THE SUM OF RECIPROCAL FIBONACCI PRIME NUMBERS CONVERGES TO A NEW CONSTANT: MATHEMATICAL CONNECTIONS WITH SOME SECTORS OF EINSTEIN'S FIELD EQUATIONS AND STRING THEORY

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Abstract

In this paper we have described a sum of the reciprocal Fibonacci primes that converges to a new constant. Furthermore, in the Section 2, we have described also some new possible mathematical connections with the universal gravitational constant G , the Einstein field equations and some equations of string theory linked to Φ and π

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1. THERE ARE AN INFINITE NUMBER OF FIBONACCI PRIMES?

In the case of the Fibonacci prime numbers always increases the distance D(x) between the Fibonacci prime numbers:

1 2
2 3
3 5
4 13
5 89
6 233
7 1597
8 28657
9 514229
10 433494437
11 2971215073

The first 10 terms are the following:

D(x) = 1, 2, 8, 76, 144, 1364, 27060, 485572, 432980208, 2537720636, …

Now since we know that the Fibonacci numbers are infinite in number this succession of D(x) it is equivalent to an infinite sum of "1", because each gap D (x) is exactly 1 because there is only and always a D(x), one for one, one for 2, one for 8, one for 76 and so on:

$1 + 1 + 1 + 1 + 1 + 1 + ... = \infty$

Even if the sum tends to infinity this very slowly, however, shows that the Fibonacci primes are infinite in number.

Since the distance D(x) between the Fibonacci numbers grows indefinitely it could be that exists always a Fibonacci prime number because Fibonacci numbers are infinite.

In fact we know that the number of Fibonacci N(x) numbers is less or equal to x:

$$N(x) \approx \frac{1}{\log \varphi} \log(x\sqrt{5}) = 2{,}078086921235027537601322606117 8 \ldots * \log(x\sqrt{5})$$

with φ golden ratio = 1,618033988749894848204586834365 6 …

From this formula we can derive that the number of Fibonacci primes F(x) less than or equal to a certain threshold x is approximately:

$$F(x) \approx 2\varphi * \log[\log(x\sqrt{5})] = 3{,}236 * \log[\log(x\sqrt{5})]$$

For $x \to \infty$ we have $F(x) \to \infty$ and the number of Fibonacci primes grows to infinity as the natural logarithm of the natural logarithm even if in a very slow but always endlessly.

2. SUM OF RECIPROCAL FIBONACCI PRIME NUMBERS

A **Fibonacci prime** is a Fibonacci number that is prime, a type of integer sequence prime.

It is not known whether there are infinitely many Fibonacci primes. The first 23 values are the following

1 2
2 3
3 5
4 13
5 89
6 233
7 1597
8 28657
9 514229
10 433494437
11 2971215073
12 99194853094755497
13 1066340417491710595814572169
14 19134702400093278081449423917
15 475420437734698220747368027166749382927701417016557193662268716376935476241
16 5298927110060956217920395567877846701971127590295345066209051628347699551344246896762623 69
17 138727712780478382711418610318624639225845035817178369007991803213602522595460259371256835 3
18 3061719992484545030554313848083717208111285432353738497131674799321571238149015933442805665949
19 105979992653014907325996436715050034125158604354094219325600096801429743471954831402932543961957698761299 09
20 3668447431608097806147361364627563045110058690119522981527024286841776806119356085790433501787954051522814377778106586 9
21 9604120061892255382394288336092486502610491741187706781682226478902901437830847886419258908418525433163764618300807462 9
22
357103560641909860720907774139063454445569926582843306794041997476301071102767570483343563518510007800304195444080518562630 9
000273864989339446192101928567683526834688317544232342179785257659210407472913166815765568614907731352148617828777165608796 8
636826611736535188492639377543192511689632234113007588028716924498069883794193124751601010163170434996358340036191080992584 7
721300802741705519412306522941202429437928826033885416656967971559902743150263252229456298992263008126719589203430407385228 2
303616284948601721297022711729264695008023426087220064207455862972679290525090591543409683485095805523071486420014384703162 2
9
23
500195636126957292905024512596972806695803345136243348970565288179435361313804956505581782637634612477979679893275103396147 3
486507620075949375108045411450023043028673410062984934043196573821232011580071882526065508066945353292322568510566563723796 4
909773530478163017381245453178151110746061951601884432033503380198480681906780256137039403673265408983882355160308329567002 4
453477589093119918386566397677610274213837391954591147603054442650326827980781140275941425217172428448698161710841740688042 5
872041612560849141667625490070127139221727482596905666145800626821966064664981025716276837267184832295780443436467376944364 0
626144436832764909740155024134110270478384161937602773776707712701003990058662584199129511148253973672517216937974044389033 2
234341104310470907449898415522414805210341138063350999730749950920147250683227798780264811215647706542511681027825390882770 7
626621854100803100452612868518426699348493305482372718383451642325605449643150903654217260041087043028543877000535919 57

