Coding Theory: Problem sheet 3

Solutions must be submitted by 12pm on Thursday 15 November 2007

1. Find the cosets and coset leaders of the linear code over Z_2 with the following generator matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

   Using coset decoding decode 01001 and 11110. [14]

2. Find the cosets and coset leaders of the linear code over Z_3 with the following generator matrix:

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}.$$

   Using coset decoding decode 1212 and 2101. [14]

3. Let C be a linear code of length n over Z_p. We say that that C is *self-orthogonal* if and only if $\mathbf{a} \cdot \mathbf{b} = 0$ for all $\mathbf{a}, \mathbf{b} \in C$. (This is equivalent to $C \subseteq C^{\perp}$.) Let A be a generator matrix for C. Prove that C is self-orthogonal if and only if $AA^t = 0$. (Here A^t denotes the transpose of the matrix A.)

   If $p = 2$ or $p = 3$ and C is self-orthogonal, prove that each word of C has weight divisible by p.

   On the other hand prove that there is a self-orthogonal linear code of length 2 over Z_5 having a word whose weight is not divisible by 5. [18]

4. The `MAPLE` syntax for producing the reduced echelon form of a matrix A computed modulo a prime p is

```
> Gaussjord(A) mod p;
```

   if one is using the old `linalg` package or

```
> ReducedRowEchelonForm(A)  mod p;
```

   if one is using the new `LinearAlgebra` package (such is progress).

Using MAPLE, find a generator matrix in standard form, for a code equivalent to the code over Z_3 with generator matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

[12]

5. Find the minimum weight of the linear code over Z_2 with the following parity-check matrix:

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

[10]

6. Find the minimum weight of the linear code over Z_3 with the following parity-check matrix:

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

[10]

7. Consider the linear code over Z_2 with parity-check matrix

$$H = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

Assuming that no more than one error has occurred in each word, correct, where possible, the following received words: (i) 10101010, (ii) 11010100, (iii) 11001011. [10]

8. Find coset leaders and their syndromes for the code in the previous question. Please do not calculate all cosets but go through in turn all words of weights 0, 1 and 2, calculating their syndromes, until you have all vectors of length 4 as syndromes. [12]

9. (Challenge problem) Let C be a self-orthogonal linear code over Z_2 with generator matrix A. Prove that if all rows of A have weights divisible by 4 then all words in C have weights divisible by 4.

RJC 23/10/2007