6^{th} **World Congress of Structural and Multidisciplinary Optimization**
Rio de Janeiro, 30 May - 03 June 2005, Brazil

Multiobjective Topology Optimization Using a Genetic Algorithm and a Morphological Representation of Geometry

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1. Abstract
In this paper, topology optimization of continuum structures is solved as a discrete optimization problem using a genetic algorithm (GA). Early efforts applying this approach have not been very encouraging due to the lack of an appropriate structural geometry representation, and a good representation scheme is essential to the effectiveness of the genetic operations and hence the success of any GA procedure. In this work, a recently developed morphological geometric representation scheme is coupled with a GA to perform topology and shape optimization. The scheme uses arrangements of skeleton and surrounding material to define structural geometry in a way that facilitates the transmission of topological/shape characteristics across generations in the evolutionary process, and will not render any undesirable design features such as disconnected segments, checkerboard patterns or single-node hinge connections.

The procedure is applied to the challenging problem of synthesizing the topological/shape design of a path generating compliant mechanism, i.e. a flexural structure that deforms/displaces through a desired path when subject to an applied load. The design problem is formulated as a multiobjective optimization problem with multiple constraints, and is solved using a multicriterion GA that is based on maintaining separate non-domination (Pareto) rankings for objectives and constraints satisfaction, thus enabling an intelligent selection of solutions for cooperative mating which eliminates the need to prescribe penalty parameters or weights. A novel multicriterion target matching problem has also been devised to test the overall methodology, and it is a problem in which a target geometry is first predefined as the optimum solution, with a corresponding multicriterion problem formulated for the GA to solve and evolve solutions that converge towards this target shape. The GA is then applied to the optimal topology design of compliant mechanisms, and optimal geometries have been obtained that are valid and practical for realization/fabrication.

2. Keywords: topology optimization, genetic algorithm, morphological representation, multicriterion optimization, target matching, compliant mechanism

3. Introduction
The most well-developed methods for topology optimization of structural continuum are those based on solving the problem as continuous optimization problems. The homogenization approach [1] and the material density (power-law) approach [2]-[4] belong to this class of methods because the design variables used for defining the structural geometry are continuous variables. However, as continuity in the variables results in a 'fuzzy' representation of material distribution with intermediate zones somewhere in between existence and non-existence of material, the final useful optimal geometry is usually obtained through imposing some filtering schemes or built-in penalizations, or with some interpretation via threshold values. Other continuous approaches like those applying the level set method [5][6] can define the geometry more directly but may not be able to adequately control the connectivity of the geometry to ensure the structure is a single structurally connected body.

The alternative to these approaches is the definition of the geometry in a discrete manner, such that individual elements in the finite element discretization of the design space are turned either on or off, with the 'on' elements forming the structural geometry while the 'off' elements form the surrounding empty space. The discrete optimization problem is then solved using a discrete optimization method, such as the genetic algorithm (GA). The GA operates on a representation of the geometry and the simplest form of representation is a (binary) bit-array representation which defines the geometry by an array of 'on' and 'off' bits (i.e. ones and zeroes) that correspondingly maps onto the design space [7][8]. Although this discrete definition eliminates the problem of fuzziness inherent in continuous representations, it does not overcome the other shortcomings like checkerboard patterns and the lack of control over structural connectivity [9].

To overcome these shortcomings, a morphological representation had been developed for defining geometry in a way that inherently ensures the design is a single connected structure and without checkerboard patterns [10]-[14]. This representation is naturally encoded into the required chromosome suitable for a GA optimization procedure, and the set of genetic operators for operating on the chromosome are devised to enhance the transmission of geometric characteristics of good designs across generations in the evolutionary process. The present work described in this paper extends the GA morphological approach to solve multicriterion topology optimization problems. The genetic operators from [14] are further refined, and a multicriterion GA from [15] for solving multiobjective problems with multiple constraints is further refined and applied here. In addition, a special test problem with a known optimal solution is formulated and solved here for the purpose of tuning and verifying the effectiveness of the overall optimization methodology. This test problem is a multicriterion version of the target matching problem developed in [14]. The methodology is then applied to optimize the topology/shape of a flexural structure that will follow as close as possible a predefined displacement path (i.e. a path generating compliant mechanism).