If the sum of reciprocal Fibonacci prime numbers converges to an irrational number, so we prove that exist infinitely many Fibonacci prime numbers.
Let's the sum as F_p, with p to denote prime number, we have:

$$F_p = \sum_{p:p\in Fp}\left(\frac{1}{p}\right) = \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{13} + \frac{1}{89} + \frac{1}{233} + \frac{1}{1597} + \frac{1}{28657} + \frac{1}{514229} + \frac{1}{433494437} + \frac{1}{2971215073} + \frac{1}{99194853094755497} + \frac{1}{10663404174917105958145721 69} + \frac{1}{19134702400093278081449423917} + \ldots = 1{,}1264472276728533386016660044138 + \ldots$$

The sum for the first 14 values gives as result:

$F_p \approx$ 1,1264472276728533386016660044138 …

The F_p value is a new constant but we don't know if it's irrational number.
If we were sure that the Fibonacci prime numbers are infinite then the constant would be irrational because each reciprocal add one decimal place to the constant and this could no longer be rational.

The reciprocal of the 15th prime Fibonacci is in fact $2{,}1 * 10^{-75}$ a very small value so that gives almost no contribution to the sum and so it goes for all other reciprocals.

The value F_p, as calculated, is approximate and we have to wait then a slightly bigger but very little than previously estimated because we must consider the sum of the reciprocals of ALL Fibonacci prime numbers.
The value F_p is a new constant irrational because we have proven in paragraph 1 that the Fibonacci primes are infinite in number.
It is interesting to note that this new irrational constant is much smaller than the Brun's constant - the sum of the reciprocals of twin primes - a value roughly equal to 1.902160583104 ...> 1.1264472276728533386016660044138, because the number of the Fibonacci primes it is significantly less than the number of the twin primes below a certain threshold x.

It has in fact, below a certain threshold x a number less than or equal to the twin primes numbers equal to

$$\pi_2(x) \approx 2C \left(\frac{x}{(\log x)^2} \right)$$

where:

C = 0,66016181584686957392781211001455 … constant of the twin primes.
We note that the value of this constant is very near **to the final spin of the black hole that is produced calculated by the observations of the gravitational waves, i.e. 0,67.**

For the Fibonacci prime numbers below a certain threshold x a number less than or equal to

$$F(x) \approx 2\varphi * \log[\log(x)] = 3.236 * \log[\log(x)]$$

Then

$$\boldsymbol{\pi_2(x) >> F(x)}$$

The value $\mathbf{F_p}$ is of course less than $\boldsymbol{\psi}$ - sum of the reciprocals of all the Fibonacci numbers - that is irrational because Fibonacci numbers are infinite, known to be approximately:

Ψ = 3,3598856662431775531720113029189271796889051337 3…

The value that would be expected to be definitely lower than:

$F_p < \log \Psi = 1{,}211906945492334408706205757167 5\ldots$

and this happen successfully.

The sum S(x) of the reciprocal of Fibonacci P(x) prime numbers is given by:

$$S(x) \approx \frac{-2\varphi}{x \log x} + 1{,}126447227672853338601666004413 8 =$$
$$= \frac{-3{,}236}{x \log x} + 1{,}126447227672853338601666004413 8 = \frac{-3{,}236}{x \log x} + F_p$$

Furthermore, we know that:

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

With the following formula, from any number *n* (imput) correspond a Fibonacci's number (output).
This formula concerning the aurea ratio. Indeed:

$$\left(\frac{1+\sqrt{5}}{2}\right)^n = \Phi^n$$

while

$$\frac{1-\sqrt{5}}{2}$$

is the negative solution of the equation from which is obtained the golden number, equivalent to:

$$-\frac{1}{\Phi}$$

NOTE 1

Examples of integrals equations containing the golden ratio $\Phi = \frac{\sqrt{5}+1}{2}$ and mathematical connections with the universal gravitational constant G , the Einstein field equations and some equations of string theory linked to Φ and π

