



ELSEVIER

Advances in Engineering Software 35 (2004) 609–617

ADVANCES IN  
ENGINEERING  
SOFTWARE

[www.elsevier.com/locate/advengsoft](http://www.elsevier.com/locate/advengsoft)

# Topology optimisation of an automotive component without final volume constraint specification

G. Chiandussi\*, I. Gaviglio, A. Ibba

*Laboratory of Applied Optimisation, Department of Mechanical Engineering, Technical University of Torino,  
Corso Duca degli Abruzzi 24, 10129 Torino, Italy*

Received 3 October 2002; accepted 25 July 2003

Available online 10 August 2004

## Abstract

The paper shows the results obtained by using a topology optimisation code to solve a three-dimensional problem concerning a real automotive component. The implemented optimisation method is based on the maximisation of the total potential energy with a volume constraint by optimality criteria. The volume of the optimal solution depends on the imposed static (displacement, stress, stiffness) and dynamic (natural frequency) constraints and has not to be specified a priori. The optimisation process converges toward a quite well defined structure made of the base material with a very little percentage of elements characterised by intermediate material properties.  
© 2004 Civil-Comp Ltd and Elsevier Ltd. All rights reserved.

**Keywords:** Topology optimisation; Optimality criteria; Volume constraint; Stress constraints; Automotive component design

## 1. Introduction

Solutions obtained by standard size and shape optimisation methods keep the same topology of the initial design. These solutions are often far from optimal because other competing topologies cannot be explored. For this reason, topology optimisation methods are becoming increasingly important as potential tools in engineering design.

In topology optimisation of continuum structures, the shape of the boundaries and the number of internal holes of an admissible design domain are considered concurrently with respect to a predefined objective function, usually the compliance minimisation or a natural frequency maximisation, and one or more constraints, e.g. a volume constraint. Various families of structural topology optimisation algorithms for generalised shape optimisation problems have been developed based on the homogenisation theory [1,2], the power-law approach (SIMP) [3–5], on evolutionary approaches like the Evolutionary Structural Optimisation (ESO) method [6–11], the soft kill and hard kill methods [12–14] and the biological growth method [15]. Other methods for topology optimisation of continuum structures have been proposed like the simulated annealing method

[16], genetic algorithms and the bubble method described in Ref. [17].

The homogenisation method is based on the modelling of a perforated material constructed from a basic unit cell consisting at a microscopic level of material and void and on the description of the structure by using a continuously varying distribution of the material density computed by invoking the formulas of the homogenisation theory. The SIMP method is based on the utilisation of constant material properties within each element and element relative densities raised to some power times the material properties of solid material as design variables. To ensure existence of solutions, the power-law approach must be combined with a perimeter constraint, a gradient constraint or a filtering technique.

Evolutionary methods have their origin in fully stressed design techniques and generate structural topologies by eliminating or adding at each iteration elements having a low value of some criterion function, such as stress, energy density (compliance) or some other response parameter. Evolutionary methods are usually intuitive methods without proof of optimality for given objective function and constraints [18].

A topology optimisation method based on optimality criteria for total potential energy maximisation with a volume constraint has been implemented. The volume of

\* Corresponding author. Tel.: +39-11-564-6935; fax: +39-11-564-6999.  
E-mail address: [giorgio-chiandussi@polito.it](mailto:giorgio-chiandussi@polito.it) (G. Chiandussi).

the optimal structure is controlled by constraints on static and dynamic responses, i.e. displacement, stress, stiffness and natural frequency constraints, through the volume Lagrange multiplier [19,20]. The results obtained by solving a three-dimensional problem concerning a real automotive component with stress and natural frequency constraints are shown.

## 2. Topology optimisation method

Let us consider a body occupying a domain  $\Omega^m$  which is part of a larger reference domain  $\Omega \in \mathbb{R}^3$ . Let us suppose that  $\Omega$  is subjected to the applied body forces  $f$ . Let us assume that  $\Omega$  has a smooth boundary  $\Gamma$  comprising  $\Gamma_d$  where displacements are prescribed and  $\Gamma_t$  where surface traction forces  $t$  are applied. It is also assumed that:

$$\Gamma_t \cup \Gamma_d = \Gamma \text{ and } \Gamma_t \cap \Gamma_d = \emptyset \quad (1)$$

Let us consider the general elasticity problem. The structural optimisation problem in its most general form can be written as:

$$\begin{aligned} & \underset{\eta}{\text{maximise}} \quad \min_{v \in V} \Pi(v) \\ & \text{subject to} \quad \int_{\Omega} \eta \, d\Omega \leq \bar{V} \\ & \quad 0 < \eta_{\min} \leq \eta \leq \eta_{\max} < \infty \\ & \quad \eta \in L^{\infty}(\Omega) \end{aligned} \quad (2)$$

where  $\eta$  is a continuous function defined on the design domain  $\Omega$  representing the effectiveness of the material in the volume  $d\Omega$ ,  $\bar{V}$  is the required volume of the optimal structure and  $v$  is a kinematically admissible displacement field.

