

# Volume visualisation

## COM3404

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# Outline

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- 1 Voxel data
- 2 Contouring
  - Marching squares
- 3 Isosurfaces: Marching cubes
- 4 Volume rendering
  - Transfer function
  - Transformation
  - Compositing
- 5 Splatting



## References

- Foley, van Dam, Feiner & Hughes. Computer Graphics.
- Watt. 3D Computer Graphics.
- Strothotte & Schlechtweg. Non-photorealistic Computer Graphics: Modeling, Rendering and Animation.
- Lorensen & Cline (1987) *Marching cubes: a high resolution 3D surface construction algorithm*. Computer Graphics, **21** (4), 163-169.

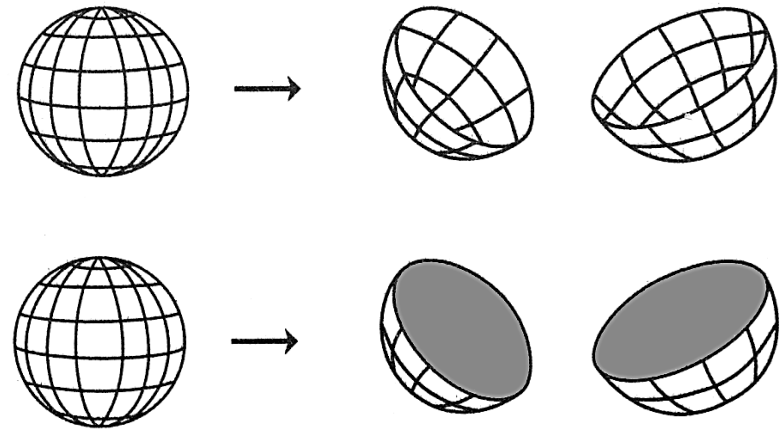
# Solid modelling

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## Solid modelling

Objects to be modelled are solid rather than surfaces.

- Often only objects are defined; eg. engineering parts



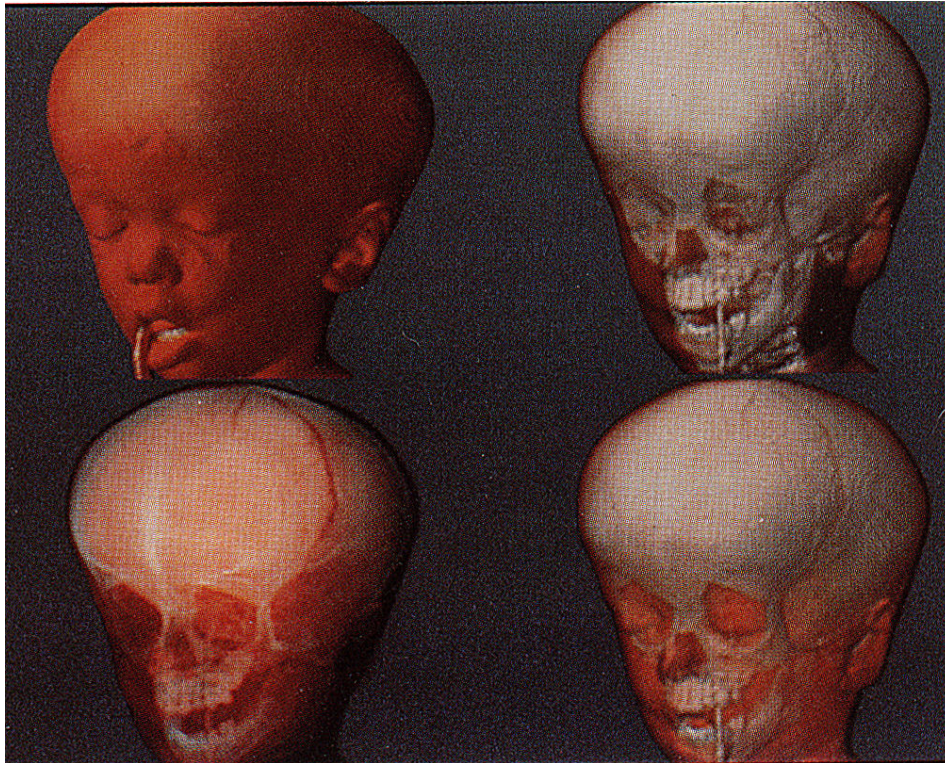
## Gridded data

Data available for every point on a 3D grid

- Usually a regular, rectilinear grid
- Data often arises from physical measurements, eg CT scanning, MRI scanning.

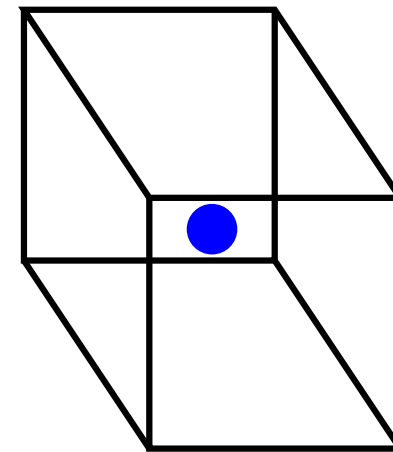
# Applications: voxel data

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Scanning produces a value for each voxel dependent on the material characteristics. Preprocessing permits different structures to be studied.

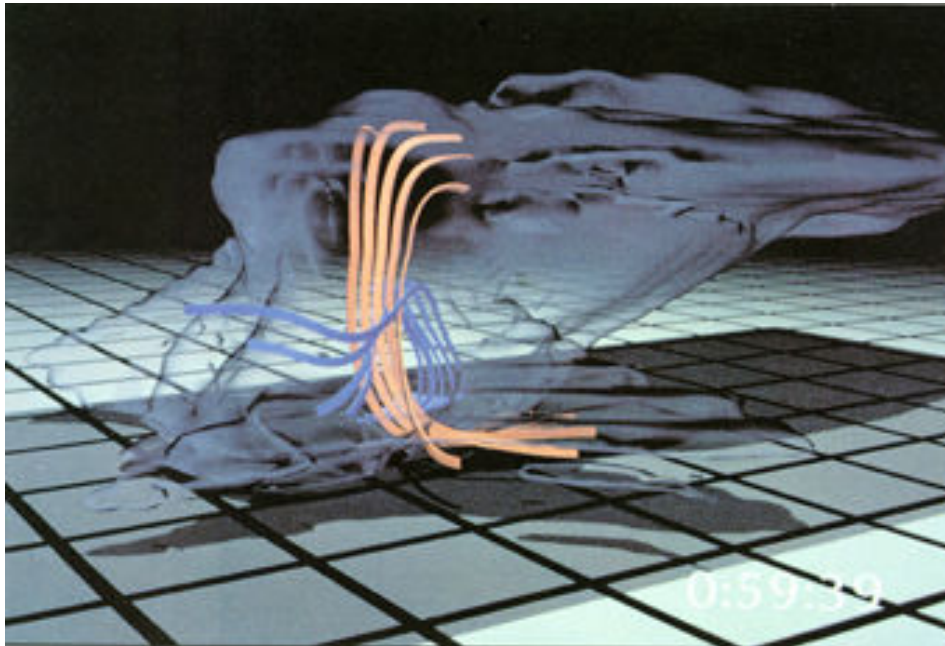
*Voxel = Volume element*



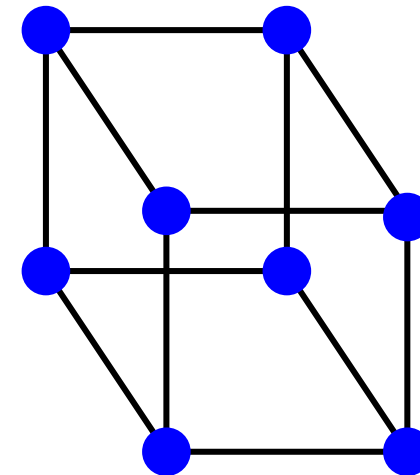
Value for each voxel represents an average value for the volume occupied by the voxel.

