Expression of electron cyclotron current drive in plasma fluid models

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Introduction

The study of the effects of electron cyclotron current drive (ECCD) on the evolution of plasma instabilities requires its proper expression in the governing fluid equations. Commonly, this is modeled by modifying Ohm's law as

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta (\mathbf{J} - \mathbf{J}_{\text{ECCD}}), \tag{1}$$

and adding a model for the spatiotemporal evolution of the EC driven current density $\mathbf{J}_{\text{ECCD}}$. One such model has been put forward by Giruzzi et al. [1]. In a recent paper the fluid closure in the presence of RF current drive has been derived more formally, identifying two major effects: an additional parallel force term in the electron momentum balance equation and a modification of the electron-ion momentum exchange term [2]. In single-fluid MHD, the first results in an additional parallel force in Ohm's law localized in the region of RF power deposition, while the second modifies the resistivity. In this contribution we apply this formal framework to the specific case of ECCD.

Electron Cyclotron Current Drive

In the formal procedure, the fluid equations are obtained by taking successively higher moments of the kinetic equation and truncating this series, typically at the second moment which represents the energy. Higher order moments appearing in these equation must then be modeled using appropriate closure relations. The effect of RF waves in the governing kinetic equation is provided by the RF quasi-linear diffusion model. The quasi-linear diffusion resulting from ECCD occurs primarily in the direction of perpendicular velocity. In the non-relativistic limit, the EC diffusion operator $Q_{\text{EC}}(f_e)$ is approximated as

$$Q_{\text{EC}}(f_e) = D\delta(v_\parallel - v_{\parallel,\text{res}}) \frac{\partial^2 f_e}{\partial v_\perp^2}. \tag{2}$$

Since the plasma is close to local thermodynamic equilibrium, the perturbation of the electron distribution function δf_{EC} generated by the EC quasi-linear diffusion is obtained by substituting a Maxwellian:

$$\delta f_{EC} = t Q_{\text{EC}}(f_M) = tD\delta(v_\parallel - v_{\parallel,\text{res}}) \left(\frac{v_\perp^2}{2v_t^2} - 1 \right) \exp\left(-\frac{v_\parallel^2 + v_\perp^2}{2v_t^2} \right), \tag{3}$$

with the thermal velocity $v_t = \sqrt{kT_e/m_e}$. This perturbation in velocity space carries no current itself, but it is subject to collisions: it represents an asymmetric collisionality that results in the so-called Fisch-Boozer current [3]. To illustrate this, we study the Boltzmann equation with a Krook-type collision operator

$$C(f_e) = -\nu(v)(f_e - f_M), \tag{4}$$

in which the collision frequency is velocity dependent: $\nu(v) = \nu_t(v_t/v)^3$, where the subscript t indicates the thermal collision frequency or velocity, respectively. The momentum loss that is implied by this operator can be regarded as the momentum transfer from electrons to ions. A simple analytical solution of the Boltzmann equation can be written for the homogeneous case:

$$f_e(v_\parallel, v_\perp; t) - f_M = Q_{\mathrm{EC}}(f_M)\frac{1}{\nu(v)}(1 - e^{-\nu(v)t}). \tag{5}$$

In the final steady-state a balance is reached between the EC quasi-linear drive and the collisional dissipation: a distribution of electrons has evolved, which carries a net parallel current, yet, transfers no net momentum to the ions. In conclusion: the steady-state, EC driven current does not contribute to the friction between electrons and ions.