In linear static problems, the equilibrium displacement field  $u$  makes the total potential energy an absolute minimum. By introducing the equilibrium equation described by the principle of virtual work, the optimisation problem (2) can be reformulated as:

$$\begin{aligned} & \underset{\eta}{\text{minimise}} \quad \frac{1}{2} l(u) = \frac{1}{2} \int_{\Omega} \varepsilon^T(u) D \varepsilon(u) \eta \, d\Omega \\ & \text{subject to} \quad a(u, v) = l(v), \quad \forall v \in V \\ & \quad \int_{\Omega} \eta \, d\Omega \leq \bar{V} \\ & \quad 0 < \eta_{\min} \leq \eta \leq \eta_{\max} < \infty \\ & \quad \eta \in L^{\infty}(\Omega) \end{aligned} \quad (3)$$

where the bilinear form for the internal work and the linear form for the external work have been introduced as:

$$a(u, v) = \int_{\Omega} \varepsilon^T(v) D \varepsilon(u) \eta \, d\Omega \quad (4)$$

$$l(v) = \int_{\Omega} f^T v \, d\Omega + \int_{\Gamma_t} t^T v \, d\Gamma \quad (5)$$

where  $\varepsilon$  is the strain field in the design domain  $\Omega$  and  $D$  the constitutive matrix for a linear elastic material. The components of the constitutive matrix can be evaluated by making reference to the relationship between the stress and strain components given by:

$$\sigma_{ij} = D_{ijkl} \varepsilon_{kl} \quad (6)$$

where:

$$D_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (7)$$

the  $\delta_{ij}$  function is the Kronecker delta and  $\lambda$  and  $\mu$  are the Lamè constants defined as:

$$\lambda = \frac{\nu_0 E_0}{(1 - 2\nu_0)(1 + \nu_0)} \quad (8)$$

$$\mu = \frac{E_0}{2(1 + \nu_0)} \quad (9)$$

where  $E_0$  and  $\nu_0$  are the Young modulus and the Poisson's ratio of the base material, respectively.

The optimisation problem described by Eq. (3) represents a classic variable 'thickness' design problem where the thickness have been substituted by an artificial variable  $\eta$ . The volume of the final structure  $\bar{V}$  as well as the compliance  $l(u)$  depend linearly on the variable  $\eta$ . The existence of solutions for this problem has already been proved and does not require a relaxation method or the introduction of materials with a micro structure [3].

The Lagrangian function of the optimisation problem described is given by:

$$\begin{aligned} L(\eta, \lambda) = & \frac{1}{2} \int_{\Omega} \varepsilon^T(u) D \varepsilon(u) \eta \, d\Omega - \lambda_1 \left[ \int_{\Omega} \eta \, d\Omega - \bar{V} \right] \\ & - \lambda_2 [a(u, v) - l(v)] \end{aligned} \quad (10)$$

where  $\lambda$  is the vector of Lagrange multipliers and the side constraints concerning the design variable  $\eta$  have been temporarily neglected. The necessary conditions for optimality can be obtained by using the Kuhn–Tucker conditions as follows:

$$\begin{aligned} \frac{\partial L}{\partial \lambda_1} &= \int_{\Omega} \eta^* \, d\Omega - \bar{V} = 0 \\ \frac{\partial L}{\partial \lambda_2} &= a(u^*, v) - l(v) = 0, \quad \forall v \in V \\ \frac{\partial L}{\partial \eta} &= \frac{1}{2} \int_{\Omega} \varepsilon^T(u) D \varepsilon(u) \, d\Omega - \lambda_1 \int_{\Omega} d\Omega = 0 \end{aligned} \quad (11)$$

where  $\eta^*$  identifies the optimal distribution of the function  $\eta$ . The Lagrange multiplier  $\lambda_1$  for the optimal solution can be obtained as

$$\lambda_1 = \frac{\frac{1}{2} \int_{\Omega} \varepsilon^T(u) D \varepsilon(u) \, d\Omega}{\int_{\Omega} d\Omega} = \bar{e} \quad (12)$$

where  $\bar{\epsilon}$  is the average value of the strain energy density of the optimal structural configuration evaluated by taking into consideration the volume of the design domain. If a discrete model is considered, Eq. (12) holds for every discrete element. Then, the elements of the discrete model should be characterised by the same strain energy density. Therefore, the optimal topology should be characterised by a uniform distribution of the strain energy density as already obtained by Venkayya [21] and Rossow and Taylor [22].

The topology optimisation problem layout described by Eq. (2) can also be used when natural frequencies have to be considered. If  $u$  and  $v$  are considered as eigenvectors, the weak form of the vibration problem can be written as:

$$a(u, v) = \Lambda b(u, v), \quad \forall v, u \in V \quad (13)$$

where  $\Lambda$  is an eigenvalue of the natural frequency eigenproblem,  $a(u, v)$  is the bilinear form for the internal work and  $b(u, v)$  represents the work done by the distributed applied loads due to the inertia effects. The bilinear form for the internal work and the work done by the distributed applied loads are:

$$a(u, v) = \int_{\Omega} \epsilon^T(v) D \epsilon(u) \eta \, d\Omega \quad (14)$$

$$b(u, v) = \int_{\Omega} \rho_0 v \cdot u \eta \, d\Omega \quad (15)$$

where  $\rho_0$  is the material density of the base material.

Eq. (13) can be substituted in Eq. (2) leading to the optimisation problem:

$$\begin{aligned} & \underset{\eta}{\text{maximise}} && \Pi(u) \\ & \text{subject to} && a(u, v) = \Lambda b(u, v), \quad \forall v \in V \\ & && \int_{\Omega} \eta \, d\Omega \leq \bar{V} \\ & && 0 < \eta_{\min} \leq \eta \leq \eta_{\max} < \infty \\ & && \eta \in L^{\infty}(\Omega) \end{aligned} \quad (16)$$

and, after the introduction of the total potential energy value at the equilibrium, to a topology optimisation problem described by a system of equations similar to that used for the static case described by Eq. (3).

