Subdivision surfaces

COM3404

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Outline

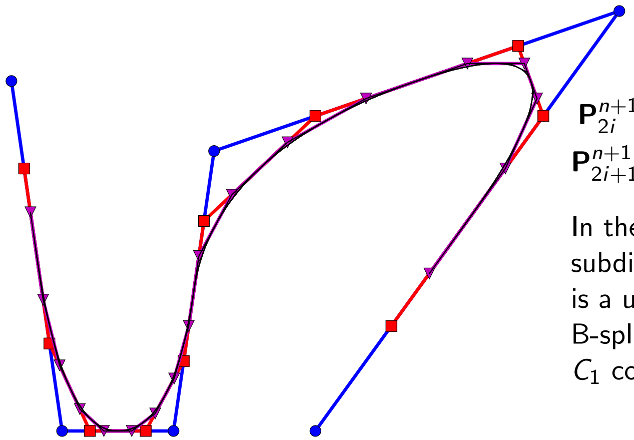
1. Subdivision curves
2. Subdivision surfaces

References

- Fundamentals of 3D Computer Graphics. Watt.
- SIGGRAPH 1999 and 2000 course notes: http://mrl.nyu.edu/~dzorin/sig00course/

Subdivision curves: Chaikin

Generate new control vertices at 1/4 and 3/4 along each line segment



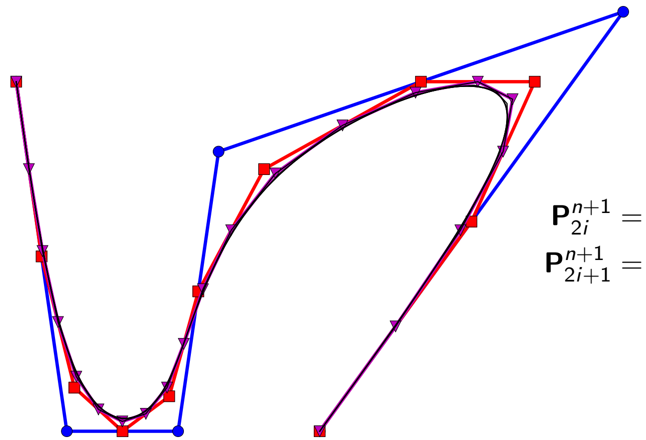
$$\mathbf{P}_{2i}^{n+1} = (3\mathbf{P}_i^n + \mathbf{P}_{i+1}^n)/4$$
$$\mathbf{P}_{2i+1}^{n+1} = (\mathbf{P}_i^n + 3\mathbf{P}_{i+1}^n)/4$$

In the limit of infinite subdivision the curve is a uniform quadratic B-spline and therefore C_1 continuous.

- Render (in screen space) by subdivision until line segments are approx 1 pixel long

Subdivision curves: C_2 continuity

Insert midpoint between vertices and adjust old vertex

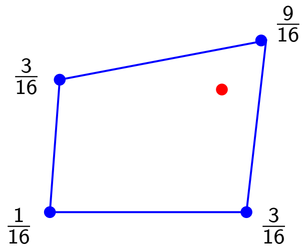


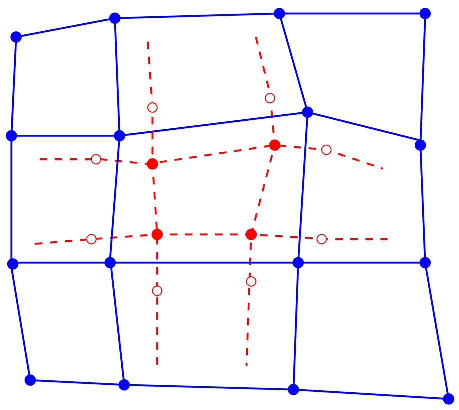
$$\mathbf{P}_{2i}^{n+1} = (\mathbf{P}_{i-1}^n + 6\mathbf{P}_i^n + \mathbf{P}_{i+1}^n)/8$$
$$\mathbf{P}_{2i+1}^{n+1} = (4\mathbf{P}_i^n + 4\mathbf{P}_{i+1}^n)/8$$

- In the limit of infinite subdivision the curve is a cubic B-spline and therefore C_2 continuous.

Doo-Sabin subdivision

- Generalisation to two dimensions of Chaikin curve subdivision.