1) We have the following expressions:

$$\Phi = 2 \cdot \cos\frac{\pi}{5}; \qquad \int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}.$$

Thence, we obtain the following relationship:

$$2 \cdot \cos\left(\frac{3}{\pi^3}\int_0^\infty \frac{x^3}{e^x - 1} dx\right) = 2 \cdot \cos\left(\frac{3}{\pi^3} \cdot \frac{\pi^4}{15}\right) = 2 \cdot \cos\frac{\pi}{5} = 2 \cdot \cos 0{,}6283185... = 2 \cdot 0{,}809017 \cong 1{,}618034...$$

2) There exist a link between Φ and π to the factor of the universal gravitational constant G (6.67 x 10^-11) by the relationship: $\pi^2 - 2\Phi = G$, with $\Phi = \frac{\sqrt{5}+1}{2}$ is the golden ratio.
The common link of Newtonian constant and that of the fine structure ($\alpha = 2\pi e^2 / (hc)$, that as we see, is linked to π) to π thus implies that between the two above mentioned constants α and G. Furthermore, by $432 / \pi = 137 = 1 / \alpha$ follows that $\alpha = \pi / 432 = 1/137$ (where 432 is the frequency of the natural La) and similarly, by $G = \pi^2 - 2\Phi$ is obtained G = 0,0155 x 432. (M. Nardelli)

The Law of Universal Gravitation states that every object in the universe attracts every other object in the universe with a force that has a magnitude which is directly proportional to the product of their masses and inversely proportional to the distance between their centers squared.

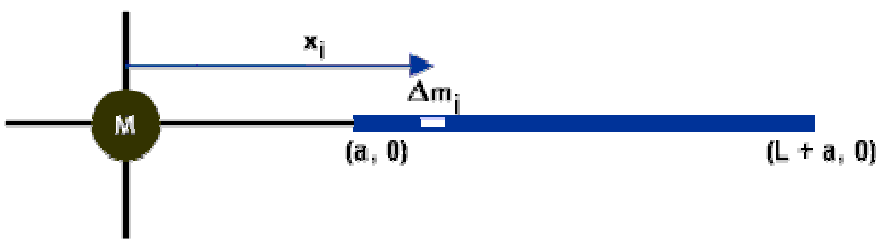
$$F = G\frac{M_1 M_2}{R^2}$$

where

- G is the gravitational constant, 6.67 x 10^{-11} Nm^2/kg^2
- M_1 is the mass of the first body in kg
- M_2 is the mass of the second body in kg
- R is the distance from the center of M_1 to the center of M_2

Now we take an example of an integral containing the gravitational constant G that is connected with ϕ by the relationship: $\pi^2 - 2\Phi = G$ above mentioned. It involves calculating the gravitational force between a point mass **M** and an extended rod of mass **m**, length L, and mass per unit length, λ.

To begin, divide the rod into a finite (countable) number of segments of mass Δm each located at a distance **x** from **M**.



Each of these segments will contribute a gravitational force of attraction.

$$F = \sum_{i=1}^{n} GM\frac{\Delta m_i}{x_i^2}$$

If we take the limit as Δm approaches zero, then our expression for **F** becomes

$$F = \lim_{\Delta m \to 0} \sum_{i=1}^{n} GM \frac{\Delta m_i}{x_i^2} \;; \qquad F = \int_a^{L+a} GM \frac{1}{x^2} dm$$

Before we can integrate we must express Δm in terms of x.

$$\lambda = \frac{dm}{dx} \;; \qquad \lambda dx = dm$$

Substituting and integrating gives us

$$F = \int_a^{L+a} GM\lambda \frac{1}{x^2} dx \;; \quad F = GM\lambda \int_a^{L+a} \frac{1}{x^2} dx \;; \quad F = GM\lambda \left. \frac{-1}{x} \right|_a^{L+a} \;; \quad F = -GM\lambda \left(\frac{1}{L+a} - \frac{1}{a} \right);$$

$$F = -GM\lambda \left(\frac{a-(L+a)}{a(L+a)} \right); \quad F = -GM\lambda \left(\frac{-L}{a(L+a)} \right); \quad F = \frac{GM\lambda L}{a(L+a)} \;; \quad F = \frac{GMm}{a(L+a)}$$

where

$$m = \lambda L$$

Thence we have that, for $\pi^2 - 2\Phi = G$:

$$F = \int_a^{L+a} GM \frac{1}{x^2} dm = \frac{GM\lambda L}{a(L+a)} = \frac{(\pi^2 - 2\Phi) M\lambda L}{a(l+a)}$$

3) The differential form of Gauss's law for gravity states

$$\nabla \cdot g = -4\pi G\rho \,,$$

where ∇ denotes divergence, G is the universal gravitational constant, and ρ is the mass density at each point.

