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Structure assembling by stochastic topology optimization

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Abstract

Topology optimization is of significant importance to the design of truss- and grillage-like structures. Conventional topology optimization procedures are usually based on the ground structure approach. Starting from highly connected structures the uneconomical links are eliminated during the course of optimization. In this paper we show that, additionally, stochastic methods offer the possibility to build-up structures starting from simple initial configurations with few elements. Stochastic optimization methods (simulated annealing, evolutionary algorithms, random cost) are applied to the topology design problem on the basis of appropriate local structure variations. Results and performance comparisons are given.

Keywords: Stochastic optimization; Structural optimization; Topology optimization; Weight optimization; Evolution strategy; Simulated annealing

1. Introduction

Attempts to apply optimization algorithms to the complex task of structural design have been made over a considerable time. One of the many problems encountered in this field is the topological design of discrete structures like trusses. The different approaches can roughly be divided into the following categories [8,21]: The most common approach is the ground structure method where the optimization is started from highly connected initial structures. During the optimization process the unnecessary structural members will be eliminated (see e.g., [4]). It is evident that in practical applications the ground structure approach necessarily leads to large matrices.

The class of optimality criteria methods covers a number of different strategies (see e.g., [13,22]). They are based upon stress criteria, displacement criteria or the Kuhn–Tucker necessary conditions of optimality. Although optimality criteria procedures in general have proven to be efficient in topology design there might be problems concerning convergence and stability.

The homogenization method [4] is based on using composite materials to model local material properties. A homogenized strain energy is utilized to formulate a material design problem whose solution can be interpreted as the topology of a discrete structure. The method is limited with respect to the choice of the objective function. This problem can be alleviated using heuristic variations of the homogenization method like the solid isotropic microstructures with penalization method (SIMP) [3]. Here the density distribution of

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the material is modified until a quasi-discrete structure is achieved.

The evolutionary structural optimization (ESO) of Xie and Steven [25,26] is a simple method based on so-called rejection criteria which are used to remove inefficient material in a structure. Despite the notion "evolutionary" the method has no biologically inspired steps based on the Darwinian mutation and selection principle and should not be mixed up with evolution strategies or genetic algorithms, see Section 2.2. As the homogenization method ESO is not capable to be used with arbitrary objective functions.

In order to overcome the above-mentioned flaws stochastic topology optimization has gained interest in the last years [2,9,10,18,19].

Stochastic optimization is a very general concept. The respective methods do not have any demand regarding the formulation of the objective function. The stochastic topology optimization methods are not restricted to mechanical problems but can be used with arbitrary objective functions, see e.g., the applications to the optimization of artificial neural networks or the digital filter design in [14].

In contrast to the classical approaches, the alternative methods described in this paper allow also for insertion of bars (see [21, p. 43–44]). This is of importance, because mechanical structures from engineering practice may have many joints and starting from highly connected structures can be computationally demanding. A completely connected initial structure is only one of many possible starting points in the search space. Almost arbitrary initial structures (for example structural models, which have emerged from a conventional design process) can be used if the structural variations allow for substantial structural rearrangements, that means addition and deletion of bar elements. The relevance of this feature increases with the number of joints if the assembling of the stiffness matrix is supported by appropriate data structures.

2. Stochastic optimization

2.1. Simulated annealing

In a famous paper Metropolis et al. [16] introduced a method, which allows the computational simulation of physical systems in thermal equilibrium. Kirkpatrick et al. [12] have taken up the Metropolis approach and adapted to the solution of complex optimization problems (simulated annealing method, SA). The idea behind simulated annealing is based on the close correspondence of energy in statistical mechanics and cost or system quality in optimization problems. Since physical systems can be forced into the energetic ground state by a careful annealing process, an optimization problem can be driven towards the global optimum by adjusting a parameter, which can be considered as the counterpart to the physical temperature. Determining the proper annealing (cooling) schedule for a given problem can be demanding.

2.2. Evolution strategies

In his pioneering evolution theory Darwin gave as the reason for the development of species the principle *survival of the fittest*. This principle states that only by natural selection an optimal adaptation of a particular species to the environment and living conditions could occur.

It is obvious to use such a selection principle as the basis for optimization methods (evolutionary algorithms). For this purpose the variable vector of the optimization problem can be interpreted as an individual of an artificial population and the selection can take place on the basis of the associated objective function value. The objective function plays the role of the fitness in a simulated environment and the adaptation to these conditions leads to the solution of the underlying optimization problem.