Eq. (12) can be used to solve the optimisation problem by using an optimality criteria approach [23]. It is necessary to identify a recursive relationship to be used in a finite element calculus as an updating procedure. The artificial variable  $\eta$  has been considered as an indicator of the local material effectiveness. It can be associated to the finite volume  $d\Omega$  (sheet thickness, beam/bar cross section) or to a material property (Young modulus and material density). The latter option have been adopted leading to the following relationship:

$$\eta = \frac{E}{E_0} = \frac{\rho}{\rho_0} \quad (17)$$

where  $E$  and  $\rho$  are continuous Young modulus and material density distributions over the design domain  $\Omega$  and  $E_0$  and  $\rho_0$  are the Young modulus and the material density of the base material, respectively. The relationship between the artificial variable  $\eta$  and the base material properties introduced by Eq. (17) transfers the role of design variable to the continuous distribution of the material properties  $E$  and  $\rho$  and do not introduces the penalties proper of the SIMP method. The volume of the final structure  $\bar{V}$  and the compliance  $l(u)$  keep depending linearly on the design variables. The solution to the optimisation problem keeps existing [3] and no filter stabilisation or perimeter control method is required to reach the convergence.

Dividing Eq. (12) by  $\bar{\epsilon}$  and multiplying it for the artificial variable as defined in Eq. (17), it is possible to define the following resizing rules to be applied in topology optimisation procedure concerning a discrete design domain:

$$E_i^{\text{new}} = E_i^{\text{old}} \frac{e_i}{\bar{\epsilon}} \quad (18)$$

$$\rho_i^{\text{new}} = \rho_i^{\text{old}} \frac{e_i}{\bar{\epsilon}} \quad (19)$$

where  $E_i^{\text{new}}$ ,  $E_i^{\text{old}}$ ,  $\rho_i^{\text{new}}$ ,  $\rho_i^{\text{old}}$  and  $e_i$  are the new and the old value of the Young modulus of element  $i$ , the new and the old value of the material density of element  $i$  and strain energy density of element  $i$ , respectively.

The application of the resizing rules described by Eqs. (18) and (19) corresponds to find the point wise optimal distribution of the material characteristics for a given fixed stress and strain field. If the structure would have been determinate, the resizing rules above described would have led to the identification of the optimal configuration in one step. Otherwise, the resizing rules affect the global behaviour of the structure and an iterative process is required until convergence is reached.

Side constraints have not been taken into consideration in the definition of the Lagrangian function of the design optimisation problem. Their satisfaction has to be verified at each iteration and for each discrete element of the design domain during the updating process of the material properties. The requirement for a structure with the base material Young modulus  $E_0$  and material density  $\rho_0$  requires the proper selection of the upper limit for the artificial variable  $\eta$ :

$$\eta_{\max} = 1 \quad (20)$$

The requirement for a positive definite stiffness matrix of the design domain leads to the selection of a lower limit for the artificial variable  $\eta$  given by:

$$\eta_{\min} = 10^{-4} - 10^{-5} \quad (21)$$

The value of  $\eta_{\min}$  is extremely low and allows to consider the elements with the corresponding value of Young modulus and material density as void.

The Lagrange multiplier of the volume constraint, a strain energy density from the dimensional point of view,

makes reference to the optimal structural configuration. It has not to be searched a posteriori in order to comply with the volume constraint. Instead, it can be calculated a priori in order to comply with the mean stress, displacement and stiffness constraints defined on the optimal solution. Therefore, the volume of the optimal solution usually unknown a priori is indirectly controlled by the imposition of a reference strain energy density evaluated by taking into account the average strain energy that should characterise the optimal solution. The imposed average strain energy density will be called in the following as reference strain energy density,  $\bar{e}_{\text{ref}}$ .

For example, if the optimal solution should be characterised by a maximum allowable stress for the base material  $\sigma_{\text{max}}$  and a truss like structure is expected, the reference strain energy could be evaluated as follows:

$$\bar{e}_{\text{ref}}(\sigma_{\text{max}}) = \frac{1}{2} \frac{\sigma_{\text{max}}^2}{E_0} \quad (22)$$

In this case, side constraints and the indeterminate problem make it necessary to update at each iteration the reference strain energy density given by Eq. (22) as follows:

$$\bar{e}_{\text{ref}}^{i+1} = \bar{e}_{\text{ref}}^i \frac{\bar{e}}{\bar{e}^i} \quad (23)$$

where  $\bar{e}_{\text{ref}}^i$  and  $\bar{e}_{\text{ref}}^{i+1}$  are the reference strain energy densities for iterations  $i$  and  $(i+1)$ ,  $\bar{e}$  is the required strain energy density given by Eq. (22) and  $\bar{e}^i$  is the average strain energy density of the structure at iteration  $i$ . The topology optimisation process can be started by using the strain energy density value given by Eq. (22) as reference.