4. Morphological Representation of Geometry

The morphological representation is a geometric representation scheme that has been developed and refined over the past few years [10]-[14]. A detailed explanation of this scheme can be found in [14], while a brief description is given here as follows. Just like most topology optimization techniques using the finite element method, the problem is defined by a predefined design space discretized into a regular mesh of finite elements. In the morphological representation, the structure is characterized by a set of input/output locations. Input locations are typically boundary points on the structure which are supported/fixed or where the load is applied, while output locations are any particular points on the structure where their structural behaviour are of interest in the optimization problem. There must exist two or more of such input/output locations in any problem because every structure must have parts that interact with its surroundings by way of at least one fixed support region and one loading region.

Fig. 1(a) shows an example problem defined with three input/output locations : one support, one loading and one output location, each made up of one element. For a valid structural design, all input/output locations must be connected to one another (either directly or indirectly) if one and only one single connected load-bearing structure is to be formed without any disconnected segments. A number of Bezier curves are required to connect the input/output elements together and in this illustration, three curves are prescribed : one curve from the loading to the output, another from the loading to the support, and finally one from the support to the output. Each Bezier curve is defined by its start and end points (i.e. the corresponding input or output locations) and a number of control points in between. The set of elements through which each curve passes forms the skeleton that connects all the input/output locations (Fig. 1(b)). Some of the elements surrounding the skeleton are then included to fill up the structure to its final geometry, and these additional elements may be considered the 'flesh' around the skeleton (Fig. 1(c)). The number of flesh elements added depends on thickness values, and this is done by considering each skeleton element in turn and adding an all-round layer of elements to it, with the thickness (number of layers) determined by the thickness value, which can range from zero to some prescribed integer maximum. How the thickness values are assigned to different segments of the skeleton will be described in a later part of this section. The union of all skeleton, flesh and input/output elements constitutes the structural geometry.

Since a GA is used for the optimization, the design variables have to be encoded into a pre-formatted data structure known as a chromosome (by analogy with evolutionary genetics). In this morphological representation, the data structure used is a directed graph (as in graph theory [16]) and an example graph that corresponds to the illustrated structure in Fig. 1 is shown in Fig. 2(a). Each vertex of the graph contains a design variable denoting either a thickness value, the location of a control point, or the location of an input/output point, as indicated by the legend in the figure. The location of any point is denoted by an element number, with the exact position located at the centre of that element. Hence every

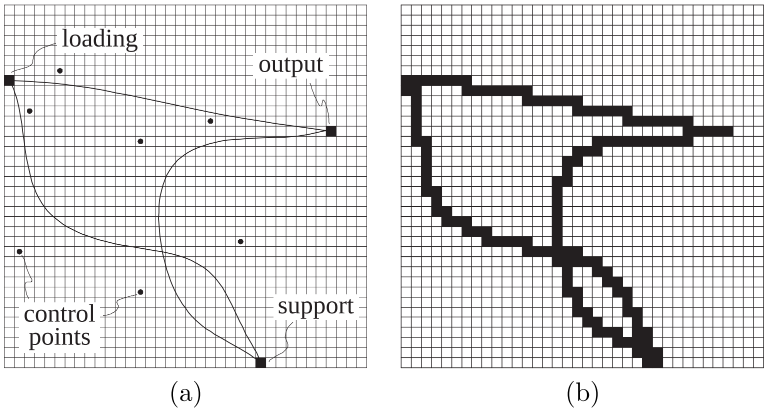


Figure 1: Geometry defined by morphological representation; (a) input/output locations connected by Bezier curves (b) skeleton formed by elements along Bezier curve (c) elements added to skeleton to form final structure

element in the design space is identified by a number, which in turn necessitates a numbering scheme for labeling each element (any numbering scheme can be used as long as a one-to-one mapping is maintained between the chromosome code and the design space). Therefore, encoded within the chromosome is a number of Bezier curves each defined by an input/output start point, a number of control points, and an input/output end point, that can be mapped onto the design space as shown in Fig. 2(b). Furthermore, as shown in Fig. 2(a), each curve is defined by control points alternating with thickness values. For example, if a curve is defined by two control points (as in curve 1), there will be three thickness values. With three thickness values, the curve will correspondingly be divided parametrically into three equal segments with each segment having its thickness determined by the respective thickness values [14].