The formulation and algorithmic realization of such evolutionary algorithms goes back to Rechenberg [20] and Schwefel [23,24] who developed the so-called evolution strategies (ES), and Holland [11] who laid the foundation for the genetic algorithms (GA). Since in a previous study [2] GA methods turned out to be not very efficient for the problems at hand they have not been given further attention in this paper. In the following a short description of the evolution strategies will be given.

Two main variants of the evolution strategies are in use. Both assume that in each iteration step a population of μ parental vectors exists. Then, with the aim of generating an offspring vector, a parental vector is chosen randomly and modified by adding a random variation (mutation). This procedure is repeated until λ offsprings have been created. In the $(\mu + \lambda)$-ES an intermediate population of the $\mu + \lambda$ individuals is the basis for the selection of the μ best vectors to be parents of the next iteration. In the so-called (μ, λ)-ES, the μ best vectors will be taken out of the λ offsprings only ($\lambda > \mu$).

Further biological phenomena like recombination, migration or the competition between populations can be easily included in such an evolutionary optimization concept.

There is an extensive literature on the theoretical background and recommendations concerning the practical application of evolution strategies (see e.g., [1,7,24]). However, these theoretical investigations are mainly devoted to continuous problems. Relatively few results concerning the strategy parameters exist with respect to discrete problems. On the other side it is

obvious that recombination tends to enhance the probability of the occurrence of kinematical structures (see Section 3.2). Therefore our simulations are based on the simple $(1 + \lambda)$- and $(1, \lambda)$-ES.

Since in genetic algorithms recombination is the main mechanism, the frequent occurrence of kinematical structures might be an explanation for the relatively poor performance of GAs found in [2].

2.3. Random cost method

The random cost method (RC) introduced by Berg [5] is based on a random walk in search space which is mapped onto a random walk in the cost or objective function (hence the name). Since the random cost method is relatively new and therefore not widely known a more detailed description of its basic principle is presented here.

Consider diffusion in one dimension (random walk). There we have a stochastic process with continuous step-size. We denote the size of the ith step by s_i and the corresponding probability density function with $w_i(s_i)$ (Fig. 1). The pth moment is defined by[1]

$$\langle s^p \rangle := \int_S s^p w(s)\,\mathrm{d}s. \tag{1}$$

If we choose $w_i(s_i)$, such that $\langle s_i \rangle = 0$ for all i, the central limit theorem states that the total displacement $x_N := \sum_{i=1}^{N} s_i$ after N steps has a Gaussian probability density function

$$W(x_N) = \frac{1}{\sqrt{2\pi\sigma_N^2}} \exp\left(-\frac{x_N^2}{2\sigma_N^2}\right), \tag{2}$$

where $\sigma_N^2 := \langle x_N^2 \rangle = \sum_{i=1}^{N} \sigma_i^2$ and $\sigma_i^2 = \langle s_i^2 \rangle$ are the variances of x_N and s_i, respectively. The width of $W(x_N)$ is increasing like $\sqrt{N}$ (the total number of steps), and the probability that x-values far off the starting point are visited is increasing steadily.

In the following we use one-dimensional continuous search spaces. The only reason for this is the sake of simplicity. High-dimensional and/or discrete search spaces can be treated in just the same way.

Assume the cost-function of the optimization problem is $y = f(x)$. A stochastic process in the search space is mapped onto a stochastic process in y, the cost- or quality-value (Fig. 2). Step-size in y and the associated probability density function are denoted by $t_i = y_i - y_{i-1}$ and $\omega_i(t_i)$. The probability density functions for the s_i and for the t_i are not independent and it is possible to choose the $w_i(s_i)$ in such a way, as to guarantee $\langle t_i \rangle = 0$. In this way we obtain the stochastic process underlying the random cost strategy. Here the expectation values

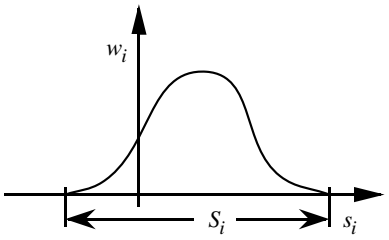

Fig. 1. Probability density function for ith step. The set of values that can be reached is S_i.

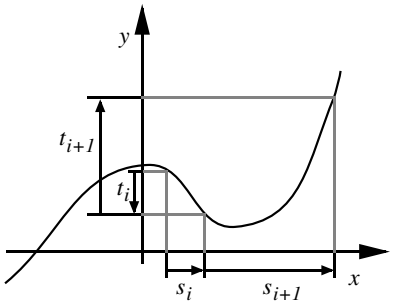

Fig. 2. Mapping of a random walk.

are defined in the usual way, that is $\langle t_p \rangle := \int_T t_p \omega(t)\,\mathrm{d}t$, where the set S is mapped onto T.

