The Effect of Intervening Plasma on Casimir Free Forces at Finite Temperatures.

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The effect of an intervening plasma on the interaction Casimir free energy between perfectly conducting surfaces is examined. We derive an asymptotic expression for the interaction free energy that is valid at any finite plasma density in both the high temperature limit and in the large separation limit.

When two objects come close together the mutual electric polarizations of the material results in an attractive force. At short distances this is the van der Waals force, at large distances retardation and the quantum nature of light becomes important and the result is the Casimir force. Casimir predicted already in 1948 an attractive interaction energy between perfect metal surfaces at zero temperature [1]: $F(l) = -\pi^2\hbar c/720 l^3$. Lifshitz recast these interactions in terms of interactions between continuous media with measurable dielectric susceptibilities [2, 3]. Considering that this force has been known for so long, and recently been measured with *claimed* accuracies down to 1 % [4–7], one may question if the field now is devoid of interest. The obvious answer to this question is that it certainly is not. Experiments aiming to understand forces induced by vacuum fluctuations, retardation effects, and influence of finite temperatures have often been compared with an incorrectly evaluated interaction energy. We are not referring to the inclusion of surface roughness, and similar important technical details, without conceptual interest. We will in this brief report extend the previous work of Ninham and Daicic [8] on the effects of an intervening plasma on the Casimir interaction between ideal metal surfaces. We will derive a simple asymptotic expression for the free energy that is valid in both the high temperature limit and in the large separation limit. An important point that we will discuss is that it is valid at any finite plasma density. Let us first briefly rehearse some flaws in the theoretical framework that have been revealed during the last few years. The high temperature asymptotic form of the Casimir interaction originally derived by Lifshitz for dielectric media has been demonstrated to also be the correct form for real dissipative metal surfaces [9, 10]. Since only transverse magnetic modes contributes to this high temperature asymptote, the magnitude is half that of the interaction between ideal metal surfaces that also receives a contribution from transverse electric modes. The high temperature asymptote between ideal metal surfaces is: $F(l) = -kT\zeta(3)/8\pi l^2$. Similarly, Ninham et al. [11, 12] have demonstrated that the incorrect separation of nonlinear electrostatic forces and linear van der Waals forces is one reason why the biological sciences for so long failed to understand a large number of vital biological phenomena (e.g. protein precipitation). So too is the separation of electrostatic and Casimir forces, as is routinely done in the interpretation of experiments aiming to measure the Casimir force, incorrect in the presence of any intervening plasma. Another example, relevant for cold molecule formation, where a so-called "well known result" turn out to be incorrect is the resonance interaction between two atoms in an excited configuration [13]. In the retarded limit the resonance interaction decays with a simple power-law, rather than oscillating as in previous incorrect treatments. As a final example, Ninham et al. [8, 14, 15] analyzed the asymptotic behavior of the Casimir interaction and the atom-atom interactions at finite temperatures. As a quite remarkable consequence of the correspondance principle they showed that these long-range interactions at any finite temperature go to classical asymptotes. The purpose of this brief report is to demonstrate that the inclusion of any finite, however small, plasma density fundamentally alters the long-range Casimir free energy of interaction.

Consider now two perfectly conducting planar surfaces separated by a free-electron plasma. The model system is chosen for demonstrational purposes, but we expect that our conclusions will be relevant for the effect of intervening plasma on the free energy of interaction between particles in general. All interactions between particles takes place in the presence of a plasma. A few examples: in biology it is an electrolyte; in Casimir force measurements between metal surfaces across a *"vacuum"* it is a very dilute electron plasma extending beyond the edges of the metallic surfaces; the plasma of fluctuating electron-positron pairs is always present. The frequency (ω) dependent dielectric susceptibility of a plasma is

$$\epsilon_2(\omega) = 1 - \frac{4\pi\rho e^2}{m\omega^2}, \tag{1}$$

where ρ is the number density of electrons (or charged particles) in the plasma, e is the unit electric charge, m is the electron mass. For future convenience we define two help variables [8]: $\overline{\rho} = [\rho/(\pi m)][e\hbar/(kT)]^2$; $x = (2kTl)/(\hbar c)$. Here $\hbar$ is Planck's constant, c is the