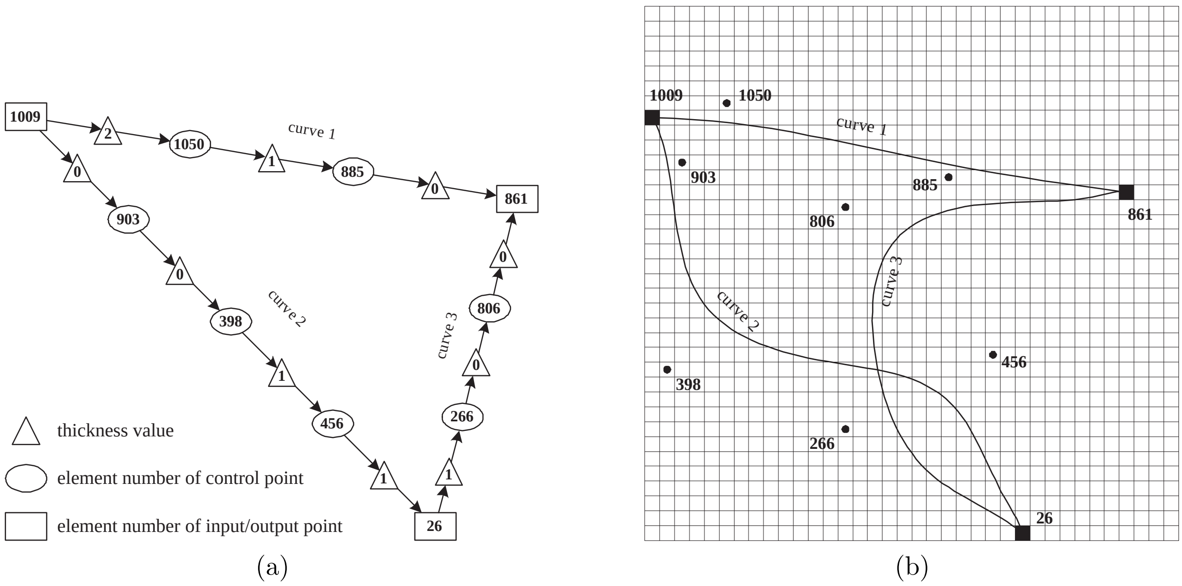


Figure 2: Example of chromosome code; (a) chromosome in the form of a graph (b) locations of input/output and control points mapped onto design space

It is evident that, in this morphological representation, the topology/shape of the structure emerges from the interaction among the skeleton and the flesh segments. The complexity of the geometry that can be defined is dependent on the user, based on the user-specified number of curves and the user-specified number of control points for each curve. In addition, the representation will not result in undesirable designs with checkerboard patterns, disconnected segments of material or single-node hinge connections (because the skeleton is ensured to have at least element edge connection). All these advantages are preserved in any offspring generated from the reproductive genetic operations of the GA procedure. The most important operation in a GA procedure is the crossover operation and in this work, a crossover operator is devised to work by randomly sectioning a single connected subgraph of a parent chromosome and swapping with a corresponding subgraph from another parent. An example illustrating this operation is shown in Fig. 3. This is a slight improvement from the crossover operator developed in [14] where not every possible subgraph (i.e. every possible combination of connected vertices of the graph) can be selected but, instead, only a portion/subgraph that emanates from an input/output point. Hence the current crossover operation is more versatile because the probability of any portion of genetic information being exchanged is now more even. Two mutation operators are implemented in this work. The first operator works by simply selecting, at random, any vertex on the graph and altering its value to another randomly generated value within the allowable range. The second operator is a local mutation operator which works in the same way as the first except that the randomly generated value is within the immediate neighbourhood of the original value (i.e. geometrically adjacent).