Multiple loading conditions can also be managed. Searching for a Pareto optimal solution, a topology optimisation problem based on the minimisation of a weighted sum of the total potential energy of each load case can be set up [24] as:

$$\begin{aligned} \bar{\Pi}(v) &= \sum_{k=1}^m w^k \Pi(v^k) \\ \sum_{k=1}^m w^k &= 1 \end{aligned} \quad (24)$$

where  $k = 1, \dots, m$  are different loading conditions,  $w^k$  is the weight corresponding to the  $k$ th loading condition and  $v$  is a kinematically admissible displacement field. In this case, the resizing rules described in Eqs. (18) and (19) have to be modified as follows:

$$\begin{aligned} E_i^{\text{new}} &= \sum_{k=1}^m w^k E_i^{\text{old}} \frac{e_i^k}{\bar{e}^k} = \sum_{k=1}^m w^k E_i^{k,\text{new}} \\ \rho_i^{\text{new}} &= \sum_{k=1}^m w^k \rho_i^{\text{old}} \frac{e_i^k}{\bar{e}^k} = \sum_{k=1}^m w^k \rho_i^{k,\text{new}} \end{aligned} \quad (25)$$

### 3. Automotive component design

The above described topology optimisation method has been implemented within the ANSYS finite element code. Several two-dimensional benchmark examples with static constraints have been solved to verify the correctness and the performance of the method [19,20].

The proposed method has been applied to solve several design topology optimisation problems with multiple loading conditions and stress constraints. The optimisation of an engine support of a mid size commercial vehicle has already been presented in Ref. [25]. In the present paper the results of the topology optimisation of a McPherson rear suspension subframe of a mid size commercial vehicle are shown. The indirect control of the volume constraint by the constraint on the maximum mean stress and by the requirement of a first natural frequency maximisation leads to a very simple layout of the topology optimisation problem. Data for the linear static and dynamic analyses have to be prepared as if the analyses should be carried out alone. The maximum allowable mean stress for the static loading conditions has to be added and the weight of each single loading condition for the multiple loading condition topology optimisation has to be specified.

#### 3.1. Topology optimisation of a rear suspension subframe

The topology optimisation problem concerns the redesign of a McPherson rear suspension subframe of a mid size commercial vehicle. The analysed structural component is linked to the wheel hub by means of two arms and to the vehicle chassis with bolts (Fig. 1). The main task of the component is to transfer the transversal loads coming from

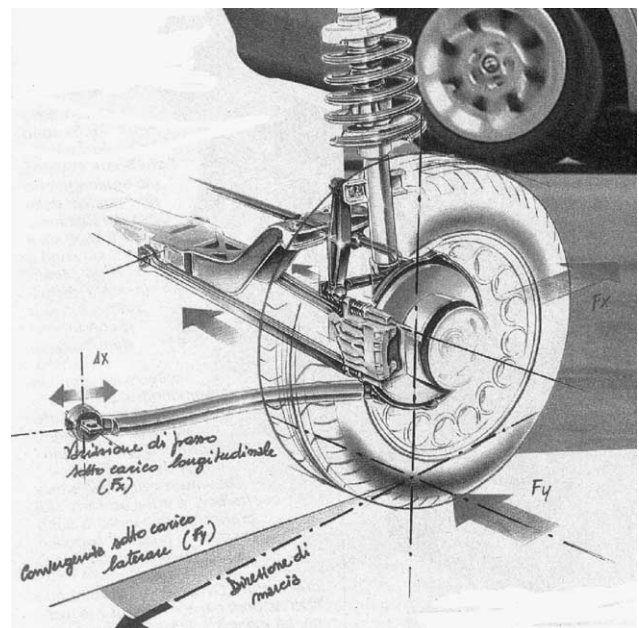


Fig. 1. Working environment of the analysed McPherson rear suspension subframe.



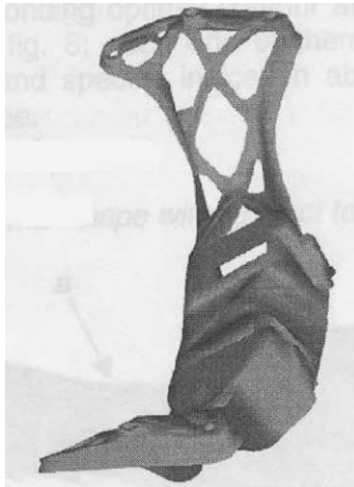


Fig. 2. Original geometry of the analysed McPherson rear suspension subframe.

the wheel hub to the vehicle chassis. The longitudinal loads coming from the wheel hub are directly transferred to the vehicle chassis by a third arm (Fig. 1). The original model of the component is shown in Fig. 2 and is characterised by a first natural frequency of  $f_0 = 120$  Hz.

The discrete model for the design optimisation process is shown in Fig. 3. Only one half of the structure have been taken into account taking advantage of its symmetry with respect to the longitudinal axis of the vehicle. The design domain of the model has been expanded as much as possible avoiding the interference with the surrounding components (Fig. 3, light grey). Bearings and fastening systems have been kept unchanged and represent the non-design domain of the model (Fig. 3, dark grey). The connecting bolts have been simulated by using a steel beam passing through the bearing. The beam has been linked to the nodes of the internal and external surfaces of the bearing by a 'star' of rigid bars. The finite element model is characterised by 14,107 tetrahedral elements, 12,668 elements for the design domain and 1439 elements for the non-design domain. Several two-dimensional elements have been used to describe the connecting bolts and the rigid bars.