Relation to the integral form

The two forms of Gauss's law for gravity are mathematically equivalent. The divergence theorem states:

$$\oint_{\partial V} g \cdot dA = \int_V \nabla \cdot g dV$$

where V is a closed region bounded by a simple closed oriented surface ∂V and dV is an infinitesimal piece of the volume V (see volume integral for more details). The gravitational field **g** must be a continuously differentiable vector field defined on a neighborhood of V.

Given also that

$$M = \int_V \rho dV$$

we can apply the divergence theorem to the integral form of Gauss's law for gravity, which becomes:

$$\int_V \nabla \cdot g dV = -4\pi G \int_V \rho dV$$

which can be rewritten:

$$\int_V (\nabla \cdot g) dV = \int_V (-4\pi G\rho) dV \ .$$

This has to hold simultaneously for every possible volume V; the only way this can happen is if the integrands are equal. Hence we arrive at

$$\nabla \cdot g = -4\pi G\rho \ ,$$

which is the differential form of Gauss's law for gravity.

The two above expressions, for the relation $\pi^2 - 2\Phi = G$, can be rewritten also as follows:

$$\int_V (\nabla \cdot g) dV = \int_V \left[-4\pi(\pi^2 - 2\Phi)\rho\right] dV$$

$$\nabla \cdot g = -4\pi\left(\pi^2 - 2\Phi\right)\rho$$

4) Also in the Einstein field equations, we have the term G and thence is possible to obtain the mathematical connection with π and Φ.

The Einstein field equations are the 16 coupled hyperbolic-elliptic nonlinear partial differential equations that describe the gravitational effects produced by a given mass in general relativity. As result of the symmetry of $G_{\mu\nu}$ and $T_{\mu\nu}$, the actual number of equations reduces to 10, although there are an additional four differential identities (the Bianchi identities) satisfied by $G_{\mu\nu}$, one for each coordinate.

The Einstein field equations state that

$$G_{\mu\nu} = 8\pi T_{\mu\nu},$$

where $T_{\mu\nu}$ is the stress-energy tensor, and

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

is the Einstein tensor, with $R_{\mu\nu}$ the Ricci curvature tensor and R the scalar curvature.

The Einstein field equations can be written also as follows:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

that include a cosmological constant term Λ. A positive value of Λ is needed to explain the accelerating universe . The Ricci curvature tensor, also simply known as the Ricci tensor (Parker and Christensen 1994), is defined by

$$R_{\mu\kappa} \equiv R^{\lambda}{}_{\mu\lambda\kappa}$$

Where $R^{\lambda}{}_{\mu\lambda\kappa}$ is the Riemann tensor.

The covariant derivative of the Riemann tensor is given by

$$R_{\lambda\mu\nu\kappa;\eta} = \frac{1}{2}\frac{\partial}{\partial x^{\eta}}\left(\frac{\partial^2 g_{\lambda\nu}}{\partial x^{\kappa}\partial x^{\mu}} - \frac{\partial^2 g_{\mu\nu}}{\partial x^{\kappa}\partial x^{\lambda}} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^{\mu}\partial x^{\nu}} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^{\nu}\partial x^{\lambda}}\right)$$

Permuting $\nu, \kappa,$ and η (Weinberg 1972, pp. 146-147) gives the Bianchi identities

$$R_{\lambda\mu\nu\kappa;\eta} + R_{\lambda\mu\eta\nu;\kappa} + R_{\lambda\mu\kappa\eta;\nu} = 0$$

which can be written concisely as

$$R^{\alpha}{}_{\beta[\lambda\mu;\nu]} = 0$$

(Misner *et al.* 1973, p. 221).

The Riemann tensor (Schutz 1985) $R^{\alpha}{}_{\beta\gamma\delta}$, also known the Riemann-Christoffel curvature tensor (Weinberg 1972, p. 133; Arfken 1985, p. 123) or Riemann curvature tensor (Misner *et al.* 1973, p. 218), is a four-index tensor that is useful in general relativity. Other important general relativistic tensors such that the Ricci curvature tensor and scalar curvature can be defined in terms of $R^{\alpha}{}_{\beta\gamma\delta}$.