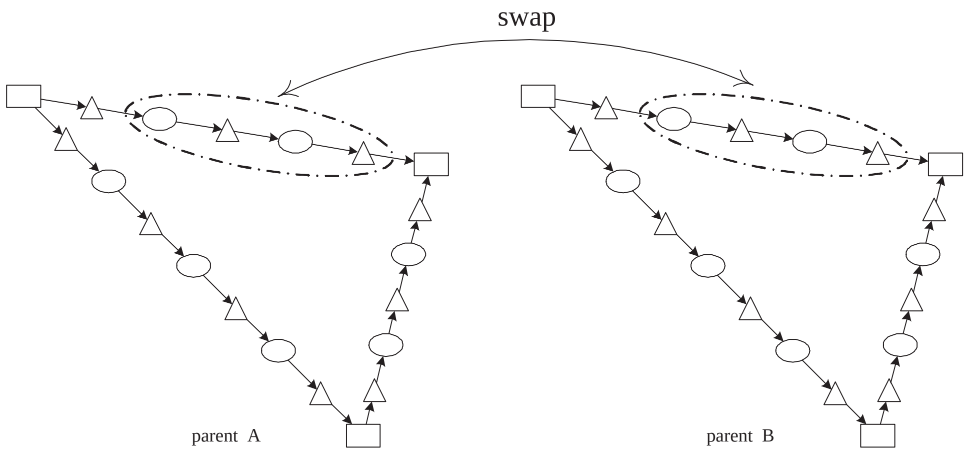


Figure 3: Example of crossover operation

5. Multicriterion Genetic Algorithm

The genetic algorithm used in this work is based on the Ray-Tai-Seow method [17] which was originally developed in [15]. This is a multiobjective algorithm with constraint handling, based on maintaining separate non-domination (Pareto) rankings for objectives and constraints satisfaction, thus enabling an intelligent selection of solutions for cooperative mating which eliminates the need to prescribe penalty parameters. This approach avoids the problems of scaling and aggregation that afflict conventional penalty function methods commonly used for constraint handling. A more detailed description of the algorithm can be found in [15]. In the current work, the refinements added to the algorithm include (a) a local mutation operation to facilitate local search, (b) a prescribed cap on the number of elites (based on diversity) moving foward to the next generation so as to prevent premature convergence, and (c) a tracking of the proportion of feasible solutions in the population so as to classify the hardness of the constraints and adaptively drive the selection of parents towards greater feasibility or better objective performance.

6. Target Matching Problem

Target matching problems are test problems that are useful for verifying the performance of the overall methodology because they are simulated topology optimization problems with known solutions. Simple single-objective unconstrained target matching problems have been demonstrated in [14] but in this current work, it is being extended into a multiobjective problem with multiple constraints.

For this problem, the design space is a square discretized into a 50×50 mesh. There is one loading, one support and one output point, connected by three curves just like in the illustration of Fig. 1. Each curve is defined by three control points, and hence each curve has four thickness values with each thickness allowed to vary between a minimum value of zero and a maximum value of 3. Note that the positions of all three input/output points are variable but confined to the boundaries, with the loading point confined to anywhere along the left boundary, the support point confined to the bottom boundary, and the output point confined to the right boundary.

In this problem, the target geometry is as shown in Fig. 4. A multicriterion topology optimization problem is then formulated such that its optimum solution is indeed this target geometry. Such a problem is defined with the help of Fig. 5.