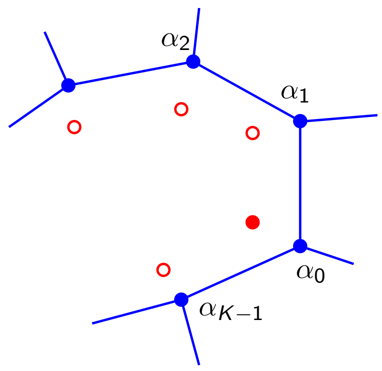
- Weights found from $[1/4, 3/4] \times [1/4, 3/4]$
- Render by sub-division until polygons are the size of pixels

Doo-Sabin extraordinary polygons

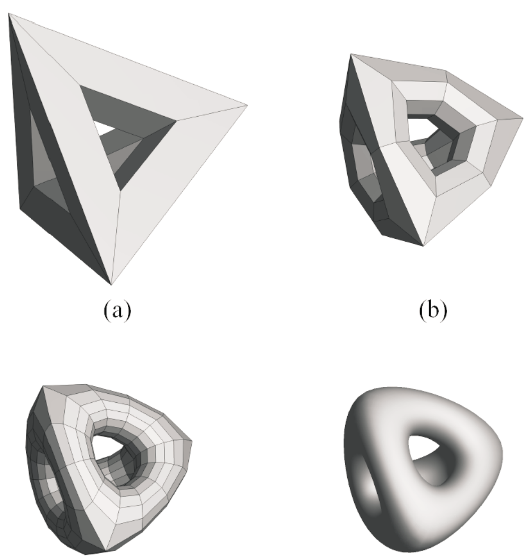
- Polygons that are not quadrilaterals are called *extraordinary*
- In order to preserve continuity of surface in extraordinary polygons Doo-Sabin subdivision requires special weights:

$$\alpha_0 = 1/4 + 5/(4K)$$
$$\alpha_i = (3 + 2\cos(2i\pi/K)/4K$$

K is the *valency*, i.e., the number of vertices

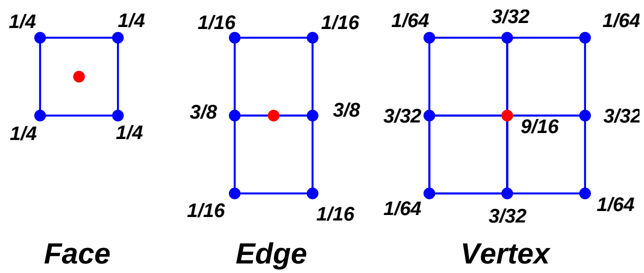


Catmull-Clark subdivision



Catmull-Clark subdivision

- Based on $[1, 4, 6, 4, 1]/8$ univariate scheme.
- New vertices may be occur in a *face*, on an *edge* or at an original *vertex*



- Obtains C_2 continuity except at extraordinary points where C_1 continuity is achieved with special choices of weight.

Subdivision for animation

Convenient for animation as:

- skeleton can be attached to coarse control points
- always yields a smooth surface (no creases) as skeleton is animated

NURBS Used for:

1995 Toy Story
1998 A Bug's Life

Subdivision surfaces

Piloted in Geri's Game (1997)
Used for:

1999 Toy Story II
2001 Monsters Inc.
2003 Finding Nemo

Animation and displacement mapping

- Rig and animate low resolution model.
- 1,700 polygons

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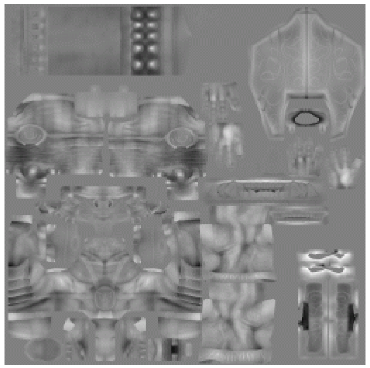
- Smooth high resolution from sub-division model
- Deforms smoothly when animated.

Animation and displacement mapping



- Displacement map describes amount to displace surface in normal direction when rendering

- Smooth sub-division surface plus displacement map
- Equivalent polygon model is 500,000 quadrilaterals

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Bézier patches, NURBS and subdivision surfaces

Bézier patches

- Difficult to stitch together

NURBS

- Continuity C_1 or C_2 can be guaranteed
- Surfaces require a *quadrilateral* mesh of control points
- Difficult to join surfaces: extraordinary points
- Very compact representation

Subdivision

- Easy to manipulate
- Handles extraordinary points well
- Not confined to quadrilateral meshes
- Requires lots of memory (but not prohibitive)