The Riemann tensor is in some sense the only tensor that can be constructed from the metric tensor and its first and second derivatives,

$$R^{\alpha}{}_{\beta\gamma\delta} = \Gamma^{\alpha}_{\beta\delta,\gamma} - \Gamma^{\alpha}_{\beta\gamma,\delta} + \Gamma^{\mu}_{\beta\delta}\Gamma^{\alpha}_{\mu\gamma} - \Gamma^{\mu}_{\beta\gamma}\Gamma^{\alpha}_{\mu\delta},$$

where $\Gamma^{\gamma}_{\alpha\beta}$ are Christoffel symbols of the first kind and A_{κ} is a comma derivative (Schmutzer 1968, p. 108; Weinberg 1972). In one dimension, $R_{1111} = 0$. In four dimensions, there are **256** components. Making use of the symmetry relations,

$$R_{iklm} = -R_{ikml} = -R_{kilm},$$

the number of independent components is reduced to **36**. Using the condition

$$R_{iklm} = R_{lmik},$$

the number of coordinates reduces to **21**. Finally, using

$$R_{iklm} + R_{ilmk} + R_{imkl} = 0,$$

20 independent components are left (Misner *et al.* 1973, pp. 220-221; Arfken 1985, pp. 123-124).

Thence, we have that :

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

for $\pi^2 - 2\Phi = G$ can be rewritten also as follows:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi\left(\pi^2 - 2\Phi\right)}{c^4} T_{\mu\nu} ;$$

or

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi^3 - 16\pi\Phi}{c^4} T_{\mu\nu} ;$$

We note that with regard the 256 components of the Riemann tensor in four dimensions 256 = 144 + 55 + 34 + 21 + 2; 36 = 2 + 13 + 21; and 21 are all Fibonacci's numbers.

Furthermore, $256 = 32 \times 8 = 2^2 \times 8^2$, where the number 8, and thence the numbers $64 = 8^2$ and $32 = 2^2 \times 8$, are connected with the "modes" that correspond to the physical vibrations of a superstring by the following Ramanujan function:

$$8 = \frac{1}{3} \frac{4\left[anti\log \frac{\int_0^\infty \frac{\cos \pi t x w'}{\cosh \pi x} e^{-\pi x^2 w'} dx}{e^{-\frac{\pi^2}{4} w'} \phi_{w'}(itw')} \right] \cdot \frac{\sqrt{142}}{t^2 w'}}{\log\left[\sqrt{\left(\frac{10 + 11\sqrt{2}}{4} \right)} + \sqrt{\left(\frac{10 + 7\sqrt{2}}{4} \right)} \right]} .$$

5) Also in string theory the term G and thence is possible to obtain the mathematical connection with π and Φ.

With regard the string theory, we know that the asymptotic value of the dilaton is:

$$\Phi_0 = \lim_{X \to \infty} \Phi(X). \quad (1)$$

The string coupling is then given by

$$g_s = e^{\Phi_0}. \quad (2)$$

With regard the action for a $D = 26$ spacetime, this is the low-energy effective action of the bosonic string and is given from the following relationship:

$$S = \frac{1}{2\kappa_0^2}\int d^{26}X\sqrt{-G}e^{-2\Phi}\left(\mathcal{R} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi\right) \quad (3)$$

where we have taken the liberty of Wick rotating back to Minkowski space for this expression.

We defined the constant part in (1); it is related to the string coupling constant. The varying part is simply given by

$$\tilde{\Phi} = \Phi - \Phi_0 \quad (4)$$

In D dimensions, we define a new metric $\tilde{G}_{\mu\nu}$ as a combination of the old metric and the dilaton,

$$\tilde{G}_{\mu\nu}(X) = e^{-4\tilde{\Phi}/(D-2)}G_{\mu\nu}(X) \quad (5)$$

One can check that two metrics related by a general conformal transformation $\tilde{G}_{\mu\nu} = e^{2\omega}G_{\mu\nu}$, have Ricci scalars related by

$$\tilde{\mathcal{R}} = e^{-2\omega}\left(\mathcal{R} - 2(D-1)\nabla^2\omega - (D-2)(D-1)\partial_\mu\omega\partial^\mu\omega\right)$$