Bearings connecting the component to the vehicle chassis have been fully constrained. Symmetry constraints

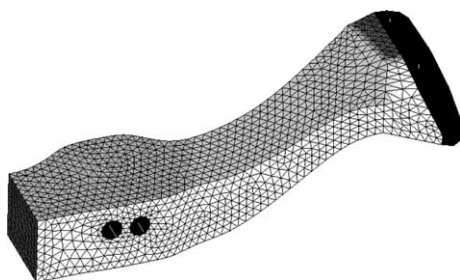


Fig. 3. Rear suspension subframe design (light grey) and non design (dark grey) domains.

Table 1

Table of applied loads in the three different loading conditions expressed as a percentage of the  $Y$  component of the steering force applied to the second arm

Loading condition	Load application	X component (%)	Y component (%)	Z component (%)
Steering	1	−0.5	−93.0	−8.4
Steering	2	−0.7	−100.0	−6.7
Braking	1	19.3	−93.5	8.1
Braking	2	23.4	93.5	87
Maximum stroke	1	0.4	75.7	10.1
Maximum stroke	2	0.3	41.3	4.8

have been applied. Loads have been applied to the central beam of the bearings connecting the component to the two arms coming from the wheel hub.

Three independent static loading conditions have been analysed. They have been defined by considering the most severe loads during steering and braking and the most severe loads when the maximum stroke of the suspension is reached. Dimensionless loads are given in Table 1 and their application point is shown in Fig. 4–6. Loads are symmetric during braking and when the maximum stroke of the suspension is reached. They are anti symmetric in the steering loading condition (Fig. 4).

An aluminium alloy Al375T5 has been considered with the following properties: Young modulus  $E = 70,000$  MPa, Poisson ratio  $\nu = 0.24$ , mass density  $\rho = 2.27$  kg/dm<sup>3</sup>, yield strength  $\sigma_y = 160$  MPa. Design requirements for the rear

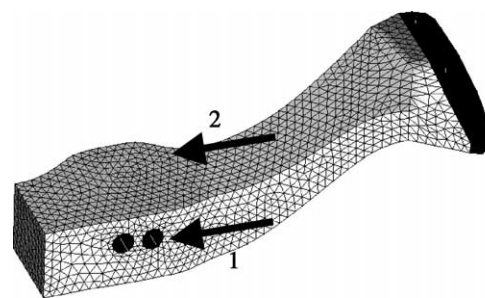
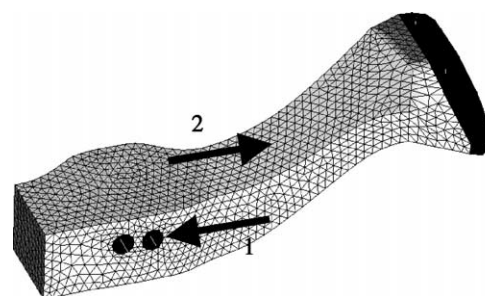


Fig. 4. Applied forces during steering.



Figs. 5. Applied forces during braking.

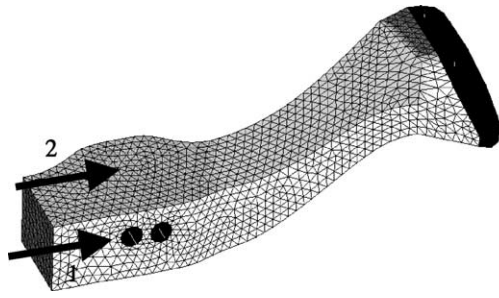


Fig. 6. Applied forces when the maximum suspension stroke is reached.

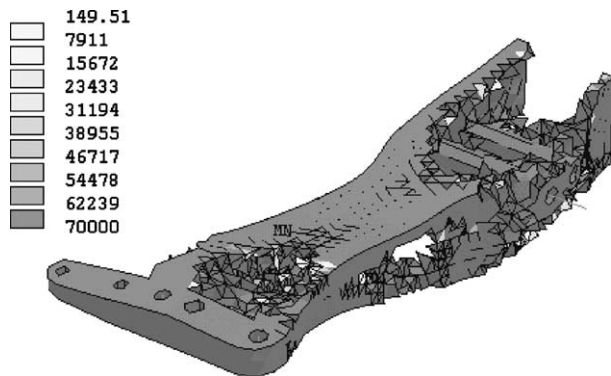


Fig. 7. Final topology for the rear suspension subframe design optimisation problem: front view.

suspension subframe are a maximum allowable stress  $\sigma_{VM,max} \approx 100$  MPa (60% of the yield strength of the base material) and a first natural frequency higher than  $f_{min} = 300$  Hz.

The multiple loading condition topology optimisation problem has been laid out as a stiffness maximisation problem with respect to the static loading conditions and as a first natural frequency maximisation problem with respect to the dynamic loading condition. Weight coefficients  $w_1 = w_2 = w_3 = 0.17$  have been used for the three static loading conditions and a weight coefficient  $w_4 = 0.50$  has been used for the dynamic loading condition. A stress constraint on the maximum mean stresses  $\sigma_{max} = 100$  MPa have been imposed. The convergence criterion of the optimisation process has been based on the relative change of the design

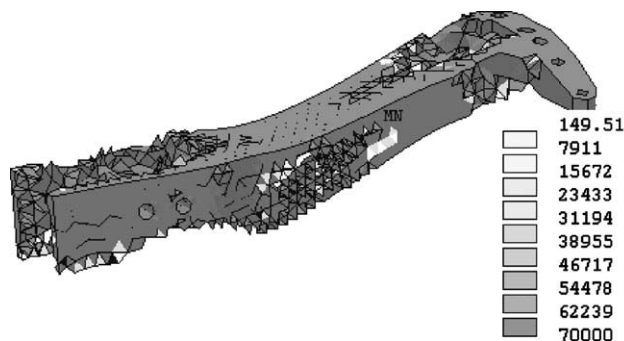


Fig. 8. Final topology for the rear suspension subframe design optimisation problem: lateral view.