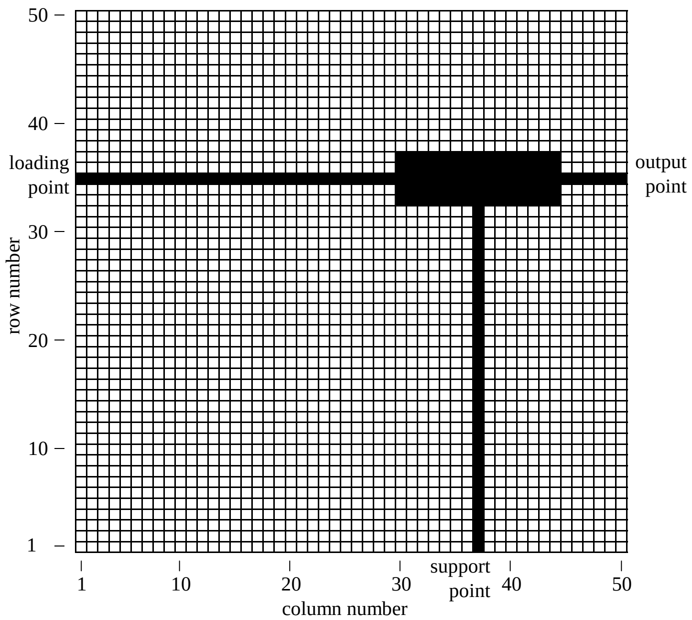


Figure 4: Target geometry

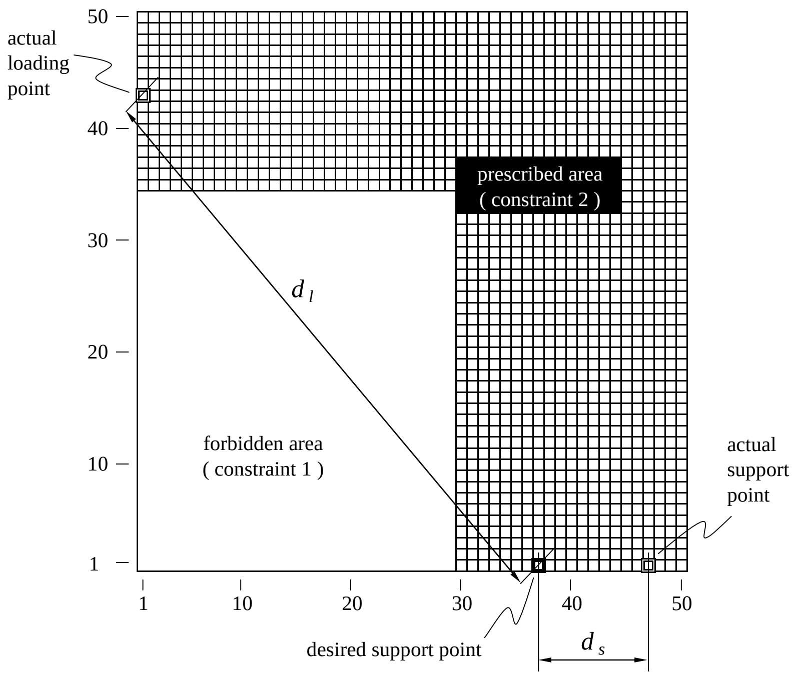


Figure 5: Formulation of target matching problem

First of all, there are two constraints in this optimization problem. The first constraint is that there cannot be any material in the forbidden area as shown in Fig. 5. Hence an inequality constraint function g_f is defined as the number of elements within this area that contains material :

$$g_\mathrm{f} = \sum_{i \in \mathrm{F}} x_i \leq 0 \tag{1}$$

where F is the set of element numbers in the forbidden area, $x_i = 0$ if element i is void and $x_i = 1$ if element i contains material. The second constraint is that the prescribed area must be fully solid, i.e. fully occupied with material. Hence an inequality constraint function g_p is defined as follows :

$$g_\mathrm{p} = N_\mathrm{p} - \sum_{i \in \mathrm{P}} x_i \leq 0 \tag{2}$$

where P is the set of element numbers in the prescribed area, and N_p is the number of elements in the prescribed area. There are two objectives in this problem. The first objective function to be minimized is a distance objective given by

$$f_\mathrm{d} = d_\mathrm{l} + d_\mathrm{s} \tag{3}$$

where d_l is the Euclidean distance between the actual loading point and the desired support point, and d_s is the Euclidean distance between the actual support point and the desired support point (note that the desired support point is a fixed predefined location). The second objective to be minimized is a material objective function given by

$$f_\mathrm{m} = \sum_{i=1}^{N} x_i \tag{4}$$

where N is the total number of elements in the design space. Hence this objective function is simply the total number of elements that contain material, and minimizing this function is equivalent to minimizing the amount of material or the 'weight' of the structure.

