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Quantum chaology, prime numbers and Riemann's zeta function

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Abstract. One problem in quantum chaology is to calculate energies of high excited states for systems whose trajectories are classically chaotic. Gutzwiller found a representation of the quantum spectrum in terms of classical periodic orbits, but this is divergent and so has to be interpreted before it can be used. The interpretation is made with the aid of an analogy between energy levels and the zeros of Riemann's zeta function; prime numbers correspond to classical periods. The quantum-Riemann analogy has led to new mathematics, as well as a new way to sum Gutzwiller's series.

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