# Applications: Cell data

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Computational modelling produces a point value at the vertices of a regular grid.



- Modelling methods are often voxel-based
- Differences between voxel and cell data often ignored



# Visualising 3D data

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Surface rendering



- Iso-surfaces: surfaces on which data value is constant
- Indirect: yields graphical primitives (polygonal mesh) which is then rendered

Volume rendering

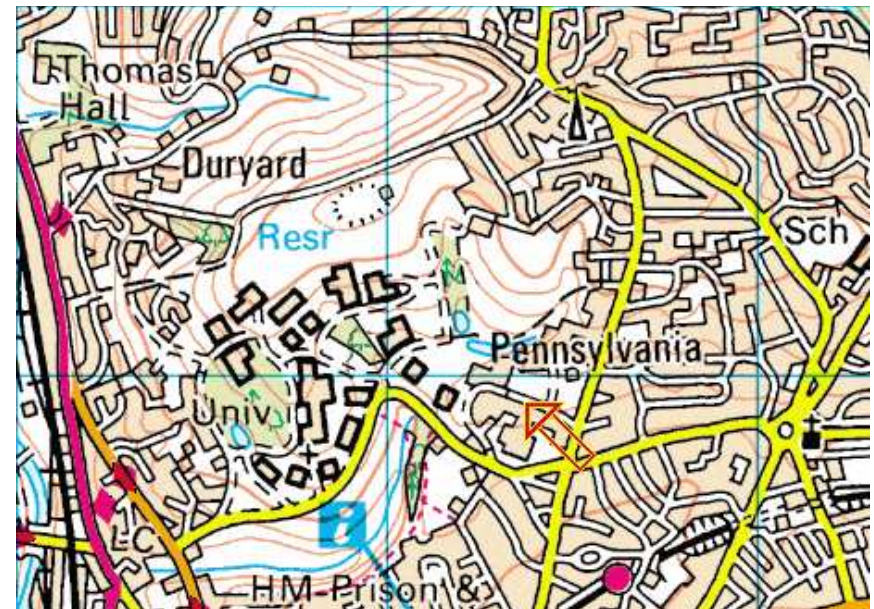


- Voxels endowed with colour and opacity
- Render the translucent data cuboid by ray casting
- Direct visualisation

# Contouring

## Two dimensions

- Construct lines on which a scalar is constant
  - Contours on OS maps: iso-height
  - Constant temperature lines: isotherms
  - Constant pressure: isobars
- Contours are the boundaries between different scalar values



## Three dimensions

- Construct iso-surfaces on which a scalar is constant
- Surfaces divide regions of differing scalar values

# Contouring

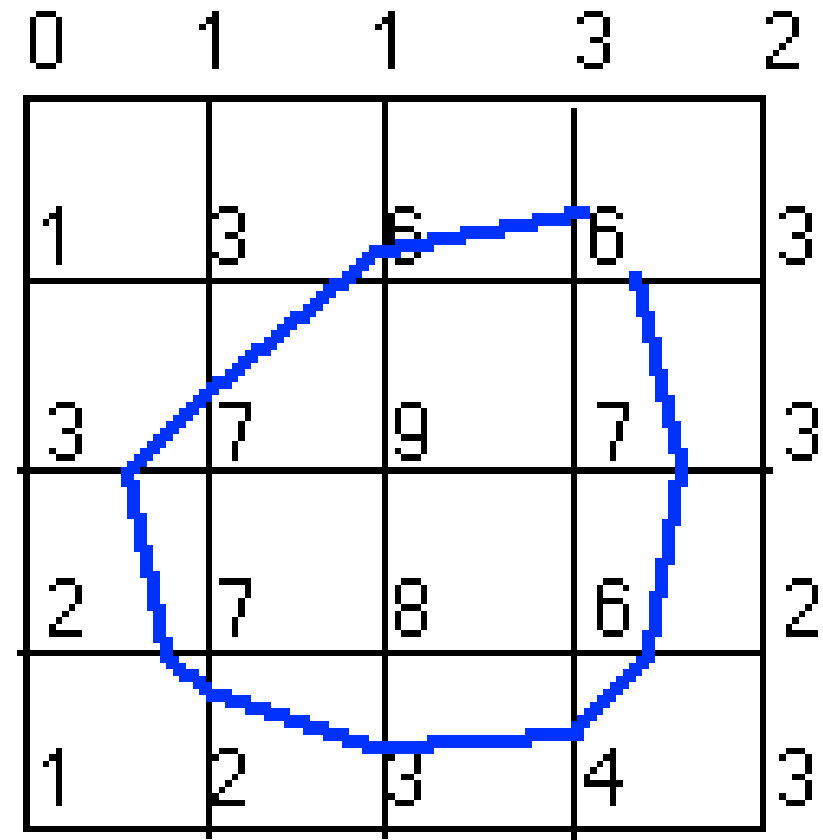
- $f(x, y)$  is the value measured at location  $(x, y)$ .
- Goal is to find the contour  $f(x, y) = c$

1 Determine the location of intersection of contour with cell edges by *linear interpolation* between vertices.

- Linear interpolation is simple, fast and usually sufficient.

2 Connect the intersections.

- Connections with straight lines: simple and fast; splines etc may lead to crossing contours.





# Contour tracking

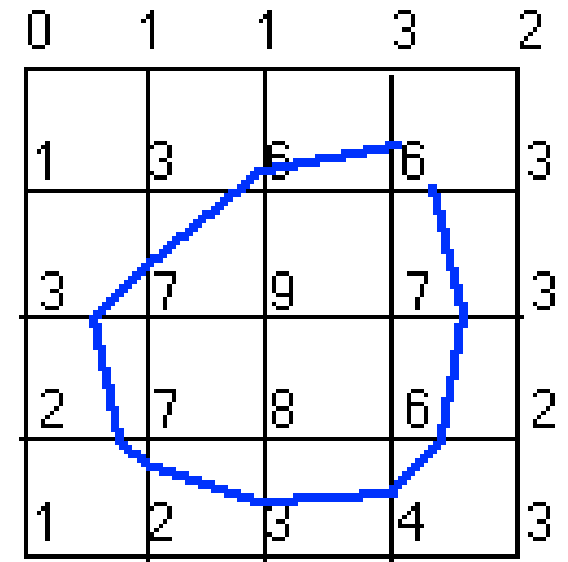
## Follow a contour from start to finish

A contour that enters a cell on one edge must exit on another edge

- ① Scan cells and edges to detect an edge intersection
- ② while not finished
- ③     find the exit-edge in current cell
- ④     mark cell as processed
- ⑤     entry-edge := exit-edge