From the previous section it follows, that $\Omega(y_N)$ (probability density function of the total displacement in y) becomes a Gaussian in the large N-limit. The width of Ω is increasing with N, and therefore the probability that the global minimum and the global maximum of the optimization problem are visited by the process increases with the number of performed steps (Fig. 3). Obviously the RC-method can be considered as a diffusion process in the quality/cost of the problem.

At the first view trapping in a local optimum does not seem to be a problem because of $\langle t_i \rangle = 0$. Unfortunately it turns out, that this is not true (see below).

In order to state the condition that leads to $\langle t_i \rangle = 0$ we need to define subsets of S:

$$S^- := \{s \in S | t(s) < 0\} \tag{3}$$

and S^+ analogously (Fig. 4).[2] It is straightforward to show that fulfilling the *master equation*

$$P(s \in S^-) = \frac{\langle t \rangle_+}{\langle t \rangle_+ - \langle t \rangle_-} \tag{4}$$

is a sufficient condition for $\langle t \rangle = 0$ (see [5]). $P(s \in S^-)$ is the probability for the next step to end up in S^- and the expectation values are defined over the subsets T^- and T^+, respectively.

[1] In order to keep the notation simple, we omit the index i whenever this seems expedient.

[2] Again we have omitted index i.

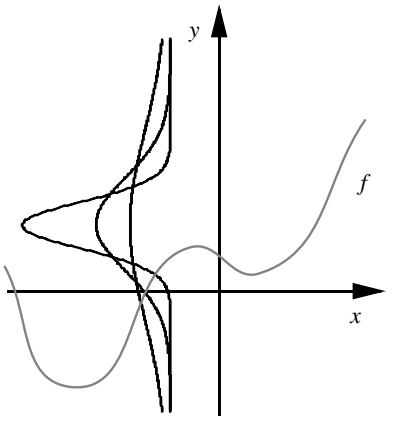

Fig. 3. Probability density functions for the total displacement in y. f is the cost-function.

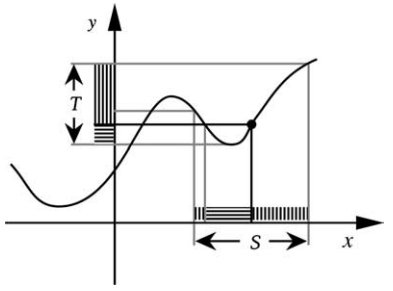

Fig. 4. Definition of subsets of S. Vertical lines: S^+ and T^+, horizontal lines: S^- and T^-.

$f(x)$ maps S^- onto T^- and S^+ onto T^+. For the example, shown in Fig. 4 it can be seen that $|\langle t\rangle_-| < |\langle t\rangle_+|$.[3] Then the master equation implies $P(s \in S^-) > P(s \in S^+)$. The process is attracted by the optimum!

At points in search space, where the cost-function is nearly linear one has $P(s \in S^-) \approx P(s \in S^+) \approx \frac{1}{2}$. Finally, if the process has reached an optimum within the range of S, one of the sets S^- or S^+ is empty and either $P(s \in S^-)$ or $P(s \in S^+)$ is equal to one—the process has been trapped! In terms of the random walk-picture the trapping is related to $\sigma_i^2 \to 0$. If this happens at a local optimum, the process has to be set free by violating the master equation. The random walk therefore does not float between the global optima but between the local optima only.

Further details concerning the RC-method can be found in [6], more about the implementation in [2].

[3] $\langle t\rangle_-$ is negative!

3. Topology optimization

3.1. Structure vector

In the following, truss structures are defined on a fixed planar grid with k nodes. The topology of the truss is specified by a structure vector $\boldsymbol{q} \in \mathbb{B}^n$, where $n = \frac{k(k-1)}{2}$ is the maximum number of connections between the nodes and $\mathbb{B} = \{0, 1\}$. The structure vector of a topology optimization problem is therefore a bit string with n binary variables where the entries $q_i = 1$ or $q_i = 0$ ($i = 1, \ldots, n$) represent the information whether a bar element in the structure is present or not.

3.2. Topology variations

The task is to find the optimal connections of the nodes with respect to a given objective function and certain constraints. The binary domain $\mathbb{B}^n$ covers 2^n possible structure vectors. Even for small values of n complete enumeration of all possibilities is out of the question. The use of stochastic optimization methods requires the creation of slightly different (neighbouring) structure vectors which will be selected by the particular algorithm.