It is clear that with the optimization problem formulated in this way, the target geometry in Fig. 4 is the optimal solution. Note that the forbidden area constraint, together with the distance objective anchored at the desired support point, will lead to an optimal geometry with its loading point and support point at the locations as defined in the target. The material objective will also lead to the optimum having the three one-element-wide straight-line segments. However, the target geometry shown in Fig. 4 is not the only optimum because there are four more geometries with the same objective function values and they are shown in Fig. 6.

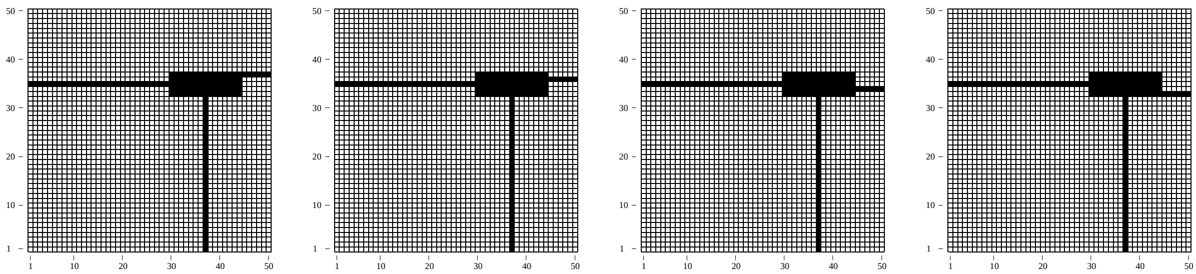
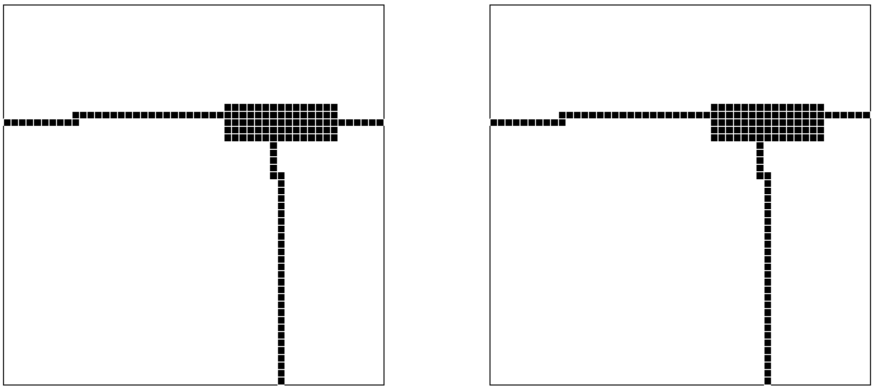


Figure 6: Four other optimum geometries in target matching problem

The problem has been solved by running the genetic algorithm for 300 generations with a population size of 100. Note that no finite element analyses are required because the objectives and constraints are purely geometric functions. By the end of the run, a total of 29492 function evaluations have been performed and two Pareto-optimal solutions obtained as shown in Fig. 7. The result in Fig. 7(a) first appeared in the 276th generation while that in Fig. 7(b) first appeared in the 292nd generation, but both have the same objective function values of $f_\mathrm{d} = 49.5$ and $f_\mathrm{m} = 144$. The structural geometries in the two results are exactly the same (except for the straight-line segment leading to the output point at the right boundary), and they are very close to the target geometry.