Terminate if

- Contour closes on itself
- Contour reaches a boundary edge

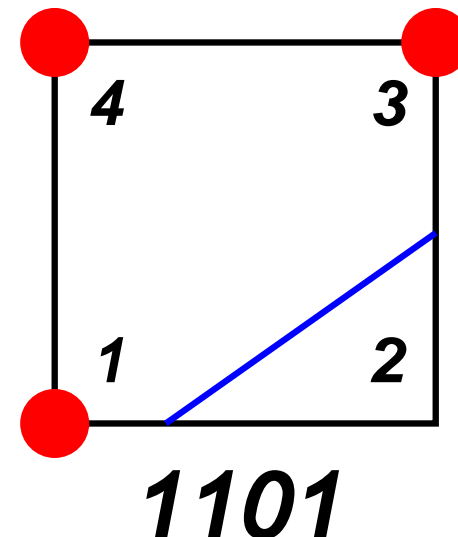
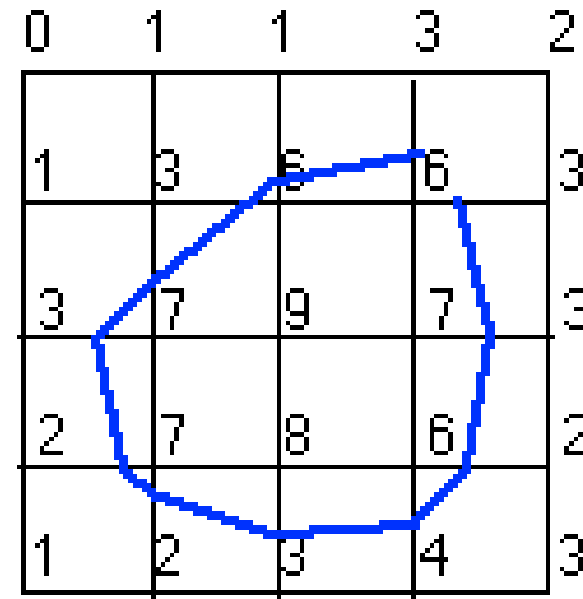


Yields contour as a single entity

Helpful for labelling, measurement

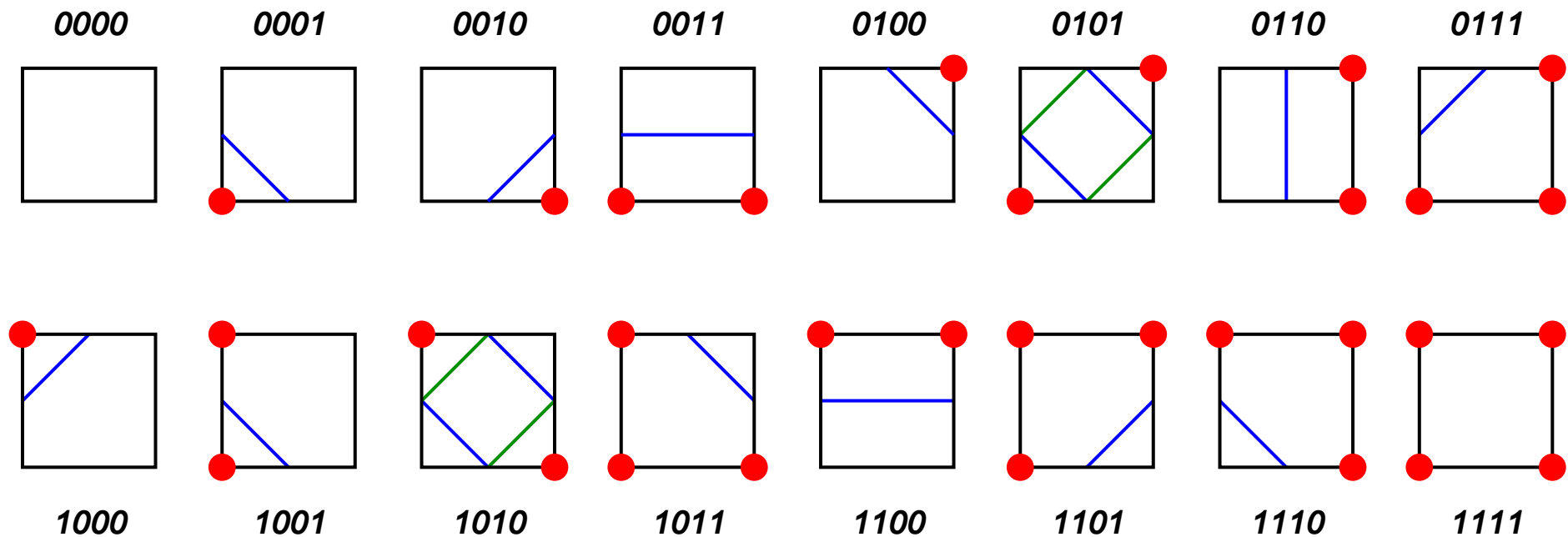
# Marching squares

- Contour tracking very difficult to extend to 3D
- Marching squares draws the contour in each cell in scanwise order
- Efficiency derived from rapid identification of the way in which contour passes through cell
- *Topological state* of cell depends on whether each of the 4 vertices is greater than or less than the contour.



# Marching squares states

$2^4 = 16$  possible states



• denotes vertices greater than contour height.

# Marching squares

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## Algorithm

- ① for each cell:
- ②     Determine whether vertices are above or below  $c$
- ③     Generate topological state index from bits
- ④     Locate intersection edges from state
- ⑤     Calculate intersection by linear interpolation
- ⑥     Draw contour segment

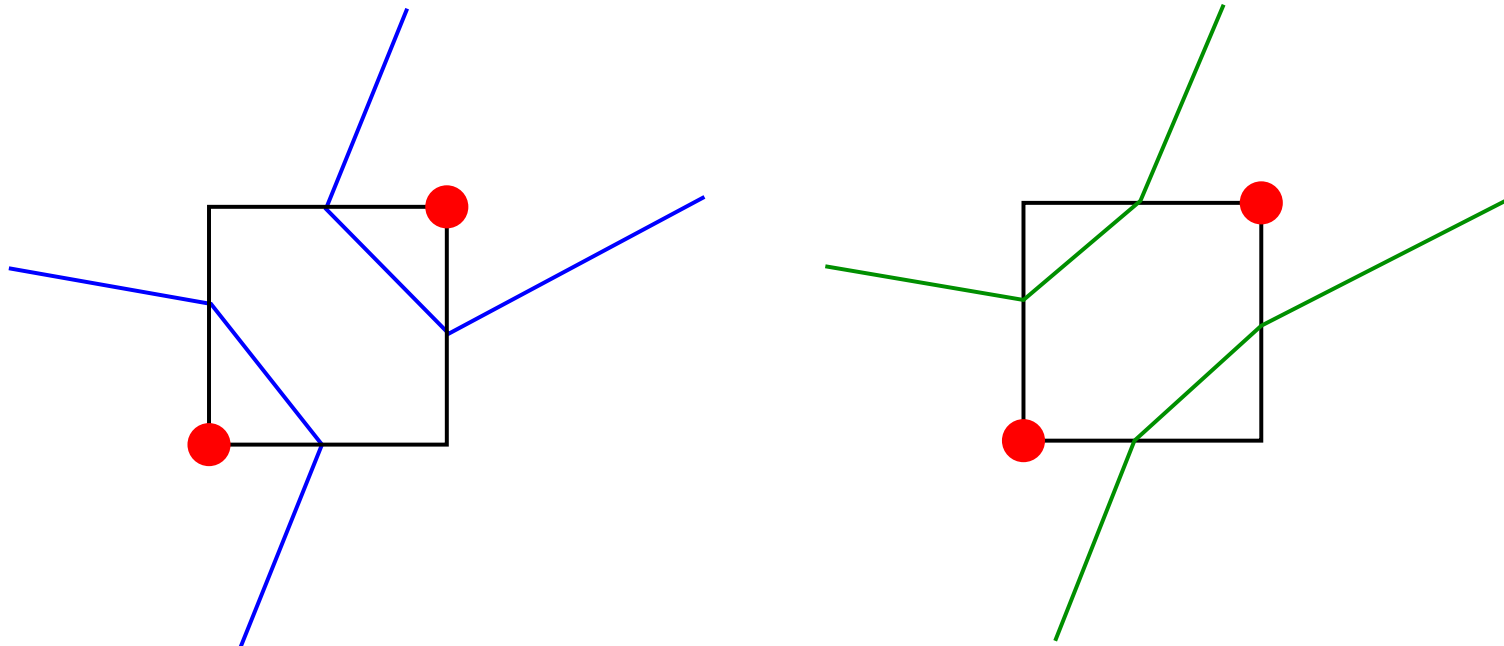
## Notes

Contour produced in segments; can be time-consuming to re-assemble.

