Glacial Maximum (LGM). (Curry and Oppo, 2005, others).
- GCM results are all over the place (Otto-Bliesner et al, 2007)
- Large implications for carbon cycle (Sigman et al 2010).

What's Next?

Implications for the real ocean, and questions for 'real' models

What's important?

- Eddies in the Southern Ocean. (Likely the Arctic is too, but that is not our focus.)
- Winds in the Southern Ocean.
- Buoyancy gradients across the North Atlantic and Arctic.

What's peripheral?

- Diapycnal mixing? There is no 'missing mixing'!
- Eddies in the North Atlantic? Important for the gyres perhaps, but not for the MOC.

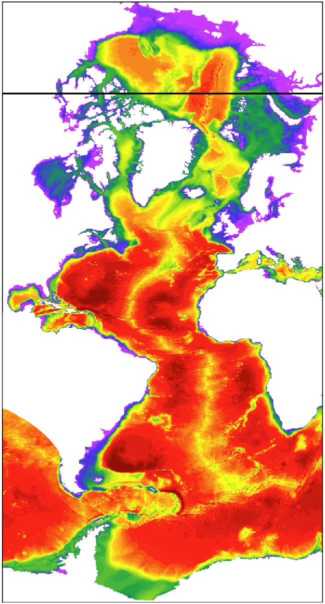
What can we expect compared to a realistic, eddy-resolving ocean model?

- Quantitative scaling predictions seem unlikely to hold.
  - Because of quantitative failure of eddy parameterizations (e.g., Gent-McWilliams).
- But qualitative features should hold (if theory is correct).

Possible GCM Experiments

1. Eddying Atlantic (North and South) with periodic ACC. No Pacific. (POP?)
2. Eddying ACC. Low resolution basins. (MPAS?)

Atlantic Experiment w/POP. M. Maltrud.



Re-entrant Atlantic, enclosed Arctic.

No imposed inflow/outflow boundary conditions.