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speed of light, k is Boltzmann's constant and T is temperature. One main result of this brief report is that the high x (i.e. high temperature or large distances) asymptotic interaction energy for **any** finite plasma density can be written as

$$F = -\frac{kT}{2\pi}\int_{\kappa}^{\infty} dttln(1-e^{-2lt}) + F_{n>0}, \quad (2)$$

$$F_{n>0} = -\frac{(kT)^2}{l\hbar c} e^{-\pi\overline{\rho}x} e^{-2\pi x}, \quad (3)$$

where $\kappa^2 = 4\pi\rho e^2/mc^2$. The first term is the $F_{n=0}$ term examined in some detail in a previous paper [8]. The second term, $F_{n>0}$, follows after some rather lengthy algebra that we for clarity outline in the appendix.

Follow with some asymptotes and consequences. End with a summary.

APPENDIX: EFFECT OF INTERVENING PLASMA ON CASIMIR FREE ENERGY

The purpose of this appendix is to outline the main steps in the derivation, rather than giving a complete derivation, of the Casimir free energy between perfect metal surfaces with an intervening plasma at high x. The complete free interaction energy of the system [8] can after some algebra be written in the following form,

$$F(l,T) = -\frac{kT}{4\pi l^2}\Phi, \quad (A.1)$$

where

$$\frac{\Phi}{\pi x^3} = I \equiv \int_0^{\infty} e^{-\pi\overline{\rho}y} y^{-5/2}\overline{\omega}(x^2/y)[1+2\overline{\omega}(y)], \quad (A.2)$$

and

$$\overline{\omega}(y) = \sum_{n=1}^{\infty} e^{-n^2\pi y} = [\frac{-1}{2} + \frac{y^{-1/2}}{2} + y^{-1/2}\overline{\omega}(\frac{1}{y})]. \quad (A.3)$$

As detailed derivation as 4 page limit allows.

[1] H. B. G. Casimir, Proc. Ned. Akad. Wet. **51**, 793 (1948)

[2] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 94 (1955) [Sov. Phys. JETP **2**, 73 (1956)]

[3] V. A. Parsegian and B. W. Ninham, Nature **224**, 1197 (1969); Biophysical J. **10**, 664 (1970).

[4] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997); **81**, 5475 (1998).

[5] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998).

[6] T. Ederth, Phys. Rev. A **62**, 062104 (2000).

[7] G. Bressi, G. Carugno, R. Onofrio, G. Rouso, Phys. Rev. Lett. **88**, 041804 (2002).

[8] B. W. Ninham and J. Daicic, Phys. Rev. A **57**, 1870 (1998). See also: B. Davies and B. W. Ninham, J. Chem. Phys. **56**, 5797 (1972).

[9] M. Boström and Bo E. Sernelius, Phys. Rev. Lett. **84**, 4757 (2000).

[10] I. Brevik, J. B. Aarseth, J. S. Hoye, Phys. Rev. E **66**, 026119 (2002).

[11] B. W. Ninham and V. Yaminsky, Langmuir **13**, 2097 (1997).

[12] M. Boström, D. R. M. Williams, and B.W. Ninham, Phys. Rev. Lett. **87**, 168103 (2001).

[13] M. Boström, J. J. Longdell, and B.W. Ninham, Europhys. Lett. **59**, 21 (2002); M. Boström, J. J. Longdell, D. J. Mitchell, and B.W. Ninham, Eur. Phys. J. D (in press).

[14] H. Wennerström, J. Daicic, and B. W. Ninham, Phys. Rev. A **60**, 2581 (1999).

[15] M. Boström, J. J. Longdell, and B.W. Ninham, Phys. Rev. A **64**, 062702 (2001).

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