(a) (b)

Figure 7: Pareto-optimal solutions for target matching problem

7. Design of Compliant Mechanism

Compliant mechanisms (sometimes also known as flexure mechanisms) are flexible structures which generate some desired motion by undergoing elastic deformation under some applied load. The relationship between the structural geometry and the load-deformation behaviour is certainly complex and can best be discovered through a process of structural optimization. Compliant mechanisms with some desired structural behaviours have been designed via the application of topology optimization of either discrete frame-like structures or continuum structures [4][11][12][14][18]-[25].

In this work, a path generating compliant mechanism is to be designed to generate a desired path which is a circular arc of radius 100mm subtending an angle of 30 degrees. The design space is a square of sides 100mm, discretized into a 50 × 50 mesh just like in the target matching problem with exactly the same geometric definition (i.e. same connectivity of three curves, three input/output points confined to boundaries, three control points per curve, thickness values ranging from zero to 3, etc.). However, this is an actual structural problem where non-linear large displacement finite element (FE) analyses (using ABAQUS) are required to compute the behaviour. A horizontal resistance force of 2.5N is first applied at the output point subject to the loading point being restrained in the horizontal direction, after which a prescribed horizontal displacement (with the vertical direction unrestrained) is applied at wherever the loading point is and the desired path is to be achieved by the output point which is still subject to the resistance force. The prescribed displacement at the input is a design variable itself, and it is allowed to vary from 10mm to 25mm. The structural material used is polypropylene with elasticity modulus 1150MPa, Poisson's ratio 0.4, yield strength 33MPa, and plane stress quadrilateral elements were used with a thickness (in the third direction) of 2mm.

There are two objectives and two constraints in this optimization problem. The first objective is to minimize the deviation between the actual output path of the mechanism and the desired path. The objective function to be minimized is formulated as a weighted average deviation between corresponding step points along the actual and desired displacement paths, weighted according to the length of the FE analysis steps :

$$f_{\text{path}} = d_{\text{ave}} = \sum_{i=1}^{N_s} \left[\frac{l_i}{\sum_{j=1}^{N_s} l_j} \left(\frac{d_{i-1} + d_i}{2} \right) \right] = \frac{1}{2\sum_{j=1}^{N_s} l_j} \sum_{i=1}^{N_s} \left[l_i \left(d_{i-1} + d_i \right) \right] \tag{5}$$

where N_s is the number of analysis steps in the non-linear FE analysis, l_j is the distance between two consecutive displacement step points along the actual path, and d_i is the distance between two corresponding step points along the actual and desired paths. The second objective is to maximize the geometric advantage of the mechanism. Hence the objective function to be minimized is formulated as the negative of the ratio of the horizontal displacement of the output (x_{out}) to that of the loading point (x_{in}) :

$$f_{\text{GA}} = -\frac{x_{\text{out}}}{x_{\text{in}}} \tag{6}$$

There are two inequality constraints for this optimization problem. The first constraint is a maximum bound on the applied load needed to cause the mechanism to achieve the desired path. Hence an inequality constraint is formulated to place a minimum bound on the mechanical advantage of the mechanism :

$$g_{\text{MA}} = 0.167 - \frac{R_{\text{out}}}{F_{\text{in}}} \leq 0 \tag{7}$$

where F_{in} is the input force needed to be applied at the loading point, R_{out} (= 2.5N) is the resistance force and 0.167 is the minimum bound on the mechanical advantage corresponding to a maximum bound of 15N on F_{in}. The second constraint is that the peak von Mises stress σ_{peak} cannot exceed the yield strength σ_{yield} of the material (which is 33MPa), so an inequality constraint function is formulated as follows :

$$g_{\text{stress}} = \frac{\sigma_{\text{peak}} - \sigma_{\text{yield}}}{\sigma_{\text{yield}}} \leq 0 \tag{8}$$

The problem has been solved by running the genetic algorithm for 200 generations with a population size of 100. By the end of the run, a total of 18099 function evaluations (i.e. finite element analyses) have been performed and 20 Pareto-optimal solutions obtained. The Pareto-optimal solution with the best path objective is shown in Fig. 8(a), with its deformed and undeformed shapes shown in Fig. 8(b). This design first appeared in the 157th generation, and its path objective function value is $f_{\text{path}} = 0.023$mm and Fig. 8(c) shows a plot of the actual path compared to the desired path.