Fluid equations and closure

In [2] the appropriate moments of the Boltzmann equation including RF quasi-linear diffusion are taken in order to show that the RF waves contribute to the electron momentum balance equation in the form of a parallel force as well as to the energy balance equation in the form of the RF absorbed power density. In single-fluid MHD this parallel force enters in generalized Ohm's law. Furthermore the RF quasi-linear diffusion is noted to affect the closure for the electron-ion friction term R_e, which in the absence of RF current drive enters generalized Ohm's law as the resistivity:

$$\frac{1}{en_e} R_e \equiv \eta J. \tag{6}$$

Since the EC quasi-linear diffusion is dominantly in the perpendicular velocity direction, this parallel force is actually negligible in the case of ECCD. Thus, the effect of ECCD enters into generalized Ohm's law only through its modification of the electron-ion friction. Because also the electron-ion momentum exchange associated with the steady-state EC-driven current is zero, the EC-driven current density J_{ECCD} must be subtracted from the total current density in the definition relating the resistivity to the electron-ion friction (6). This implies the commonly-used modification of Ohm's law (1) as given in the introduction.

The closure of the fluid equations then requires a model for the EC driven current. Taking the appropriate moment of the Boltzmann equation including only the collision and EC quasi-linear diffusion operators, we find

$$\frac{\partial}{\partial t}\mathbf{J}_{\mathrm{ECCD}} = \frac{e}{m_e}\mathbf{R}_e^{\delta f_{\mathrm{EC}}} - v_{\|}\nabla_{\|}\mathbf{J}_{\mathrm{ECCD}}, \tag{7}$$

where $R_e^{\delta f_{\mathrm{EC}}}$ represents the transient electron-ion friction associated with the EC-driven quasi-linear modification of the distribution function: within the approximations of the model set out in the previous paragraphs

$$R_e^{\delta f_{\mathrm{EC}}} = -\int d^3\mathbf{v}\; m_e \mathbf{v}\nu(v)\delta f_{\mathrm{EC}}. \tag{8}$$

This formulation shows once more that ECCD is not a direct product of EC quasi-linear diffusion, but requires the intermediate step of collisions to set up a current by a transient momentum transfer between electrons and ions. As a consequence, the localization of the source term for the EC-driven current is not identical to the extremely-localized EC deposition region, but instead will be spread out over a larger region: effectively the entire flux tube passing through the power deposition region as a consequence of fast convection with the parallel velocity. To close the system of equations we must express $R_e^{\delta f_{\mathrm{EC}}}$ in terms of known quantities. We model $R_e^{\delta f_{\mathrm{EC}}}$ as the sum of a current generation term and the collisional decay of the current density, $\nu\mathbf{J}_{\mathrm{ECCD}}$. The current generation is taken to be proportional to the flux tube averaged power deposition, $\langle p_{\mathrm{ECCD}}\rangle$. The constant of proportionality is determined by the condition that the steady-state driven current I_{ECCD} equals the total driven current as predicted, for example, by bounce averaged quasi-linear Fokker-Planck calculations. Results of the latter are commonly presented in terms of the current drive efficiency η_{ECCD} relating the driven current to the total absorbed power: $I_{\mathrm{ECCD}} = \eta_{\mathrm{ECCD}} P_{\mathrm{ECCD}}$. The final result then is

$$\frac{\partial}{\partial t}\mathbf{J}_{\mathrm{ECCD}} = -\nu\mathbf{J}_{\mathrm{ECCD}} + \nu\eta_{\mathrm{ECCD}}\langle p_{\mathrm{ECCD}}\rangle 2\pi R - v_{\|}\nabla_{\|}\mathbf{J}_{\mathrm{ECCD}}, \tag{9}$$

which differs from the model as proposed originally by Giruzzi et al. [1] in the averaging applied to the power deposition profile and in the parallel convection term.

Implementation of ECCD in JOREK

To simulate the effects of ECCD on the growth and stabilization of neoclassical tearing modes, we have implemented the modified Ohm's law (1) and equation (9) for the evolution of the EC driven current into the 3D nonlinear, reduced MHD code JOREK [4]. In a fast rotating plasma, an appropriate approximation for the flux tube averaged power density is a narrow toroidal annulus. Figure 1 shows the EC driven current density distribution when this averaged EC power distribution is applied in a plasma with a large 2/1 magnetic island. During stabilization, the parallel convection term produces a complex flow of EC current within the island.