Interpolate in the same direction (eg low to high) to avoid round-off errors and ensure consistency of interpolated intersection.

# Marching squares ambiguities

- States 0101 and 1010 are ambiguous: there are two possible contours at the saddle point.
- In two dimensions choose either interpretation.



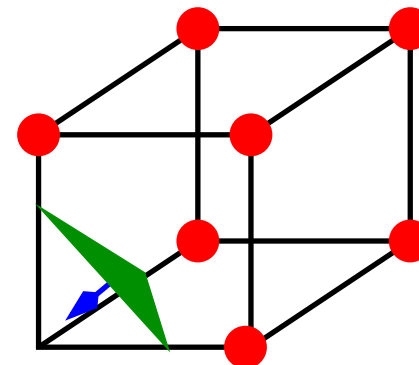
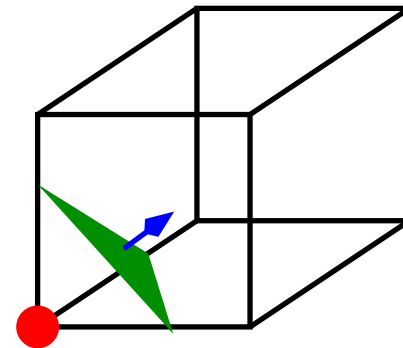
- Either interpretation is consistent with dataset.
- Rate of change of height can be used to infer the correct interpretation.



# Marching cubes

Generalisation of marching squares to 3D  
to draw an isosurface  $f(x, y, z) = c$

- $2^8$  arrangements of vertices
- Inside/outside choice is arbitrary
- Other symmetries reduce the number of unique configurations to 15



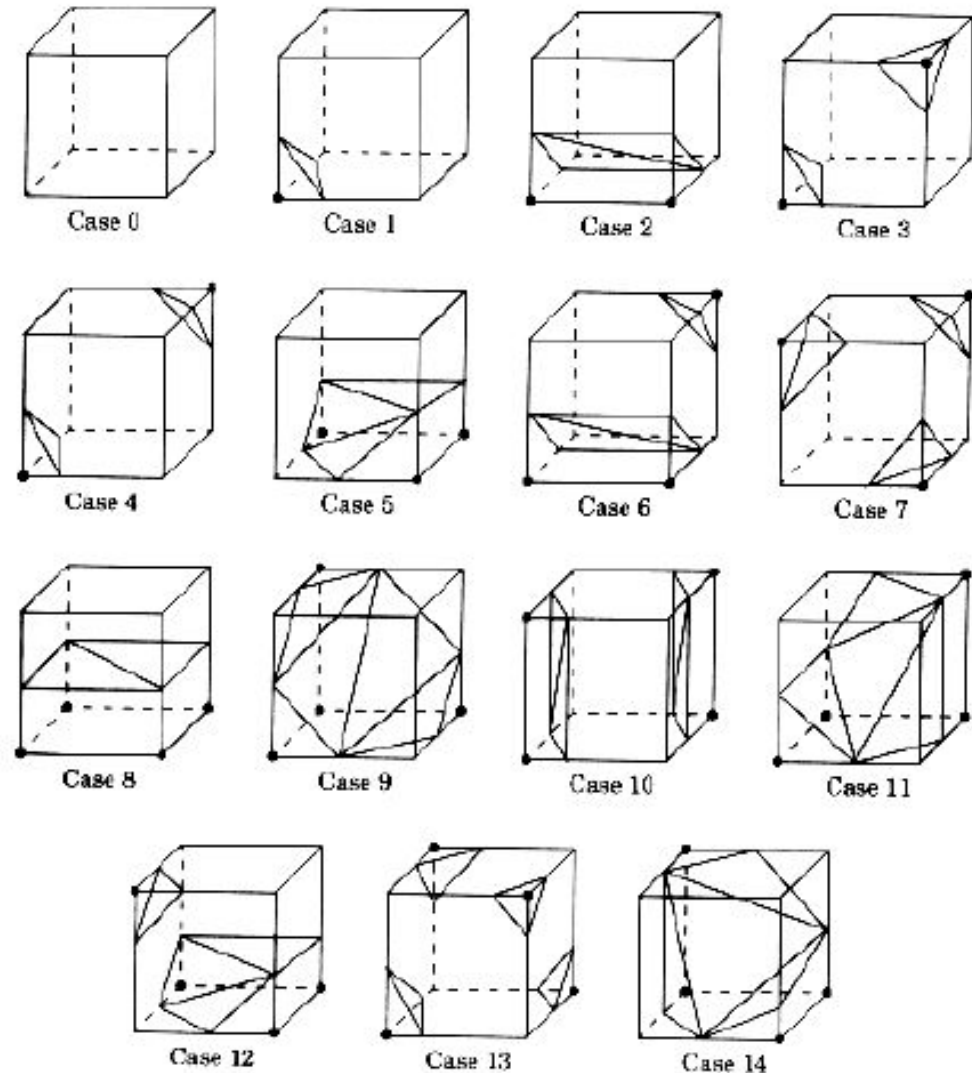
# Marching cubes

For each cell:

- 1 determine state of cell from data values at vertices
- 2 find intersection of edges by inverse linear interpolation
- 3 draw triangular patches

## Ambiguities

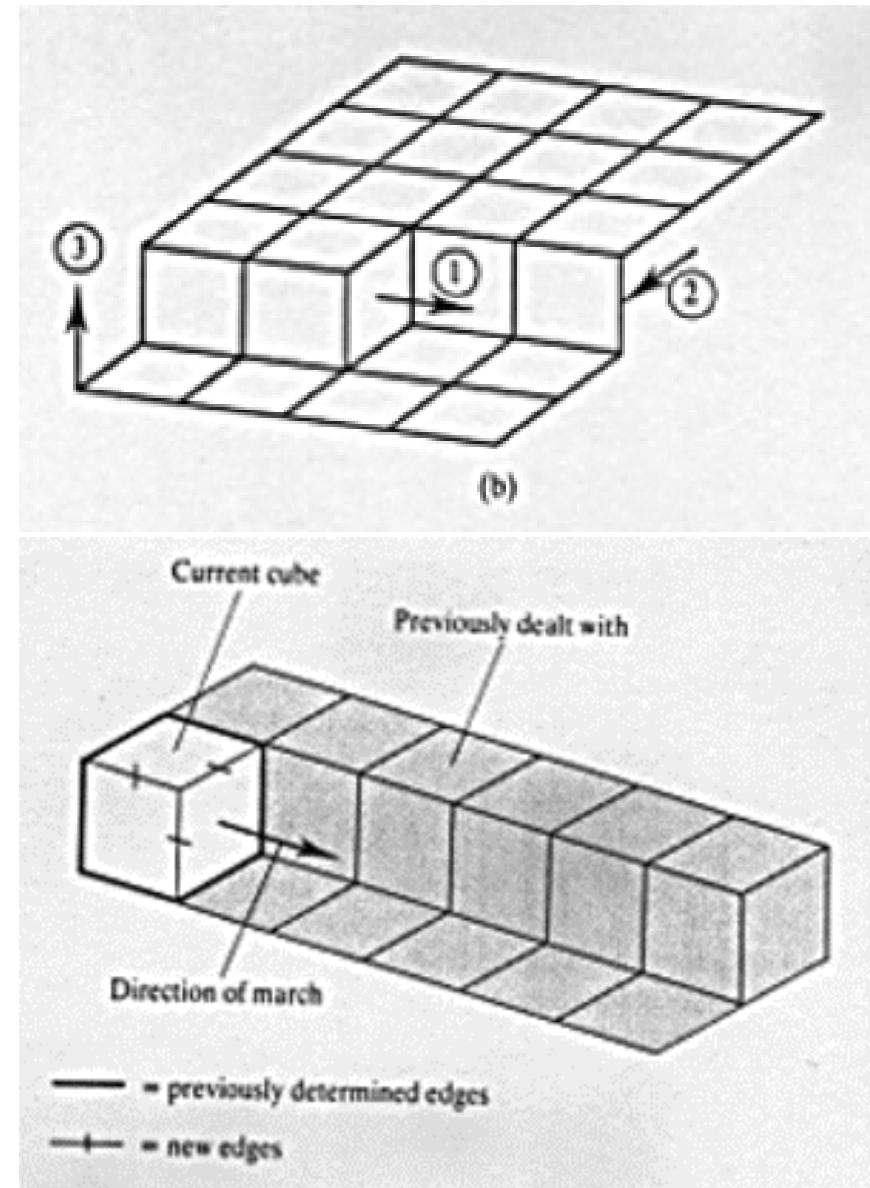
Resolved by reference to neighbouring cells to prevent holes in isosurface