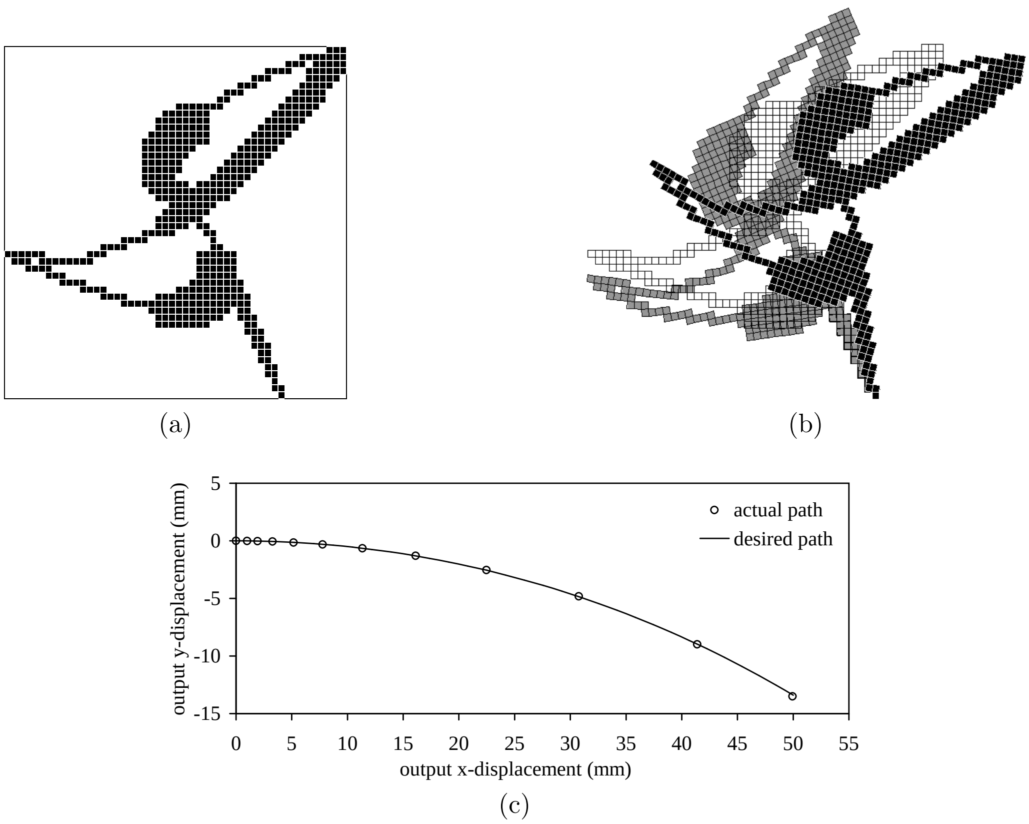


Figure 8: Best path objective solution; (a) optimum geometry (b) undeformed and deformed geometries (c) plot of actual path compared to desired path

8. Concluding Remarks

The versatility and effectiveness of the topology optimization methodology developed in this work rest on three key components : an efficient morphological geometry representation that defines practical and valid structural geometries, a compatible graph-theoretic chromosome encoding and reproduction system that embodies topological and shape characteristics, and a multiobjective genetic algorithm with proficient constraint handling. A multicriterion target matching problem with known solutions has been formulated and solved to illustrate the capabilities of the methodology. A path generating compliant mechanism has been designed by solving a complex structural topology optimization problem. Good results can be obtained within reasonable number of generations of the evolutionary procedure and the geometries are inherently free from checkerboard patterns, disconnected segments or poor connectivity.

9. Acknowledgements

Part of this work was supported by the Singapore Ministry of Education Academic Research Fund through research grant RG92/98, which the authors gratefully acknowledge.

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