The Hamming-distance

$$d_{\mathrm{H}}(\boldsymbol{q}, \boldsymbol{r}) = \sum_{i=1}^{n} | q_i - r_i | \tag{5}$$

can be considered as a measure of structural difference. It denotes the number of different bits in a component-wise comparison of two strings $\boldsymbol{q} \in \mathbb{B}^n$ and $\boldsymbol{r} \in \mathbb{B}^n$. On the basis of the Hamming-distance a neighbourhood

$$\mathcal{N}_m(\boldsymbol{q}) = \{\boldsymbol{r} \in \mathbb{B}^n \mid d_{\mathrm{H}}(\boldsymbol{q}, \boldsymbol{r}) = m\}, \quad m \in \mathbb{N} \tag{6}$$

can be defined which describes the feasible structural topology variations.

In the present investigation the following structure mutation method has been used:

(1) One of the k nodes is selected at random.
(2) A second node (different from the first) is also selected randomly.
(3) In case, that an element of the actual structure is connecting these two nodes, this element is deleted from the structure. Otherwise an element is inserted.
(4) Continue at 2. until a certain random criterion is fulfilled.

The random criterion used in the fourth step is chosen in such a way, that on average a pre-specified number of elements m is inserted/deleted during one structure mutation step. We denote the resulting neigh-

bourhood as $\mathcal{N}_{\langle m \rangle}$. In the language of evolutionary algorithms the topology variations can be called (on average) m-point-mutation of the binary structure vector without recombination.

Since one objective of the present paper is to compare the relative performance of different optimization methods, the design of mutation procedures has not been given much attention. Considerable improvement should be achievable by using an optimized mutation scheme when confronting stochastic optimization methods with practical applications.

If a kinematical structure occurs during the course of optimization it will be associated a high objective function value (in case of minimization) and will be sorted out by the selection process (lethal structure). To keep the probability of lethal structures small it has to be ensured that $m = \mathcal{O}(1)$.

3.3. Weight optimization problem

In the following we will take the minimization of the truss weight as a test problem for the assembling of discrete structures.

In topology optimization the objective function concerning the structure weight can be written as

$$W = \gamma \sum_{i=1}^{n} q_i A_i l_i, \tag{7}$$

where γ is weight per unit volume, A_i the cross-section and l_i the length of the structural member i. This is a coupled formulation with continuous size variables A_i and discrete (binary) structure variables q_i. For the components $\{j|q_j = 0\}$ of the structure vector the processing of the structural members (element matrices) may be suppressed in the finite element analysis.

Using a simplified approach

$$\widehat{W} = \gamma \sum_{i=1}^{n} q_i A_i(\boldsymbol{q}) l_i \tag{8}$$

for the objective function one gets a pure discrete optimization problem with structure variables q_i. In order to avoid side effects, which may occur when including the continuous size variables, our simulations are based on this approach. The resulting approximation of the weight evaluation implies that the structure is treated like a statically determinate one. Thus the cross-sections can be calculated by means of a prescribed maximum stress value $\sigma_{\max}$. The objective function value is correct at least for statically determinate optimal structures.

4. Results and comparisons

4.1. Initial structures

Our experiment to demonstrate the structural assembly by stochastic topology optimization methods is