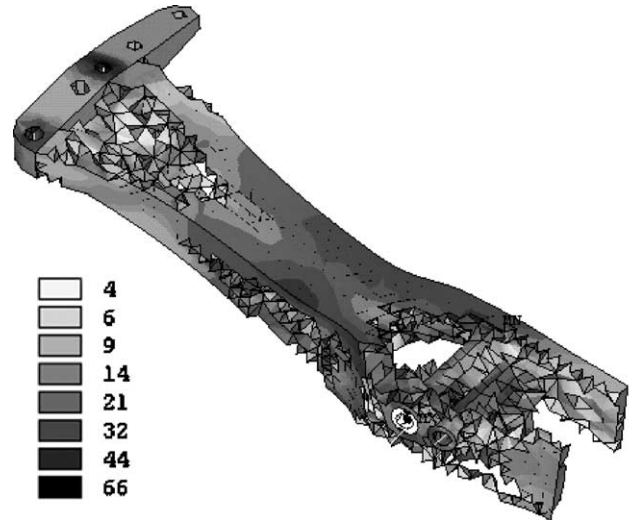


Fig. 9. Von Mises stress distribution on the optimal configuration of the rear suspension subframe during steering.

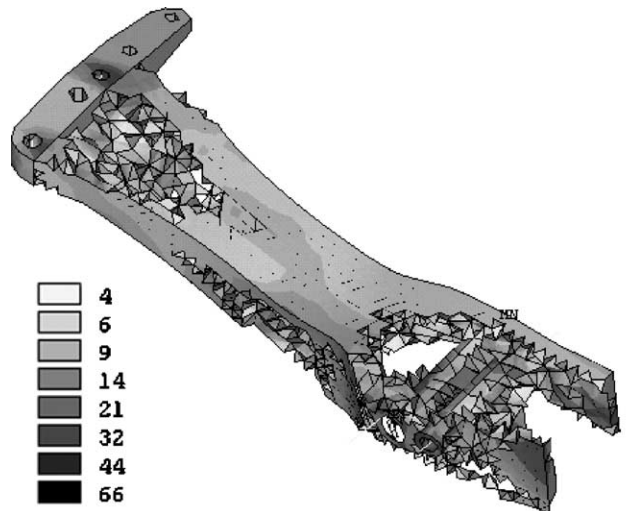


Fig. 10. Von Mises stress distribution on the optimal configuration of the rear suspension subframe during braking.

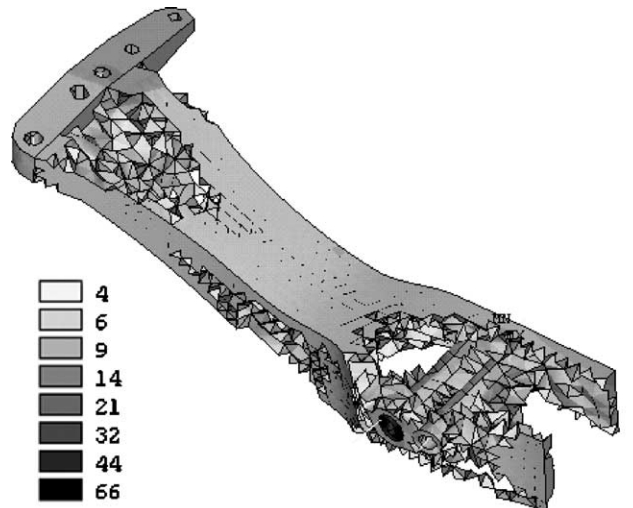


Fig. 11. Von Mises stress distribution on the optimal configuration of the rear suspension subframe when the maximum suspension stroke is reached.

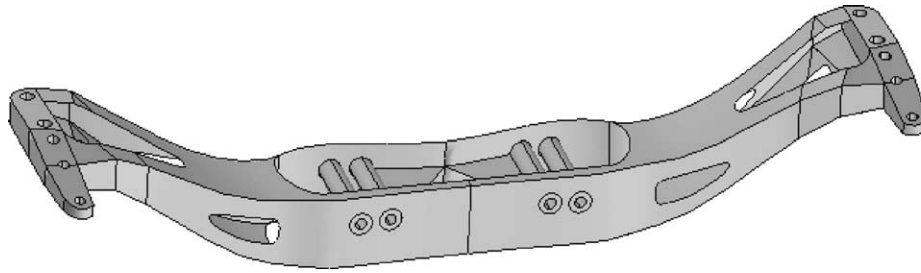


Fig. 12. The proposed geometrical configuration for the rear suspension subframe.

domain volume. The optimisation process has been stopped when a relative change of the design domain volume of less than 1% has been achieved.

The optimisation process converged in 25 iterations. The final topology and the Young modulus distribution in the design domain are shown in Figs. 7–8. The implemented method leads to the identification of a quite well defined structural topology. Less than 60% of the elements of the design domain have been kept (5103 elements). Only 156 of these elements (1% of the design domain elements) show intermediate material properties with a Young modulus value between 100 and 60,000 MPa. Therefore, the material properties of

the elements of the design domain (Young modulus and mass density) are equal to those of the base material or almost zero (void).