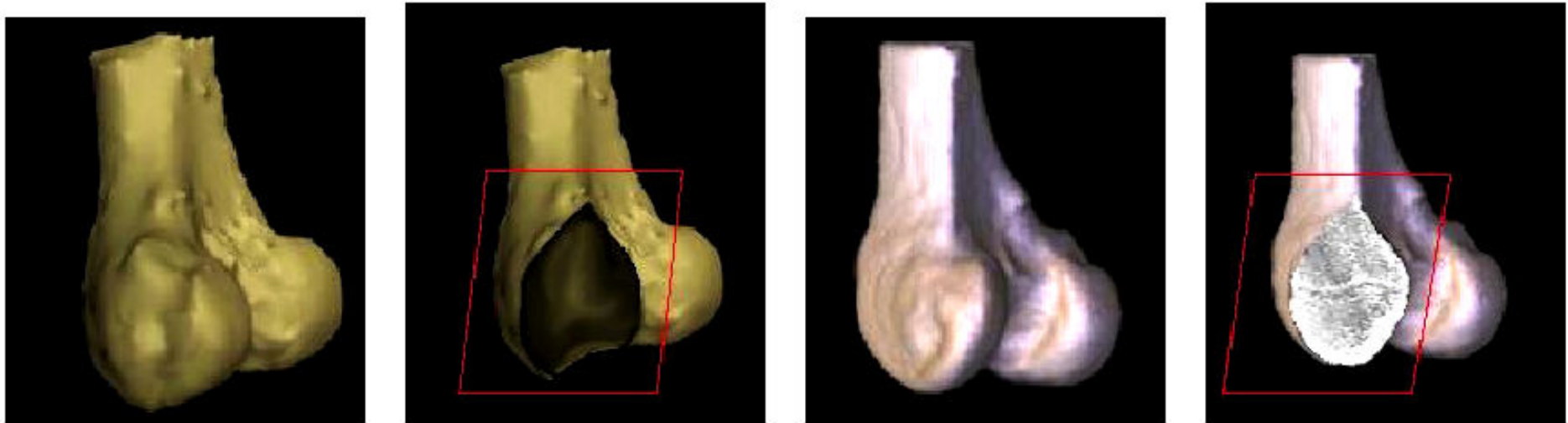
# Efficiency

- Use *coherence of cells* to avoid expensive intersection calculations by
  - Storing edge intersections for reuse
  - Ordering the march to minimize necessary storage

Buffers to retain intersections required until next slice is complete.

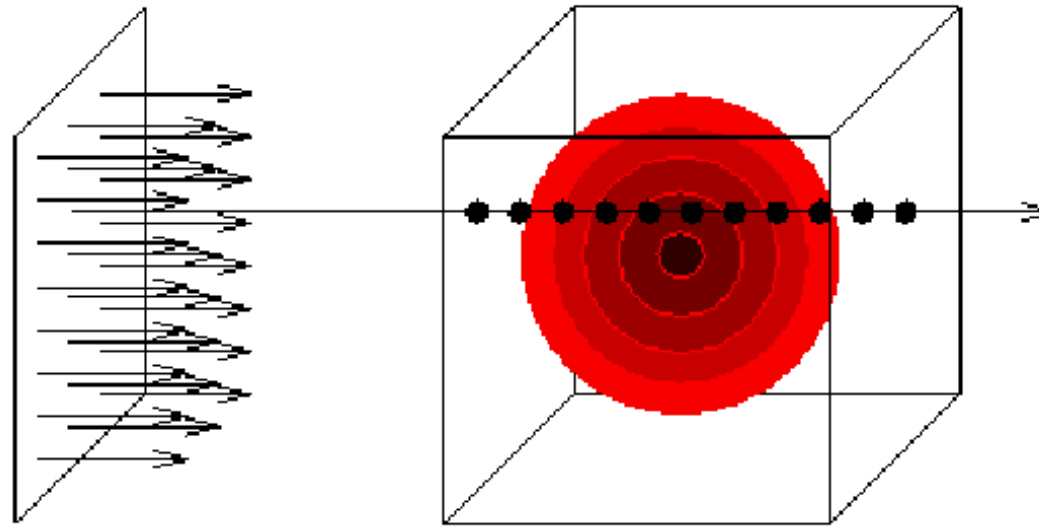


# Limitations



- Visualisation uses intermediate geometrical primitives
- Poor resolution for small objects
- Unlike 2D data, outer iso-surfaces obscure inner ones: no information about the inside.

# Volume rendering



- Endow each voxel with an *opacity* and *colour*
- View data cube by casting parallel rays from the image plane

## References

- Examples: <http://www.fovia.com/gallery.php>
- Levoy's website: <http://graphics.stanford.edu/projects/volume>



# Volume rendering

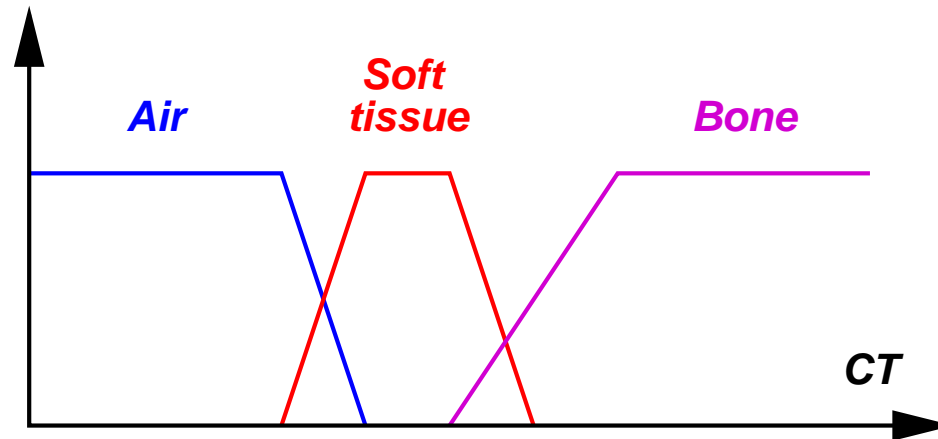
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- Classify** each voxel into a particular material and thereby assign colour and opacity.  
Assume that each voxel is composed of a single material
- Transform** classified volume data into the viewing direction
- Cast rays** from each pixel in the image plane find the overall pixel colour and intensity

# Voxel classification: transfer function

**Transfer function** maps the data value (e.g., X-ray absorption coefficient) to RGBA.

- Frequently determined by mapping data value to a material and then material to RGBA.



- Choice of material to RGBA mapping defined by the user.

# Transfer function

- Transfer functions often chosen interactively by specifying a colour for a region/organ of interest, but difficult unintuitive and slow.