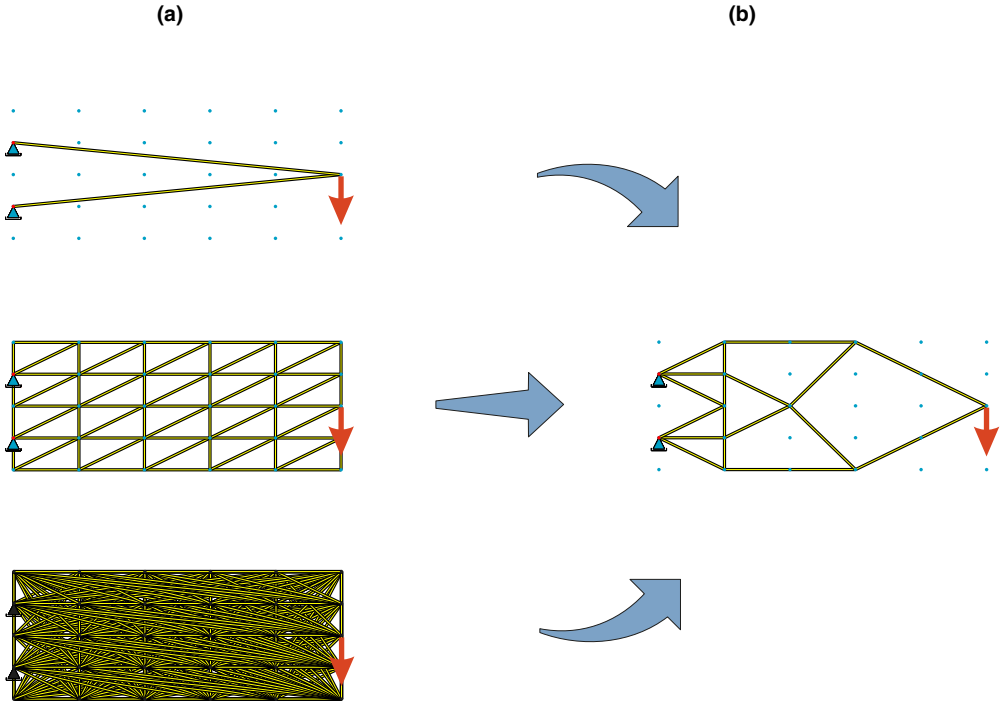

Fig. 5. (a) Initial structures, (b) optimal structure.

based on a regular grid with $k = 30$ nodes (Fig. 5). Thus the maximum number of structural members is $n = 435$ and the stock of structures is $2^{435} \approx 10^{131}$. On the left-hand side of the structure are two fixed supports and one node on the right side is loaded by a single force. This set-up was especially devised because the global minimum on this grid, an approximation of a Michell structure (see [17]), is known.

In Fig. 5a three different initial structures are depicted representing starting points in the search space. At the bottom a completely connected structure is shown, thus a thin out process, similar to the conventional ground structure approach, has to be performed to obtain the solution. In contrast, the net-like triangular structure in the middle needs substantial reorganization by the optimization process. First results concerning the performance of evolution strategies and the random cost method, based on these two initial structures, have been discussed in [2,15].

The structural build-up launched from simple initial configurations, like the 2-bar-structure on top of Fig. 5a, is a more difficult problem. Stochastic methods allow the complete rearrangement of structural members. This is a prerequisite for an automatic assembly of complex structures. In our simulations the optimal solution, Fig. 5b, could be achieved starting from all three configurations.

4.2. Performance considerations

Figs. 6 and 7 allow comparisons of the three different types of stochastic algorithms (SA, ES and RC) and of the influence of the structural variation neighbourhood. Each strategy had to solve the problem 10 times using different random number sequences. The figures show how many optima have been detected and how many function evaluations have been needed for the particular run. The results are based on the net-like initial structure (see Fig. 5a). The parameter setting of the simulated annealing runs has been chosen as close as possible to the values given by Kirkpatrick [12]. The random cost data have been obtained using a maximum number of structural variations per iteration step $\lambda = 50$.

The optimization runs, the results of which are shown in Fig. 6, have been carried out on a basis of a structural neighbourhood $\mathcal{N}_1$. Fig. 7 is founded on $\mathcal{N}_{\langle 2 \rangle}$. Thus the influence of the two different structural neighbourhoods on the topology optimization process can be seen by comparing the graphs of both diagrams. It turns out that the use of the larger neighbourhood $\mathcal{N}_{\langle 2 \rangle}$ improves the reliability of finding the optimal solution. On the other hand, with the smaller neighbourhood $\mathcal{N}_1$, the successful runs on average needed less iterations. This means that at the beginning of the optimization small changes in the structure vector are more useful to get a faster

