ECM3404: Logic and Computation

Exercise on Gödel Numbering

We'll use the numbering system given in the notes, i.e.,

0	s	$=$	$\neg$	$\vee$	$\forall$	$($	$)$	$+$	$\times$	x	$'$
1	3	5	7	9	11	13	15	17	19	21	23

(It is important to remember that many different schemes for this are possible; in fact Gödel himself used a different formal language and a different numbering scheme—but here we adopt the same general principles as Gödel did.)

The formula ϕ consisting of the symbols $a_1a_2a_3\dots a_n$ has Gödel number

$$g(\phi) = 2^{g(a_1)}3^{g(a_2)}5^{g(a_3)}\dots p_n^{g(a_n)},$$

where p_n is the nth prime number. The Gödel number assigned to a sequence $\phi_1, \phi_2, \dots, \phi_n$ of formulae, is

$$2^{g(\phi_1)} \times 3^{g(\phi_2)} \times \cdots \times p_n^{g(\phi_n)}.$$

There follows a sequence of simple questions to help you consolidate your understanding of how Gödel numbering works.

1. Write down a formula which says that $0 \neq 1$, and compute its Gödel number.
2. Which well-formed formula has the smallest Gödel number, and what is the number?
3. What string of symbols has Gödel number 3000? Is it a well-formed formula?
4. Explain why 288 and 4000 are not Gödel numbers.
5. Write down necessary and sufficient conditions for the number n to be the Gödel number of a string of symbols.
6. Show that the Gödel number of any formula must be divisible by 30.
7. What is 6^{2430} the Gödel number of?
8. What is the highest power of 2 that can divide the Gödel number of a formula?
9. What is the highest power of 2 that can divide the Gödel number of a *closed* formula?
10. Provide a recipe for deriving $g(\neg\phi)$ from $g(\phi)$. Illustrate for the case $g(\phi) = 2430$.