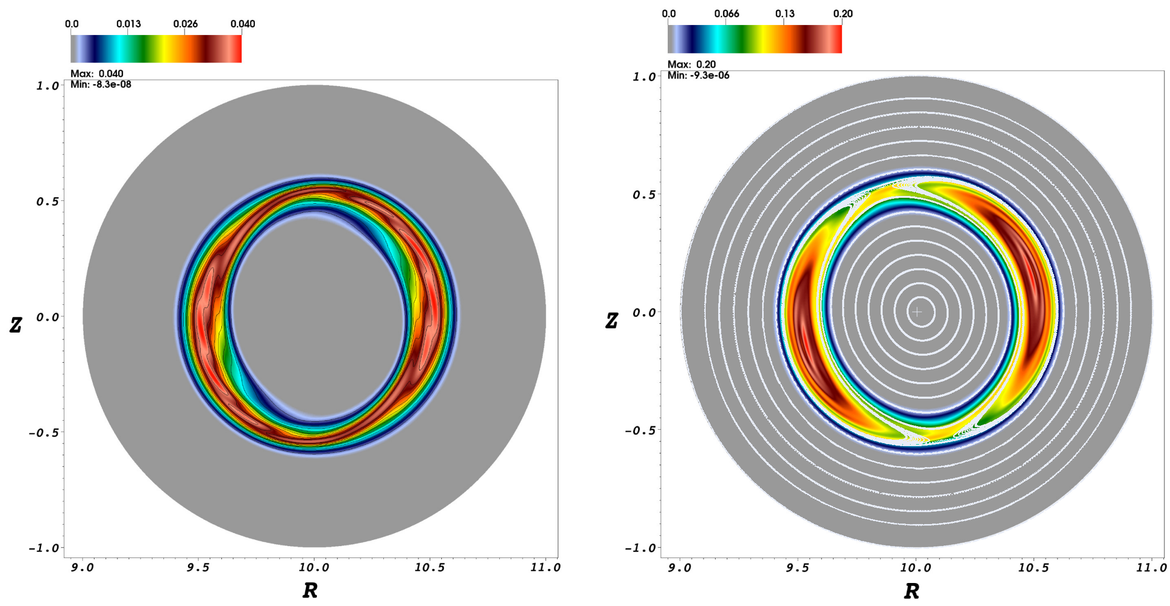

Figure 1: (Left) Distribution of EC current after 0.15 ms, when radial shearing of the current begins to produce flow structures associated with convection. (Right) Distribution of EC current after 1.0 ms. Steady flow structures emerge that fill the magnetic islands and follow the flux surfaces. A Poincaré map shows the position of the 2/1 magnetic islands. Toroidal simulations are performed for a cold circular tokamak with major radius $R = 10\mathrm{m}$, minor radius $a = 1\mathrm{m}$, density $n_0 = 6 \cdot 10^{-19}\mathrm{m}^{-3}$, mass density $2 \cdot 10^{-7}\mathrm{kg/m}^3$, toroidal magnetic field $B_0 = 1.945\mathrm{T}$, viscosity $4 \cdot 10^{-9}\mathrm{kg/m\ s}$, and resistivity $2.5 \cdot 10^{-6}\Omega\mathrm{m}$. 5 toroidal harmonics are simulated. ECCD current is deposited in a narrow annulus centered at the resonant surface.

Conclusion

We have revisited the closure of single fluid MHD in the presence of ECCD. A typical feature of ECCD is the two step process characterizing the Fisch-Boozer type current drive [3]: the EC waves create an asymmetry in the collisionality of the electron distribution which then results in the creation of a net current with negligible momentum transfer between electrons and ions. This implies the commonly-used modification of Ohm's law (1), which is closed by an equation for the evolution of the EC driven current (9) reflecting the delayed nature of its source and its convection with the parallel velocity of resonant electrons.

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