

UNIVERSITY OF EXETER  
SCHOOL OF ENGINEERING, COMPUTER SCIENCE  
AND MATHEMATICS

**ECM3404**

*Logic and Computation*

**Continuous Assessment**

Date set: 27th February 2009  
Hand-in date: 19th March 2009  
Return date: 27th April 2009

This CA comprises 20% of the overall module assessment.

This is an **individual** exercise, and your attention is drawn to the guidelines on collaboration and plagiarism in the School handbook.

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This is an exercise in Logic, using the material covered in lectures 1–7. To obtain full marks, you must demonstrate a full understanding of all topics, the application of this understanding to particular problems, and the ability to set out your answers clearly and concisely using correct notations, as taught in the module.

**Please Turn Over**

1. A Propositional Calculus inference to validate using Natural Deduction. You will first need to translate the sentences into logical formulae; please be sure to provide a key to your translation (i.e., a table showing what sentence each schematic letter you use stands for):

If you do not have an umbrella, then if it rains you will get wet.

If it snows and you do not have a coat, then you will freeze.

If you freeze or get wet, then you will catch a cold.

Therefore, if rains or snows and you do not catch a cold, you have an umbrella or a coat.

**(50 marks)**

2. Three theorems of Peano Arithmetic to prove. You should use the style of proof used in the lectures and in the answers to the tutorial exercise on Peano Arithmetic.

(a)  $\forall x(0 * x = 0)$

(b)  $\forall x \forall y (sx * y = x * y + y)$

(c)  $\forall x \forall y (x * y = y * x)$

You may make use (without proving them again) of the results that were proved in the tutorial exercise, and also the following result (which can of course be proved, but you are not being asked to prove it):

$$\forall x \forall y \forall z (x + (y + z) = (x + y) + z).$$

Note: Part (b) is more difficult than parts (a) and (c). If you get stuck with part (b), you should move on to part (c), assuming the result of part (b) as necessary.

**(50 marks)**