With the choice $\omega = -2\tilde{\Phi}/(D-2)$ in (5), and restricting back to $D = 26$, the action (3) becomes

$$S = \frac{1}{2\kappa^2}\int d^{26}X\sqrt{-\tilde{G}}\left(\tilde{\mathcal{R}} - \frac{1}{12}e^{-\tilde{\Phi}/3}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{1}{6}\partial_\mu\tilde{\Phi}\partial^\mu\tilde{\Phi}\right) \quad (6)$$

The gravitational part of the action takes the standard Einstein-Hilbert form. The gravitational coupling is given by

$$\kappa^2 = \kappa_0^2 e^{2\Phi_0} \approx l_s^{24} g_s^2 \quad (7)$$

The coefficient in front of Einstein-Hilbert term is usually identified with Newton's constant

$$8\pi G_N = \kappa^2$$

From Newton's constant, we define the $D = 26$ Planck length $8\pi G_N = l_p^{24}$ and Planck mass $M_p = l_p^{-1}$.

For the relationship $\pi^2 - 2\Phi = G$ eq. (3) can be rewritten also in the following equivalent form:

$$S = \frac{1}{2\kappa_0^2}\int d^{26}X \sqrt{-\left(\pi^2 - 2\left(\frac{\sqrt{5}+1}{2}\right)\right)} e^{-2\Phi}\left(\mathcal{R} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi\right) \quad (8)$$

We know that the value of the Planck constant is:

$$h = 6{,}626070040\times10^{-34}\ \mathrm{J\ s} = 4{,}135667662\times10^{-15}\ \mathrm{eV\ s}$$

and that the value of the reduced Planck constant is:

$$\hbar = \frac{h}{2\pi} = 1{,}054571800\times10^{-34}\ \mathrm{J\ s} = 6{,}582119514\times10^{-16}\ \mathrm{eV\ s}$$

In physics, the **Planck length**, denoted ℓ_P, is a unit of length, equal to $1.616199(97)\times10^{-35}$ metres. It is a base unit in the system of Planck units, developed by physicist Max Planck. The Planck length can be defined from three fundamental physical constants: the speed of light in a vacuum, the Planck constant, and the gravitational constant.

The Planck length ℓ_P is defined as

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1{,}61619 \times 10^{-35} \text{ m}$$

where c is the speed of light in a vacuum, G is the gravitational constant, and $\hbar$ is the reduced Planck constant.

We note immediately, as the number 1,61619 is very near, about equal, to the value of the golden ratio 1,61803398… = (√5 + 1) / 2

The Planck length is about 10^{-20} times the diameter of a proton.

In physics, the **Planck mass**, denoted by m_P, is the unit of mass in the system of natural units known as Planck units. It is defined so that

$m_P = \sqrt{\frac{\hbar c}{G}}$ ≈ $1{,}2209 \times 10^{19}$ GeV/c^2 = $2{,}17651 \times 10^{-8}$ kg = 21,7651 µg (microgram) = $1{,}3107 \times 10^{19}$ amu.

where c is the speed of light in a vacuum, G is the gravitational constant, and $\hbar$ is the reduced Planck constant.

We note that the value 21,7651 is very near to 21 that is a Fibonaccì's number.

The Planck time is defined as:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 5{,}39106 \times 10^{-44} \text{ s}$$

We have that **F_p ≈ 1,126447227672853338601666004413 8 …** Now **1,126447× 5 = 5,632235 value very near to the value of t_P that is also very near to the number 5, that is a Fibonacci's number. Furthermore, we know that the constant of the twin primes is C = 0,66016181584686957392781211001455. Now: C × 8 ≈ 5,28129 value very near to the value of the Planck's time. Note that 8 is a Fibonacci's number.**

In physics, **Planck energy**, denoted by E_P, is the unit of energy in the system of natural units known as Planck units.

$$E_P = \sqrt{\frac{\hbar c^5}{G}} \approx 1{,}956 \times 10^9\,\mathrm{J} \approx 1{,}22 \times 10^{28}\,\mathrm{eV}$$

We have that Ψ = 3,35988566624317755317201130291892717968890513373…

Furthermore, we have log Ψ = 1,211906945492334408706205757167 5…value that **is very near to the value in eV of E_P and of m_P in GeV/c^2. Furthermore, 1,956 is a value very near to the Brun's constant** $B_2 \approx$ 1,902160583104. (The sum of the reciprocals of the twin prime numbers tends to a constant B_2. We note that the Brun's constant 1,902160 / 3 = 0,6340 value very near **to the final spin of the black hole that is produced calculated by the observations of the gravitational waves, i.e. 0,67.**). **Also here the twin prime constant C × 3 = 1,98048 value very near to the Planck's energy. Note that 3 is a Fibonacci's number.**

An equivalent definition is:

$$E_P = \frac{\hbar}{t_P},$$

where t_P is the Planck time.

