Answer to Herrington's Objection

November 24, 2018

Dear Dr Garry Herrington,

I had very recently attended and studied Dr. Eswaran's Lecture.

I also read your comment of Eswaran's Paper. (Your comments are pasted at the end for your convenience)

Your main objection is that Eswaran treats the sequence $\lambda1, \lambda2, \lambda3, \ldots \lambda n$ as a Random walk, where in actuality it is a perfectly deterministic series.

My Comment:

The point is: Eswaran knows that the $\lambda1, \lambda2, \lambda3, \ldots \lambda n$ is a fixed sequence which is unalterable because each λn (= $\lambda(n)$) has been obtained by actually factorizing the integer n. What he means to say is that the sequence can be treated as an **instance of a random walk** provided for large and arbitrary n the, $\lambda(n)$ satisfy the following **three criteria viz. (i) equal probabilities of being +1 or -1 , (ii) the λ-sequence has no cycle** and (iii) **unpredictability** that is there is no fixed integer k, such that for an arbitrary large n, $\lambda(n)$ is predictable from only its previous k values of λ i.e. from $\{\lambda(n-1), \lambda(n-2), \lambda(n-3), \ldots \lambda(n-k)\}$. If the three criteria is satisfied by $\lambda(n)$, then it is reasonable to expect that L(N) satisfy the same bounds as s(N)- the distance travelled by a random walker in N steps - and then since s(N) is bounded by $N^{1/2+\varepsilon}$ one can then deduce L(N) will be similarly bounded by $N^{1/2+e}$ from this result RH can be deduced as a consequence of Littlewoods Theorem which is proved in Eswaran's paper.

In his paper Eswaran goes about proving the above three criteria and thus proving RH.

End of My Comment

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I had the opportunity to attend Eswaran's Endowment Lecture see the news item:
https://www.thehansindia.com/posts/index/Young-Hans/2018-11-20/Hyderabad-based-mathematicians-endeavour-generates-interest/444502

His Lecture Slides can be read, it has some more information regarding Borwein's condition:

https://www.researchgate.net/publication/329072432_Memorial_Lecture_On_the_Final_and_Exhaustive_Proof_of_the_Riemann_Hypothesis_at_the_Institute_of_Engineers_Khairatabad_Hyderabad_India

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Regards

R. Raghavan

P.S. For your convenience a pdf version of this email is attached.

Herrington’s Email

From garry.herrington@gmail.com , 1 November 2018

Herrington’s Objection

Problem with a Claimed Proof of the Riemann Hypothesis > Eswaran 2018

Eswaran 2018 claims a proof of the Riemann Hypothesis under the title “The Final and Exhaustive Proof of the Riemann Hypothesis from First Principles”. This is a bold claim given the history of attempts to prove the Riemann Hypothesis and the absence in Eswaran 2018 of any evidence of independent expert review.

§ 1 Theorems on which the claimed proof is based

Theorem 1

If s1, s2, s3, …, sn is a 1-D random walk with steps sn = ± 1 and a probability p that sn = +1 then E(|s1 + s2 + s3 + … + sn |) $\leq$ 1 n ½ where E(x) is the expected value of x (Stillwell 2016, p.285).

Theorem 2

The Riemann Hypothesis is equivalent to Lim n$\rightarrow\infty$ (λ1 + λ2 + λ3 + … + λn) / n ½ + ε =2 0 ∀ε > 0 where ε is independent of n (Borwein et al 2008, Theorem 1.2, p.6). λn is the “Liouville function” and is defined as -1 if Ω(n) is odd and +1 if Ω(n) is even where Ω(n) = m1 + m2 + m3 + … + mk when n = p m 1 1 p m 2 2 p m 3 3 …p m k k (Borwein et al 2008, Definition 1.1, p.6).

§ 2 A problem with the claimed proof

If p = ½ in Theorem 1 then relation (1) may be written,

-2n½ < s1 + s2 + s3 + … + sn < 2n ½

and so Lim n$\rightarrow\infty$ (s1 + s2 + s3 + … + sn) / n ½ + ε =3 0 ∀ε > 0 where ε is independent of n.

If λ1, λ2, λ3, … λn was a 1-D random walk with p= ½ ,as claimed in Eswaran 2018, then relations(2,3) would imply the Riemann Hypothesis. However λ1, λ2, λ3, … λn is not a 1-D random walk because the value of λn is determined by n whereas in a 1-D random walk s1, s2, s3, …, sn the value of sn is independent of n. Therefore the claim λ1, λ2, λ3, … λn is a 1-D random walk is false because λn does not satisfy an essential defining property of a 1-D random walk.

§ 3 Some History

An argument similar to the one in Eswaran 2018 that uses the Liouville function λn is made in Good and Churchhouse 1968 using the Möbius function μ(n) but those authors do not claim a proof of the Riemann Hypothesis.

Quote from Good and Churchhouse 1968, p. 857

"The aim of the present note is to suggest a "reason" for believing Riemann's hypothesis.

The Möbius function is defined by $\mu(n) = (-)^k$ if the positive integer n is the product of k different primes, $\mu(1) = 1$, and $\mu(n) = 0$ if n has any repeated factor. It is known (see, for example, Titchmarsh [9, p. 315]) that a necessary and sufficient condition for the truth of the Riemann hypothesis is that $M(x) = O(x^{1/2 + \varepsilon})$, for all $\varepsilon > 0$, where $M(x) = \Sigma\, \mu(n)\ (n \leq x)$. The condition $M(x) = O(x^{1/2 + \varepsilon})$ would be true if the Möbius sequence $\{\mu(n)\}$ were a random sequence, taking the values -1, 0, and 1, with specified probabilities, those of -1 and 1 being equal.

More generally, if we first select a subsequence from $\{\mu(n)\}$ by striking out all the terms for which $\mu(n) = 0$, and if this subsequence were 'equiprobably random', i.e. if the value -1 and 1 each had (conditional) probability ½ , then the condition $M(x) = O(x^{1/2 + \varepsilon})$ would still be true. Of course a deterministic sequence can at best be 'pseudorandom' in the usual incompletely defined sense in which the term is used, and of course all our probability arguments are put forward in a purely heuristic spirit without any claim that they are mathematical proofs."

End quote

The reference to Titchmarsh in the quote is given in the references below.

Good and Churchhouse 1968 is discussed in Davis and Hersh 1988 pp. 363 - 369 which mentions that an earlier paper, Denjoy 1931, "uses similar but less detailed probabilistic arguments". Denjoy 1931 is one of the references in Eswaran 2018 but Good and Churchhouse 1968 is not.

References

1. Borwein, P., Choi,S., Rooney,B., and Weirathmueller,A., "The Riemann Hypothesis: A Resource for the Aficionado and Virtuoso Alike", Springer, 2008

2. Davis,P. and Hersh, R., "The Mathematical Experience", Pelican Books 1988

3. Denjoy, A., "L'Hypothèse de Riemann sur la distribution des zeros de ζ(s), reliée à la théorie des probabilités", Comptes Rendus Acad. Sci. Paris 192, 656-658, (1931).

4. Eswaran, K., "The Final and Exhaustive Proof of the Riemann Hypothesis from First Principles", www.researchgate.net, May 2018

5. Good,I. and Churchhouse,R., "The Riemann Hypothesis and Pseudorandom Features of the Möbius Sequence", Mathematics of Computation. 22, 857 – 864, 1968

6. Stillwell, J., "Elements of Mathematics: From Euclid to Gödel", Princeton University Press, 2016

7. Titchmarsh,E., "The Theory of the Riemann Zeta-Function", Clarendon Press, Oxford,1951. MR 13, 741.

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