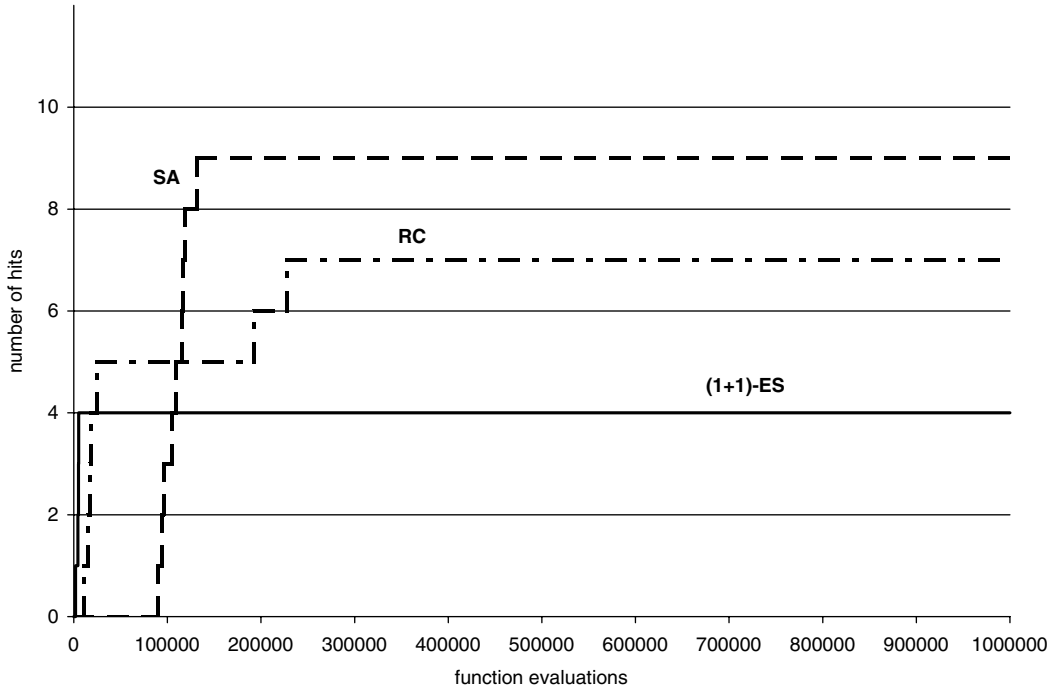

Fig. 6. Comparison of methods starting from the net-like structure (Fig. 5a, middle) based on $\mathcal{N}_1$. Ten runs have been performed for each strategy.

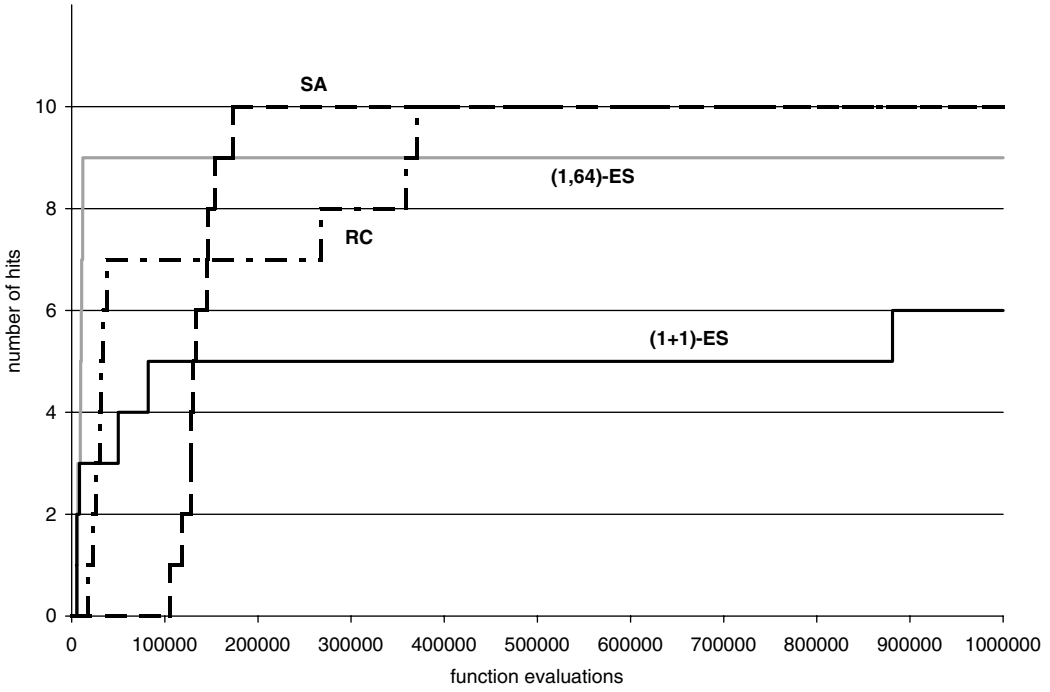

Fig. 7. Comparison of methods starting from the net-like structure (Fig. 5a, middle) based on $\mathcal{N}_{\langle 2\rangle}$. Ten runs have been performed for each strategy.

descend of the objective function value than larger structural variations. One reason might be the enhanced probability of creating kinematical structures in the $\mathcal{N}_{\langle 2\rangle}$ case. It is clear, however, that in situations where the optimization gets stuck in a local minimum using the $\mathcal{N}_1$ neighbourhood, a wider structural variation environment could lead to a continuation of the process; therefore the better prospect in finding solutions.