Low data  $\mapsto$  high  $\alpha$

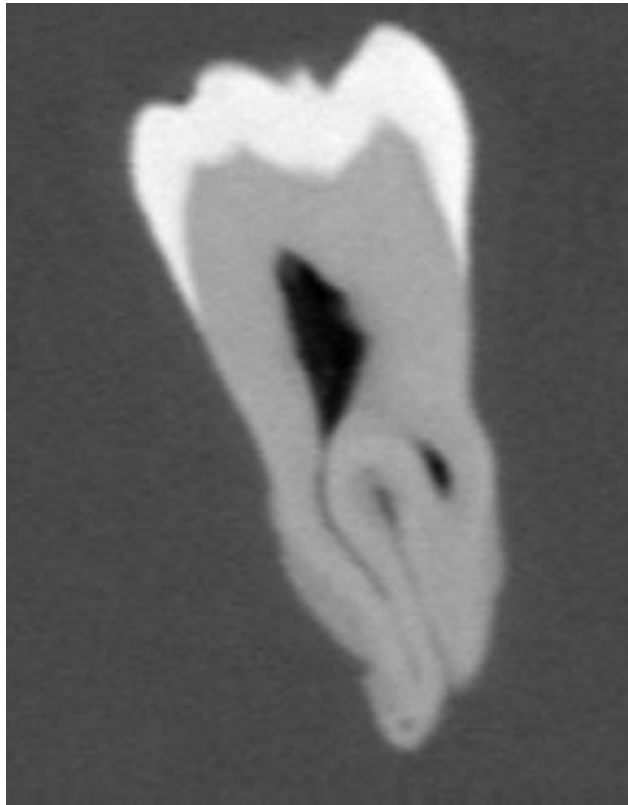


High data  $\mapsto$  high  $\alpha$

- Semi-automatic methods to incorporate spatial information, such as edges or the curvature of isosurfaces.

<http://www.cs.utah.edu/~gk/papers/vis03/>

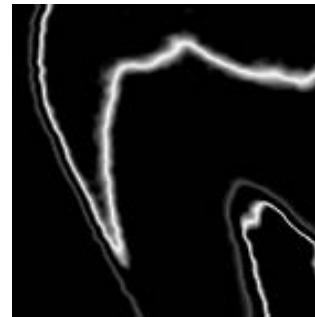
# Transfer function



CT adsorption,  $f$



$RGB(f)$



$\alpha(f)$



Composited tooth

# Transformation into viewing direction

Either

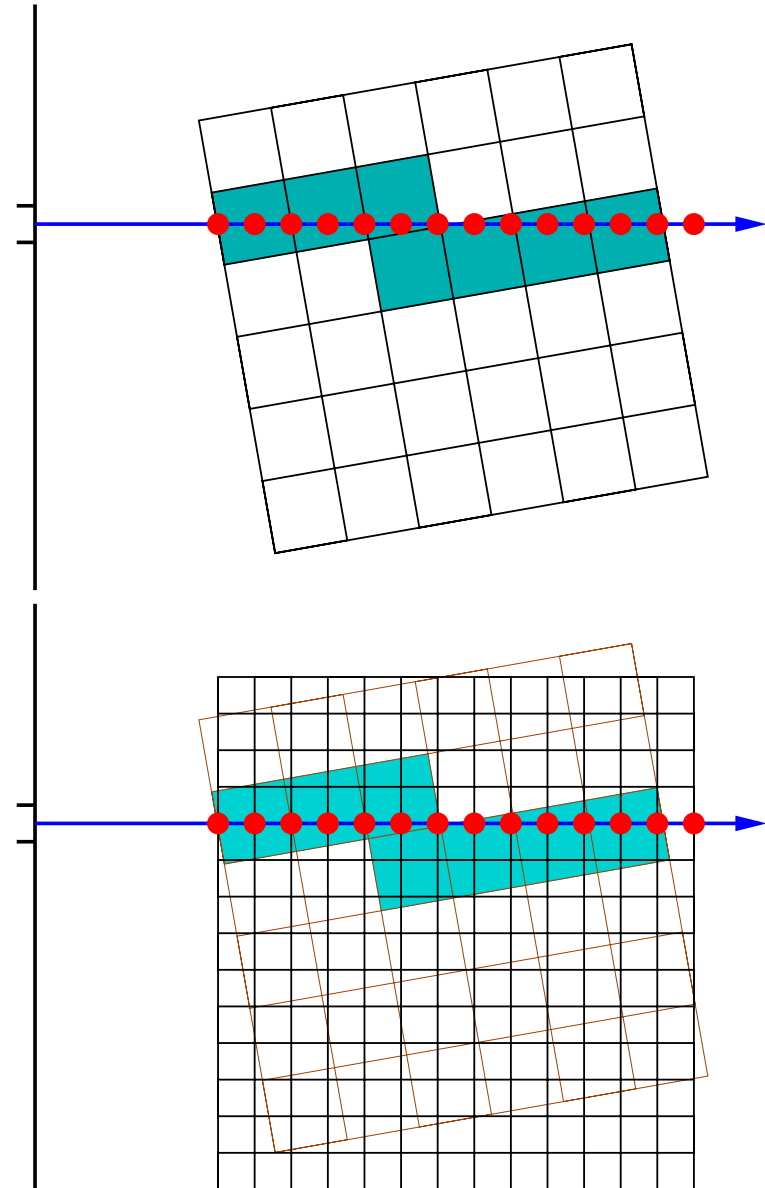
Sample the data at equal intervals along the untransformed data

Linear interpolation to find the value at each sample point.

Or

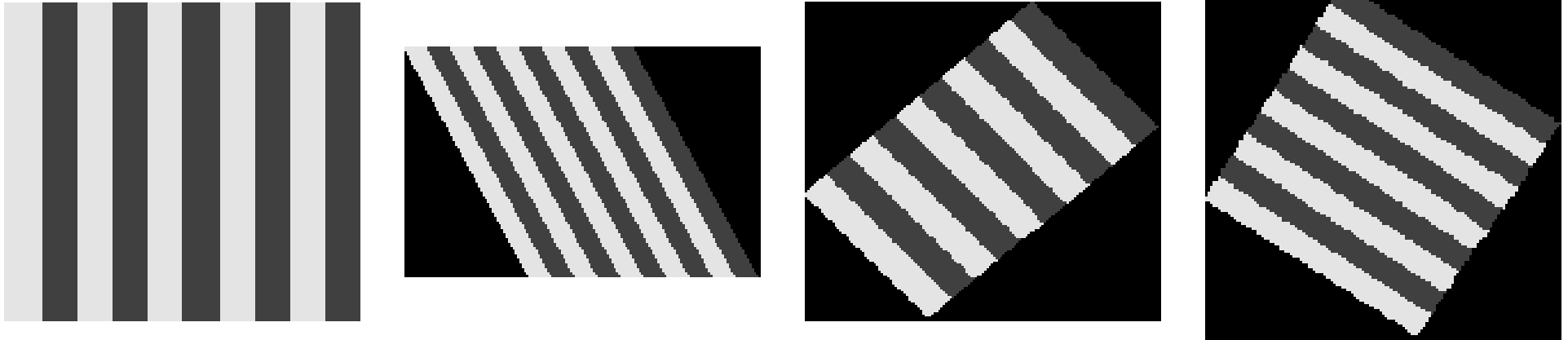
Pre-transform the data so that ray is parallel with transformed pixels.

Rotation about each axis efficiently accomplished by 3 shears in hardware.





# Three shear rotation



$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin(\theta) & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & 1 \end{bmatrix}$$

- Each transformation is just a shift along rows or columns.
- Filtering to reduce aliasing can be applied at each stage.