We have Ψ = 3,35988566624317755317201130291892717968890513373…

We note that Ψ / 5 = 0,671977, value very near **to the final spin of the black hole that is produced calculated by the observations of the gravitational waves, i.e. 0,67. Interesting also that the divisor, i.e.** 5 , **is a Fibonacci's number .**
Also $\frac{\sqrt{(1{,}61803398)^3}}{\pi} = 0{,}6551$ **is very near to** 0,67 **and in this formula there is also** π**.**
Furthermore, log Ψ = 1,211906945492334408706205757167 5… **is very near to the** 36 / 29 = 1,2413793…, **where 36 and 29 are the values of the solar masses of the two initial black holes.**

The Hilbert-Einstein action of the gravitational field is:

$$S_g = -\frac{c^3}{16\pi G}\int d^4x\sqrt{-g(x)}R(x), \quad (9)$$

where g is the determinant of $g_{\mu\nu}$ and R is the scalar curvature,

$$R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu}\left(\frac{\partial \Gamma^{\rho}_{\mu\nu}}{\partial x^{\rho}} - \frac{\partial \Gamma^{\rho}_{\mu\rho}}{\partial x^{\nu}} + \Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho} - \Gamma^{\rho}_{\sigma\nu}\Gamma^{\sigma}_{\mu\rho}\right). \quad (10)$$

The variation to volume fixed from the integral (9) is

$$\delta\int d^4x\sqrt{-g}R = \int d^4x\sqrt{-g}R_{\mu\nu}\delta g^{\mu\nu} + \int d^4xR\delta\sqrt{-g} + \int d^4x\sqrt{-g}\,g^{\mu\nu}\delta R_{\mu\nu}\,. \quad (11)$$

We have that

$$\delta\sqrt{-g} = -\frac{1}{2\sqrt{-g}}\delta g = -\frac{1}{2\sqrt{-g}}\frac{\partial g}{\partial g^{\mu\nu}}\delta g^{\mu\nu}\,. \quad (12)$$

now

$$g_{\mu\nu} = \frac{1}{g}\frac{\partial g}{\partial g^{\mu\nu}}\,, \quad (13)$$

we have

$$\frac{\partial g}{\partial g^{\mu\nu}} = g g_{\mu\nu}\,, \quad (14)$$

thence, the eq. (12) become

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\,g_{\mu\nu}\delta g^{\mu\nu}\,. \quad (15)$$

Substituting this expression in the (11), we obtain

$$\delta\int d^4x\sqrt{-g}R = \int d^4x\sqrt{-g}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\delta g^{\mu\nu} + \int d^4x\sqrt{-g}\,g^{\mu\nu}\delta R_{\mu\nu}\,. \quad (16)$$

The second term of the eq.(16) is a surface term which does not contribute to the variation of the action. Then, we have

$$\delta\int d^4x\sqrt{-g}R = \int d^4x\sqrt{-g}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\delta g^{\mu\nu}\,. \quad (17)$$

The principle of stationary action

$$\delta S_g = -\frac{c^3}{16\pi G}\delta\int d^4x\sqrt{-g}R = 0, \quad (18)$$

implies, for the (17),

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0. \quad (19)$$

that is the Einstein's equation of the gravitational field in the absence of the sources. Consider now a source of energy represented from a generic field φ. The action of φ has the form

$$S_\varphi = \frac{1}{c}\int d^4x\sqrt{-g}\mathcal{L}, \quad (20)$$

where $\mathcal{L}$ is the density of Lagrangian. Putting equal to zero the variation of S_φ with respect to φ, we obtain the equation of motion for the field φ. We are interested only the variation of S_φ with respect to $g^{\mu\nu}$, that is

$$\delta S_\varphi = \frac{1}{c}\int d^4x\left[\frac{\partial\left(\sqrt{-g}\mathcal{L}\right)}{\partial g^{\mu\nu}}\delta g^{\mu\nu} + \frac{\partial\left(\sqrt{-g}\mathcal{L}\right)}{\partial\partial_\rho g^{\mu\nu}}\delta\partial_\rho g^{\mu\nu}\right] =$$
$$= \frac{1}{c}\int d^4x\left[\frac{\partial\left(\sqrt{-g}\mathcal{L}\right)}{\partial g^{\mu\nu}} - \partial_\rho\frac{\partial\left(\sqrt{-g}\mathcal{L}\right)}{\partial\partial_\rho g^{\mu\nu}}\right]\delta g^{\mu\nu}. \quad (21)$$