With respect to a comparison of the optimization methods used, the general tendency is similar in both diagrams. All in all the simple (1 + 1)-evolution strategy found the optimum most early but failed in about half of the cases. Simulated annealing was the most reliable method and found a solution in 19 out of 20 runs (9 of 10 in Fig. 6 and 10 of 10 in Fig. 7) but needed a great deal more of computation time. The computational performance of the random cost method is somewhere in between and it turns out to be a very reliable procedure (17 hits of 20 runs). Additionally, in Fig. 7, the data obtained by a $(1, 64)$-evolution strategy are plotted. The strategy proved to be comparable with the (1 + 1)-ES with regard to the number of function evaluations needed, but was more successful.

The preceding statements lead to the question how the computational performance depends on the type of strategy and how it scales with the number of structural updates per iteration (population size) λ. Fig. 8 shows the best objective function value found after 10^5 function evaluations. Each point represents the average of five independent runs. The graphs are based on the net-like initial structure and $\mathcal{N}_{\langle 2\rangle}$. Only data for the evolution strategies and the random cost method can be taken into account because there is no such parameter like λ in the conventional simulated annealing procedure. The results show that λ has a strong influence on the random cost method and the $(1, \lambda)$-ES. The quality of the solution increases considerably with the use of a larger number of structural updates per iteration. In contrast, the $(1 + \lambda)$-ES is nearly not affected by the population size. This, of course, is convenient from the practical point of view.

4.3. Structural assembly

The results from the previous section were utilized in the simulations which have been performed with the aim of a structural build-up (initial structure on top of Fig. 5a). The performance of the different strategies can be extracted from Fig. 9, which is based on $\mathcal{N}_{\langle 2\rangle}$. The simple $\mathcal{N}_1$ neighbourhood cannot be used with the 2-bar-initial structure because it exclusively produces kinematical configurations at the beginning of the simulation. Hence, the optimization process cannot be initiated.

The first point to note is, that simulated annealing shows a similar performance as in the previous simulations based on the net-like initial structure (Figs. 6 and

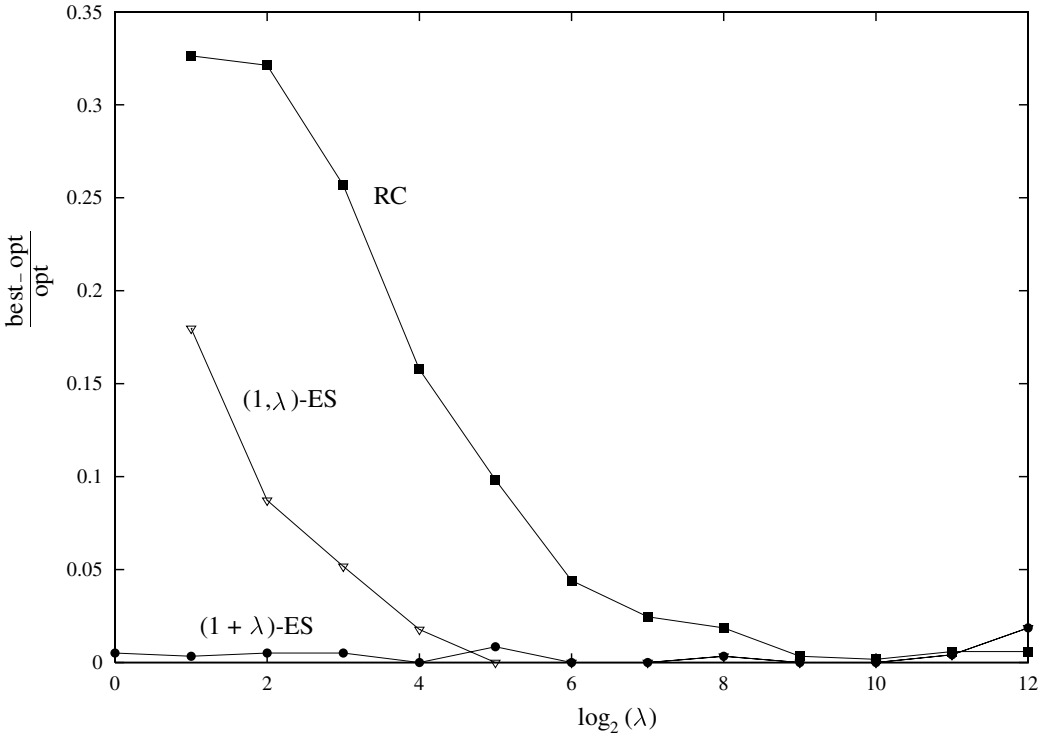

Fig. 8. Influence of λ—the results are based on the net-like initial structure (Fig. 5a, middle) and the $\mathcal{N}_{(2)}$ neighbourhood. Each data point represents the average of five runs.