The optimal configuration identified is characterised by a total mass of  $m_f = 4.76$  kg and a first natural frequency of about 518 Hz. The optimal structural configuration complies with the requirements concerning the Von Mises stress limits and the minimum first natural frequency of the component. The Von Mises stress distribution for the optimal configuration is shown in Figs. 9–11 for the three static loading conditions, respectively. Stress distribution is quite uniform in each of the three cases.

The structural topology shown in Figs. 7–11 has been used in order to define the geometrical configuration of an hypothetical optimal component as shown in Fig. 12. The proposed geometrical model is characterised by the same mass of the structural configuration identified by the optimisation procedure. The first natural frequency of the proposed component reduced to about  $f_f = 318$  Hz. Fig. 13 shows the first natural mode of the component corresponding to the first natural frequency.

The constraint on the mean stress is satisfied leading to maximum Von Mises stresses lower than required. Figs. 14–16 show the Von Mises stress distribution evaluated by the application of the loads during steering, braking and when the maximum stroke of the suspension is reached. The only exception is a small area around the bearing connecting the suspension subframe to the wheel hub where the Von Mises stresses reach a peak value of

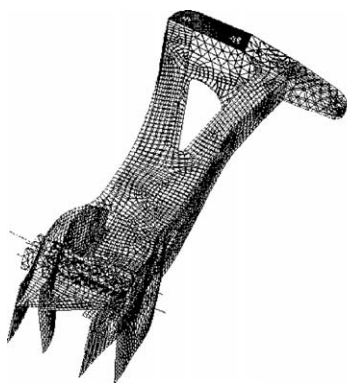


Fig. 13. First mode of the proposed design.

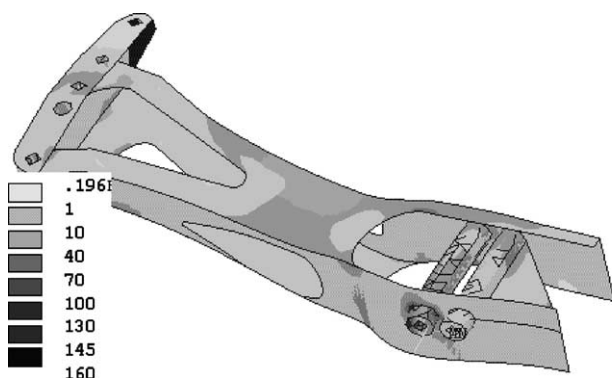


Fig. 14. Von Mises stress distribution on the proposed geometrical configuration of the rear suspension subframe during steering.

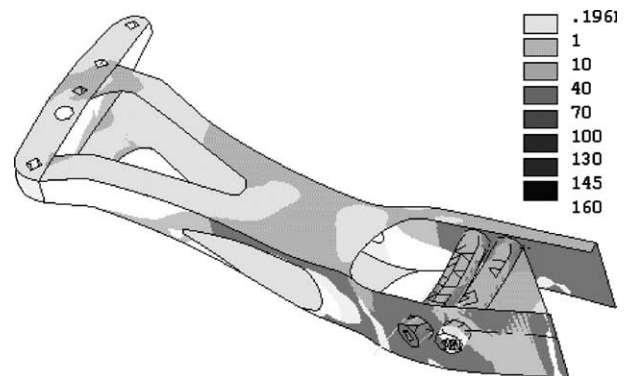


Fig. 15. Von Mises stress distribution on the proposed geometrical configuration of the rear suspension subframe during braking.

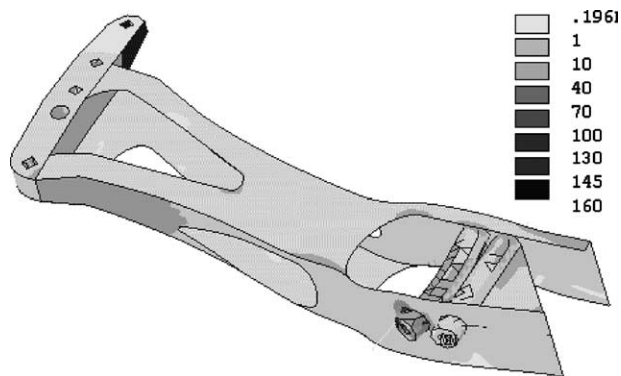


Fig. 16. Von Mises stress distribution on the proposed geometrical configuration of the rear suspension subframe when the maximum suspension stroke is reached.

about 200 MPa. This stress concentration effect is partly due to the fastening system that have been kept as it was in the original geometrical configuration of the component and has not been adapted to the new geometry.

#### 4. Conclusions

The proposed SIMP-like topology optimisation method is based on compliance minimisation with a constraint on the volume of the optimal solution. The search procedure avoids the introduction of a penalisation coefficient in order to preserve the linear relationship between the design variable and the material stiffness and, consequently, in order to preserve the existence of a solution to the problem. The Lagrange multiplier of the volume constraint, a strain energy density from the dimensional point of view, has not to be searched a posteriori in order to comply with the volume constraint. Instead, it can be calculated a priori in order to comply with the structural constraints defined on the optimal solution (mean stress, displacement, stiffness constraints). The Lagrange multiplier is already available before each new material density layout modification and effectively interprets the structural status of the optimal solution. The usual global re-scaling of the material density distribution at the end of each iteration can be avoided.