Introducing the tensor $T_{\mu\nu}$ defined by

$$\frac{1}{2}\sqrt{-g}T_{\mu\nu} = \frac{\partial\left(\sqrt{g}\mathcal{L}\right)}{\partial g^{\mu\nu}} - \partial_\rho\frac{\partial\left(\sqrt{-g}\mathcal{L}\right)}{\partial\partial_\rho g^{\mu\nu}}, \quad (22)$$

the eq. (21) can be rewritten as follows

$$\delta S_\varphi = \frac{1}{2c}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}. \quad (23)$$

The full action of the system constituted by the field φ and by the gravitational field $g_{\mu\nu}$

is

$$S = S_g + S_\varphi = -\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}R + \frac{1}{c}\int d^4x\sqrt{-g}\mathcal{L}. \quad (24)$$

The interaction between the two fields is incorporated in the second term. Using the eq. (17), we find that the stationarity condition for S is

$$\delta S = \int d^4x\sqrt{-g}\left[-\frac{c^3}{16\pi G}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{2c}T_{\mu\nu}\right]\delta g^{\mu\nu} = 0, \quad (25)$$

and from this are obtained the Einstein's equations in the presence of sources,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}. \quad (26)$$

For the relationship $\pi^2 - 2\Phi = G$, we have that

$$\delta S = \int d^4x\sqrt{-g}\left[-\frac{c^3}{16\pi(\pi^2 - 2\Phi)}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{2c}T_{\mu\nu}\right]\delta g^{\mu\nu} = 0, \quad (27)$$

with $\Phi = \frac{\sqrt{5}+1}{2}$ that is the golden ratio.

Furthermore, we have the following mathematical connection between the eq. (27) and the eq. (6):

$$S = \frac{1}{2\kappa^2}\int d^{26}X\sqrt{-\tilde{G}}\left(\tilde{\mathcal{R}} - \frac{1}{12}e^{-\tilde{\Phi}/3}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{1}{6}\partial_\mu\tilde{\Phi}\partial^\mu\tilde{\Phi}\right) \Rightarrow$$

$$\Rightarrow \int d^4x\sqrt{-g}\left[-\frac{c^3}{16\pi\left(\pi^2 - 2\left(\frac{\sqrt{5}+1}{2}\right)\right)}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{2c}T_{\mu\nu}\right]\delta g^{\mu\nu} = 0. \quad (28)$$

For the relationship $\pi^2 - 2\Phi = G$, we have finally that

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi\left(\pi^2 - 2\Phi\right)}{c^4} T_{\mu\nu}; \quad (29)$$

From this new expression we can to obtain a new way of obtaining Φ. Indeed, we have that:

$$\Phi = -\frac{\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) c^4}{16\pi T_{\mu\nu}} + \frac{1}{2}\pi^2$$

or:

$$\frac{\sqrt{5}+1}{2} = -\frac{\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) c^4}{16\pi T_{\mu\nu}} + \frac{1}{2}\pi^2. \quad (30)$$

Thence, the golden ratio Φ is directly proportional to the gravitational field $g_{\mu\nu}$, the $R_{\mu\nu}$, that is the Ricci curvature tensor and R, that is the scalar curvature and c that is the speed of the light; and inversely proportional to the $T_{\mu\nu}$, that is the stress-energy tensor.

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2.1 *APPROXIMATION OF THE VALUE OF NEW CONSTANT Fp*

A good approximation of the value of the new constant **Fp** is given by:

$\sqrt{\sqrt{\Phi}} = \sqrt[4]{\varphi}$ 1,127838485561682260264835483177 , with $\Phi = (\sqrt{5} + 1) / 2$

$F_p \approx$ 1,126447227672853338601666004413 8 …

with a relative error of 0.1235% and an absolute error of about only 0.00139, about a thousandth and a half.

WARNING: this approximate value does not tend to the new constant Fp that is independent of the golden ratio and is a new irrational number probably also transcendental.

3. REFERENCES

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