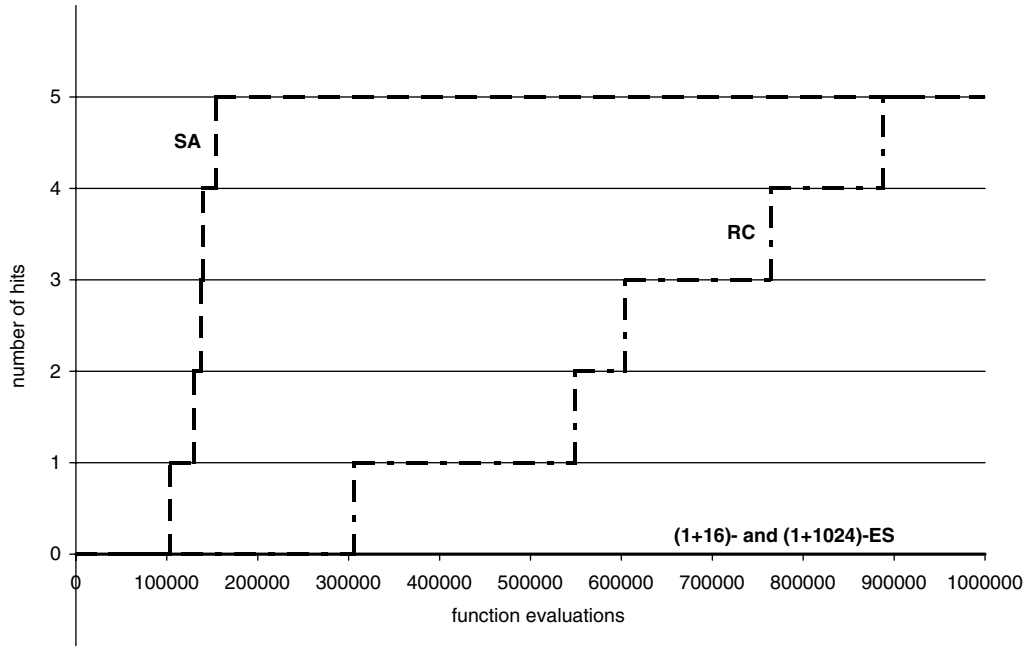

Fig. 9. Results for the structural build-up starting from the 2-bar-structure (Fig. 5a, top) based on $\mathcal{N}_{(2)}$. Five runs have been performed for each strategy.

7). Simulated annealing is not a fast method, however this strategy seems to be completely unimpressed by the fact that the problem at hand is more difficult than the previous ones. This can be explained by the analogy with statistical mechanics. At high temperatures simulated annealing creates configurations with a high degree of disorder (entropy) and the information concerning the starting configuration gets lost.

In the light of the results of Section 4.2 the random cost simulations have been performed with $\lambda = 1024$. Like simulated annealing random cost exhibits a rate of success of 100%. However, random cost needs much more function evaluations until the optimal structure is found.

The $(1 + \lambda)$-evolution strategies ($\lambda = 16$ and $\lambda = 1024$ have been used) do not show even one single success within the prescribed number of function evaluations. On average the best objective function results obtained were about 10% above the optimal value. The advantage of the $(1 + \lambda)$-ESs is their high initial progress rate, but once trapped in a local optimum they are not able to escape from it.

The $(1, \lambda)$-evolution strategy, in contrast, is able to leave a local optimum. Despite this and the fact that this strategy looks very promising with regard to Fig. 7 this type of strategy is not included in Fig. 9. The reason is that the $(1, \lambda)$-ES has to find a non-lethal structure within λ structure proposals. Otherwise a lethal structure has to be accepted as the parent for the following generation. In a sparsely occupied initial configuration, like the 2-bar-structure, the creation of a non-kinematical structure is a rare event and therefore the $(1, \lambda)$-ES has very low chances for success. It might be possible to circumvent this problem by certain measures. However, this measures stand in contradiction to the spirit of a direct comparison of the methods and therefore the $(1, \lambda)$-ES has been excluded.

5. Conclusions

In the present paper it was shown that stochastic methods are able to build-up topological optimal constructions starting from very simple structures. The use of appropriate structural variation procedures is essential.

All the investigated methods (simulated annealing, evolution strategies and random cost) are generally suited for topology optimization problems. However, there are substantial differences in the computational performance: simulated annealing turns out to be most reliable in finding the optimum. The evolution strategies exhibit an ambiguous nature. If successful they are fast, but their usability depends on the choice of an appropriate initial structure. The performance of the random cost method shows an intermediate behaviour.

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