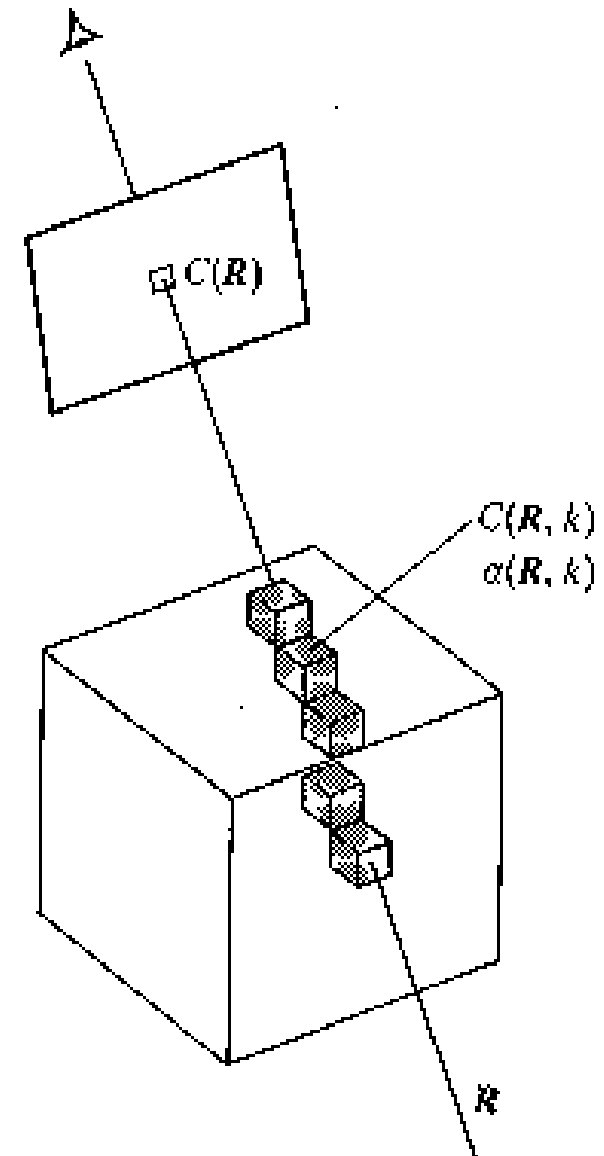
# Compositing

- Ray intensity is modified by each voxel/sample it passes through on the way to the eye:

$$C_{out} = C_{in}(1 - \alpha) + \alpha C$$

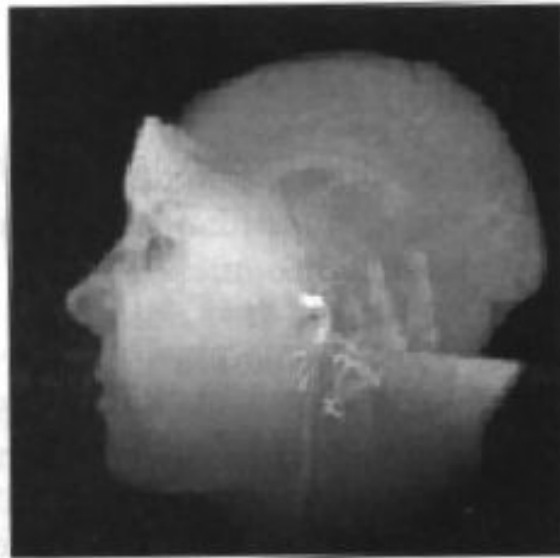
$C_{in}$  R/G/B for incoming ray  
 $C_{out}$  R/G/B for outgoing ray  
 $C$  R/G/B for this sample/voxel  
 $\alpha$  opacity for this sample/voxel

- High  $\alpha$  voxels are visible: they obscure voxels behind them and low- $\alpha$  voxels in front are relatively transparent.

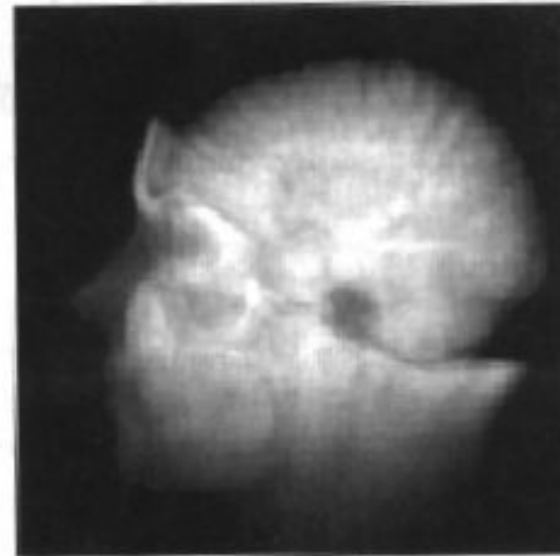


# Alternative ray functions

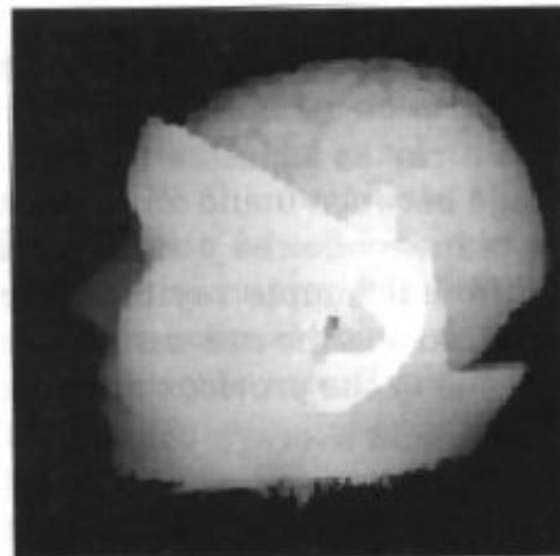
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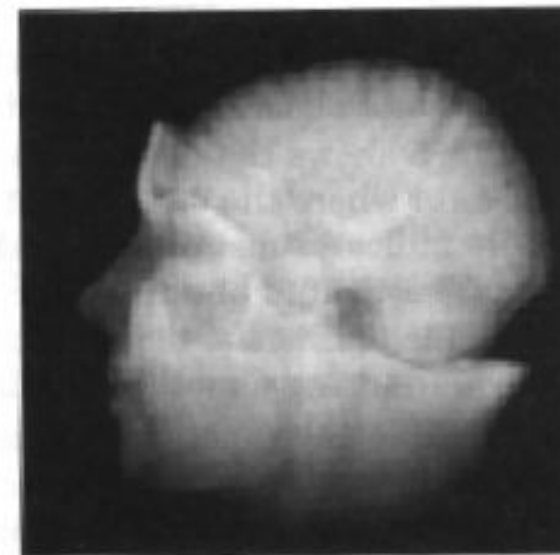
Maximum value