The optimal structural configurations identified by using the described search procedure are always continuous and show a very small number of elements with intermediate material characteristics (0–1 solution). The procedure does not require additional constraints or other techniques to converge and convergence is usually reached with a small number of iterations (10–30 iterations depending on the optimisation problem).

The proposed topology optimisation method has been used to solve a multiple loading condition problem concerning the McPherson rear suspension subframe of a mid size vehicle. The geometry of the optimal topology

is quite well defined with less than 1% of the design domain elements with an intermediate value of material properties (Young modulus and material density). The optimal structural configuration complies with the requirements concerning the maximum Von Mises stresses and the minimum first natural frequency of the component.

The results obtained from the topology optimisation problem have been used to define the hypothetical optimal shape of the component. The mechanical characteristics of the component with its final geometrical configuration are slightly different from those obtained at the end of the optimisation problem due to a large reduction of the first natural frequency. The mechanical property variation is strictly linked to the number of still bulk elements and their layout into the design domain.

#### References

- [1] Bendsøe MP, Kikuchi N. Generating topologies in structural design using a homogenization method. *Comput Meth Appl Mech Eng* 1988; 71:1197–224.
- [2] Suzuki K, Kikuchi N. A homogenization method for shape and topology optimization. *Comput Meth Appl Mech Eng* 1991;93: 291–318.
- [3] Bendsøe MP. Optimization of structural topology, shape, and material. Heidelberg: Springer; 1995.
- [4] Bendsøe MP. Optimal shape design as a material distribution problem. *Struct Optim* 1989;1:193–202.
- [5] Rozvany GIN. Aims, scope, methods history and unified terminology of computer-aided topology optimization in structural mechanics. *Struct Multidisciplinary Optim* 2001;21:90–108.
- [6] Xie YM, Steven GP. A simple evolutionary procedure for structural optimization. *Comput Struct* 1993;49:885–96.
- [7] Xie YM, Steven GP. Evolutionary structural optimization. Berlin: Springer; 1997.
- [8] Querin OM, Steven GP, Xie YM. Evolutionary structural optimisation (ESO) using a bi-directional algorithm. *Eng Comput* 1998;8: 1031–48.
- [9] Zhao C, Hornby P, Steven GP, Xie YM. A generalized evolutionary method for numerical topology optimization of structures under static load conditions. *Struct Optim* 1998;15:251–60.
- [10] Young V, Querin OM, Steven GP, Xie YM. 3D and multiple load case bi-directional evolutionary structural optimization (BESO). *Struct Optim* 1999;18:183–92.
- [11] Liang QQ, Xie YM, Steven GP. Optimal selection of topologies for the minimum-weight design of continuum structures with stress constraints. *Proc Inst Mech Engrs, Part C* 1999;213:755–62.
- [12] Hinton E, Sienz J. Fully stressed topological design of structures using an evolutionary procedure. *Eng Comput* 1995;12:229–44.
- [13] Sienz J, Hinton E. Reliable structural optimisation with error estimation, adaptivity and robust sensitivity analysis. *Comput Struct* 1997;64:31–63.
- [14] Papadrakakis M, Tsompanakis Y, Hinton E, Sienz J. Advanced solution methods in topology optimization and shape sensitivity analysis. *Eng Comput* 1996;13:57–90.
- [15] Mattheck C, Burkhardt S. A new method of structural shape optimisation based on biological growth. *Int J Fatigue* 1990;12: 185–90.
- [16] Shim PY, Manoochemri S. Generating optimal configurations in structural design using simulated annealing. *Int J Numer Meth Eng* 1997;40:1053–69.



- [17] Eschenauer HA, Kobelev VV, Schumacher A. Bubble method for topology and shape optimization of structures. *J Struct Optim* 1994;8: 42–51.
- [18] Zhou M, Rozvany GIN. On the validity of ESO type methods in topology optimisation. *Struct Multidisciplinary Optim* 2001;21: 80–3.
- [19] Chiandussi G, Gaviglio I, Ibba A. Evolutionary method for topology optimisation with maximum stress control. In: *Proceedings of the European Conference on Computational Mechanics*; 2001.
- [20] Chiandussi G, Gaviglio I, Ibba A. Topology optimisation with optimality criteria and a given volume Lagrange multiplier. In: *Proceedings of the XXX National Conference of the Italian Association Stress Analysis*; 2001. p. 1103–12.
- [21] Venkayya VB. Design of optimum structures. *Comput Struct* 1971;1: 265–309.
- [22] Rossow MPJ, Taylor JE. A finite element method for the optimal design of variable thickness sheets. *AIAA J* 1973;11:1566–9.
- [23] Venkayya VB. Optimality criteria: a basis for multidisciplinary design optimization. *Comput Mech* 1989;5:1–21.
- [24] Bendsøe MP, Díaz A, Kikuchi N. Topology and generalized layout optimization of elastic structures. In: Bendsøe MP, Mota Soares CA, editors. *Topology design of structures*, Dordrecht: Kluwer; 1993.
- [25] Chiandussi G, Gaviglio I, Ibba A. Optimality criteria with a ‘fixed’ Lagrange multiplier for topology optimisation. In: *Proceedings of the Fifth World Congress on Computational Mechanics*; 2002.