Average value



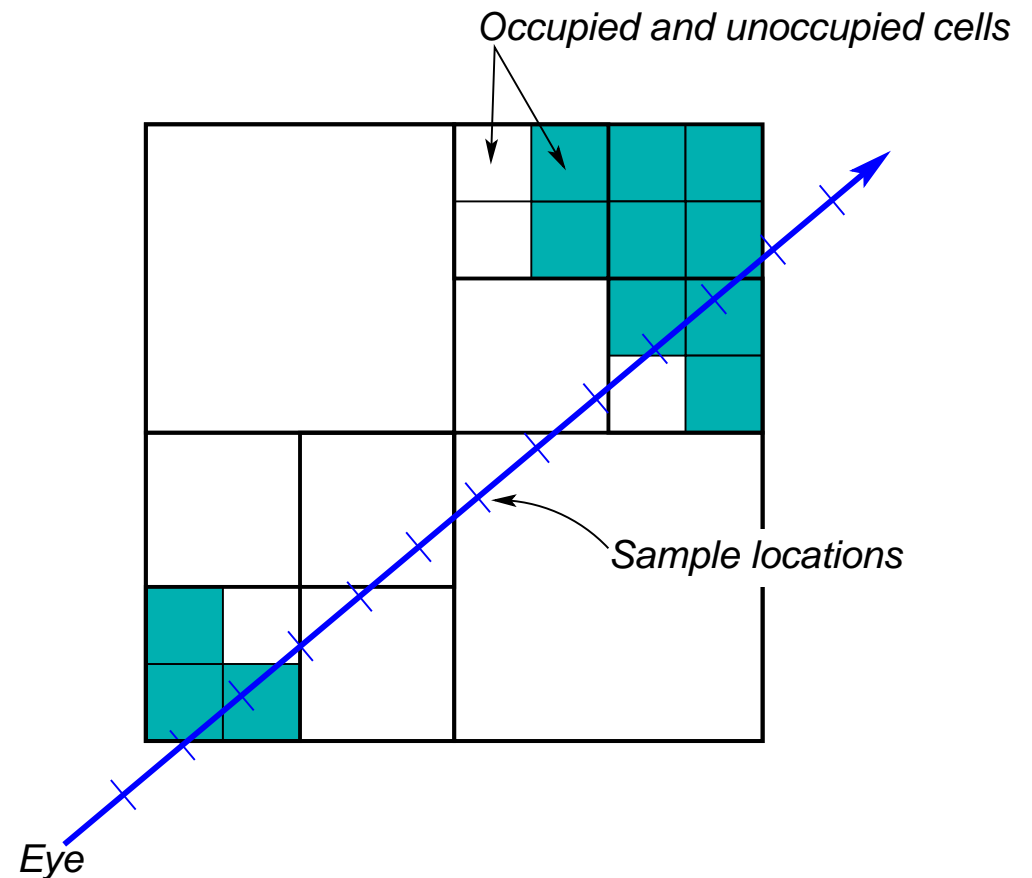
Distance to value 30



Composite

# Efficiency

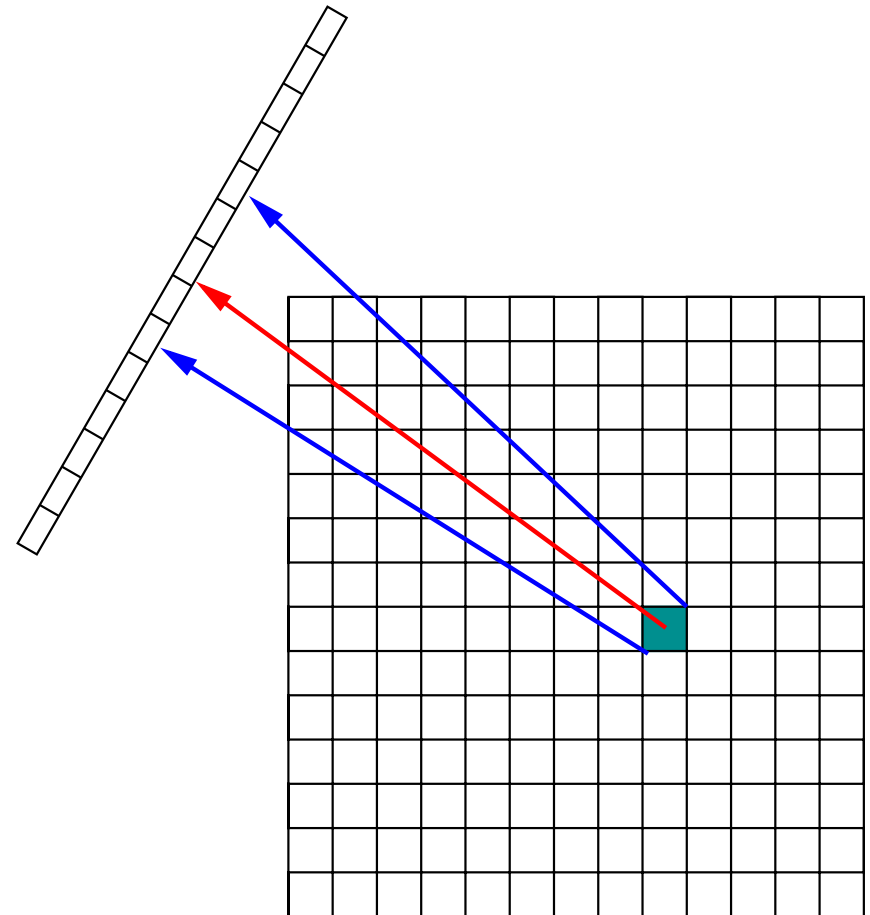
- Build pyramidal data structure (eg. octree) with 1s at high levels indicating non-zero  $\alpha$  in lower levels. Only visit voxels with non-zero  $\alpha$ .
- Early ray termination: when compositing from front to back, stop if the ray is too dark to be visible.
- Hardware: utilisation of GPU hardware permits rapid volume visualisation.



*After: 'Efficient Ray Tracing of Volume Data',  
Marc Levoy, ACM Transactions on Graphics,  
1990.*

# Splatting

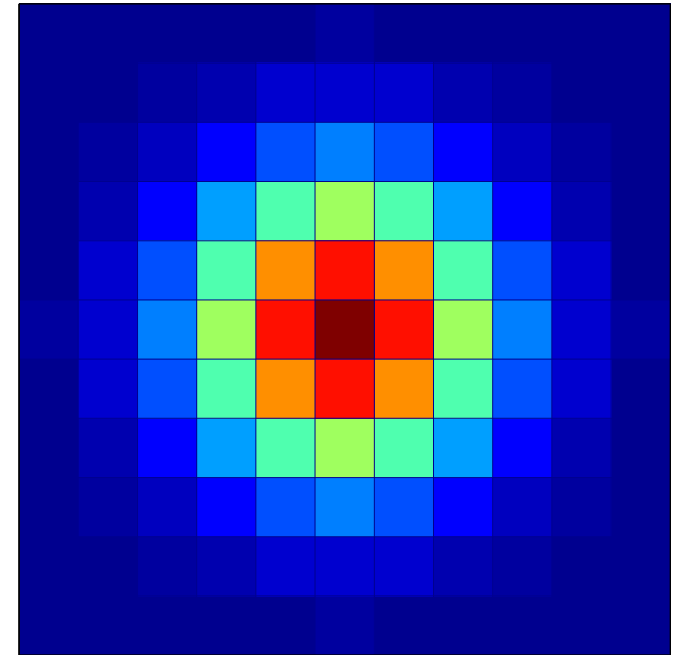
- Project voxels forward from data cube to the image plane.
  - Accumulate total pixel intensities by working from front to back of data cube.
- 
- Each voxel projects to more than one pixel.
  - Kernel or spreading function determines the contribution of each voxel to the central pixel and neighbours.





# Splatting

- Kernel function often a discretised Gaussian function



- Splatting operations very fast
- Advantageous to rotate (shear-warp) before splatting.
- Splatting can be performed in parallel and can take advantage of GPUs; memory access is the key to a fast algorithm.
- Care to avoid aliasing required when more than one voxel maps into a pixel
- *L. Westover 'Footprint evaluation for volume rendering', Computer Graphics, 24, 1990*

# Volume Graphics

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## Advantages

- Insensitive to scene complexity  
Objects are preconverted into voxel form.
- Insensitive to object complexity
- Complexity is viewpoint independent
- Interior information is available

## Disadvantages

- Memory requirements:  $512^2 \times 2$  bytes per voxel = 256 Mbytes
- Processing power
- Discrete form: resolution is limited by voxel resolution
- Geometric form is lost on conversion to voxel representation